MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

COMPARATIVE SIUDY OF SEVERAL PARAMETER ESTIMATION TECHNIQUES FOR A CERTAIN CLASS OF DYNAMIC SYSTEMS

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ABSTRACT

The desire of the practical engineers has been to obtain better knowledge of specific plants, thus facilitates the design of improved control systems and consequently lower operating costs.
In general the identification problem is characterized by three elements :

- Mathematical models of the systems
- Input signals
- Criteria for best estimation

Regarding to these elements, the auther has chosen the most common mathematical models (and the related algorithms) suited for linear time invariant systems with finite memory. At the same time, the used algorithms do not make strict limitations on the input signals. With respect to the criteria, it was chosen to be a functional of the error of the output response.

A comparative measures of parameter estimation for three algorithms have been introduces. These algorithms are :

- The algorithms derived by the auther
- The algorithms derived by. R. Subbayyan; R.Nagarajan
- The algorithms derived by. L.C. Westphal.

1. INTRODUCTION :

In generol we can say that the experimental system identification can almost and always be considered as a problem of finding the extremal of functionals. The form of the functional is given by the criterion accepted for the system identification and by the mathematical model . To achieve the desired extremal, different computing techniques can be applied. From this point of view, two large groups of methods can be distinguished. The method of the first group can be denoted as direct methods. They do not use the physical realization of the mathematical model of the system and handle the identification simply as a mathematical problem of finding an extremal. According to the form of the functional they produce one of the explicit mathematical

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relation of the unknown parameters. The model adjusting methods form the second group. The physical realization of the model is often implemented in connection with analog computing technique.
The experimental identification of a physical system is to great extent dependson the kind of the input signal which excites the unknown system. Where it is possible to choose the input signal, the choice can markedly influence the accuracy of the identification or simplify the whole procedure.

In this paper, we are concerned with the methods use the model adjusting technique, to evaluate their comparative measures. The comparison by theoritical anaysis is rather complicated, that is urged the usage of simulation techniques to carry out the comparative study.

2. IDENTIFICATION TECHNIQUES FOR COMPARATIVE STUDY :

Linear systems naturally represent the most extensively developed area in the field of systems, since they can approximate a wide class of processes. Making use of thisfact, we shall concentrate the study of identification techniques used for linear systems.

2.1 Discrete- Time Identification By R.Sub.And R.Nag. :

The technique employs time-varying gain matrices to reduce the high overshoots and large settling time which may be observed in the other techniques that use time-invariant gain matrices .
The identification algorithm is derived through defining a model in the form,

$$X_m(k+1) = A_m(k+1)\, X_p(K) + B_m(k+1)\, U(K)$$

where, A_m and B_m are the model parameters (to be determined)

X_m... model state vector with n - elements

U ... input vector with r - elements

A_m... n x n matrix

B_m... n x r matrix

Defining the parameter alignment error matrix,

$\theta(k)$ and the state error vector are to be, respectively

$$\theta(k) = [\, A_m(k) - A_p \;\vdots\; B_m(k) - B_p \,] \,,$$

and

$$e(k) = X_m(k) - X_p(k) \,,$$

where,

A_p... process coefficient matrix

B_p... process input matrix

X_p... process state vector with n-elements

Let, $Z(k)$ is given by

$$Z(k) = \begin{bmatrix} X_p(k) \\ \cdots\cdots \\ u(k) \end{bmatrix}$$

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The parameter alignment error vector, $\emptyset(k)$ is written as,

$$\emptyset^T(k) = [\ \theta_{11}(k)\ \theta_{12}(k)\ \dots\ \theta_{1f}(k)\ \ \theta_{21}(k)\ \dots\ \theta_{2f}(k)\ \theta_{n1}(k) \dots\ \theta_{nf}(k)]$$

Where,

$f = n + r$, then defining the performance index to be,

$$J(\emptyset) = \emptyset^T(k) . \emptyset(k)$$

The algorithm for modifying $\theta(k)$ is written in a more comvenient form using the element by element matrix product,

$$\theta(k+1) = \theta(k) - \frac{2R(k) \odot [e(k) .\ Z^T(K-1)]}{Z^T(k-1)\ Z(k-1)}$$

Where, $R(k)$ is ($n \times f$) matrix gain sequence.

2.2. The Improved adaptive Identifier by L.C. Westphal :

This algorithm is a modification of the kudva-Narendra method. Basically the modification entails orthogonal projection of the vector system states and inputs used for excitation the adaptive equation and does not impair the stability of the orignal identifier. The algorithm is given in the form,

$$e(k) = \hat{\theta}(k)\ Z(k-1) - X_p(k)$$

$$y(k-1) = Z(k-1) - \sum_{i=1}^{q} \frac{Z^T(k-1) y(k-1-i)}{y^T(k-1-i)\ y(k-1-i)} \cdot y(k-1-i)$$

$$\theta(K+1) = \hat{\theta}(k) - \frac{e(k)\ y^T(K-1)}{y^T(k-1)\ y(k-1)}$$

Where,

$e(.)$; $X_p(.)$ and $Z(.)$ as defined before.

$O(.)$ .... the estimated state error vector

q .... integer equals $n + r - 1$

2.3. Identification Algorithm Through Liapunov Method :

This algorithm is derived by the auther. The model is described by,

$$X_m(k+1) = A_m(k) X_p(k) + C(k)\ [X_m(k) - X_p(k)] + B_m(k)\ U(k)$$

Where, $C(k)$ is any choosen stable matrix.

The error vector and the liapunov function are defined to be,

$$e(k) = X_m(k) - X_p(k)$$

$$V(k) = 1/2\ [\ e^T(k)\ P\ e(k) + \sum_{i=1}^{n} \emptyset_i^T(k) . \emptyset_i(k) + \sum_{i=1}^{r} \Psi_i^T(k) . \Psi_i(k)]$$

Where,

P .... positive definite symmetric matrix.

$\emptyset_i(k)$ ... i th column of matrix $\emptyset(k) = A_m(k) - A_p$

$\Psi_i(k)$ ... i the column of matrix $\Psi(k) = B_m(k) - B_p$

Let, $\Delta V(k) = V(k+1) - V(k)$

$$= -1/2 \left\{ e^T(k+1)\, Q\, e(K+1) + [\,\emptyset(k) X_p(k) + \Psi(k).\, U(k)\,]^T C^{-T} P\, C^{-1} [\,\emptyset(k)\, X_p(k) + \Psi(k)\, U(k)\,] \right\}$$

Where,

$Q = C^{-T} P\, C^{-1} - P$ Positive definite matrix.

The algorithm will be in the form,

$$\emptyset_i^T(k+1) = \emptyset_i^T(k) - e^T(k+1)\, C^{-T} P\, C^{-1} X_i(k), \quad i = 1,2,\ldots n$$

$$\Psi_i^T(k+1) = \Psi_i^T(k) - e^T(k+1)\, C^{-T} P C^{-1}\, U_i(k), \quad i = 1,2,\ldots r$$

Where,

$X_i(k)$ .... the i^{th} element of the state vector $X_p(k)$

$U_i(k)$ .... the i^{th} element of the input vector $U(k)$

3. COMPARATIVE STUDY :

The identification techniques that have been considered are simulated on the computer. A six parameter second-order unknown plant is selected to identify its parameters. The exact plant

$$A_p = \begin{bmatrix} 0,5 & 0,1 \\ 0,1 & 0,5 \end{bmatrix} \quad \text{and} \quad B_p = \begin{bmatrix} 0,5 \\ 1 \end{bmatrix}$$

3.1. Discrete - Time Identification :

The initial values of model parameters are taken zeros. The input, $U(k)$ equals ± 1 with a frequency crossponding to three sample periods. The elements of the matrix sequencas are ,

$$r_{ij}(k) = \begin{cases} 0,4 & 0 \leqslant k < 3 \\ 0,9 & k \geqslant 3 \end{cases} \qquad \begin{array}{l} i = 1,2,\ldots n \\ j = 1,2, \ldots f \end{array}$$

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Parameters $a_{m_{22}}$; b_{m_2} are shown in fig. 1,2. From which we can observe that the time needed to reach steady- state is high, also high harmonic oscillations exist till steady state .

3.2. Improved Adaptive Identifier :

The input signal u(k) = sgn (sin(k)) with sgn(0) = 1. The apriori information is that $a_{p_{11}} = a_{p_{22}} = 0,5$. The initial values for parameters $a_{m_{12}} = a_{m_{21}} = b_{m_1} = b_{m_2} = 0$. Results are shown in fig.3,4. We can observe that the model parameters reaches steady state in few number of steps. However the computational time needed for each step is relatively high. This limits the usage of the algorithm in systems with time varing parameters.

3.3. Modified Liapunov Method :

The input U(k) is taken to be switched between + 1 and - 1 with a frequency corresponding to three sampling periods. Initial values of the model parameters are zeros. Results are realized in fig.5,6. Parameters converge to steady state in reasonable number of steps. Overshoot is acceptable.

4 . CONCLUSION :

We can conclude the results in table - 1

Indentification technique	No.of operation in one step	No.of iterations for convergence	total no of operations	overshoot	steady state error
Discrete time identification	16	200	3200	acceptable	No
Improved adaptive identifier	23	10	230	acceptable	No
Modified Liapunov method	18	12	216	acceptable	may exist

The above results illustrate that the modified liapunov method has the advantage of faster convergence and requires less computational time to reach steady state. Steady-state error may exist if the input signal U(k) is not enough random.

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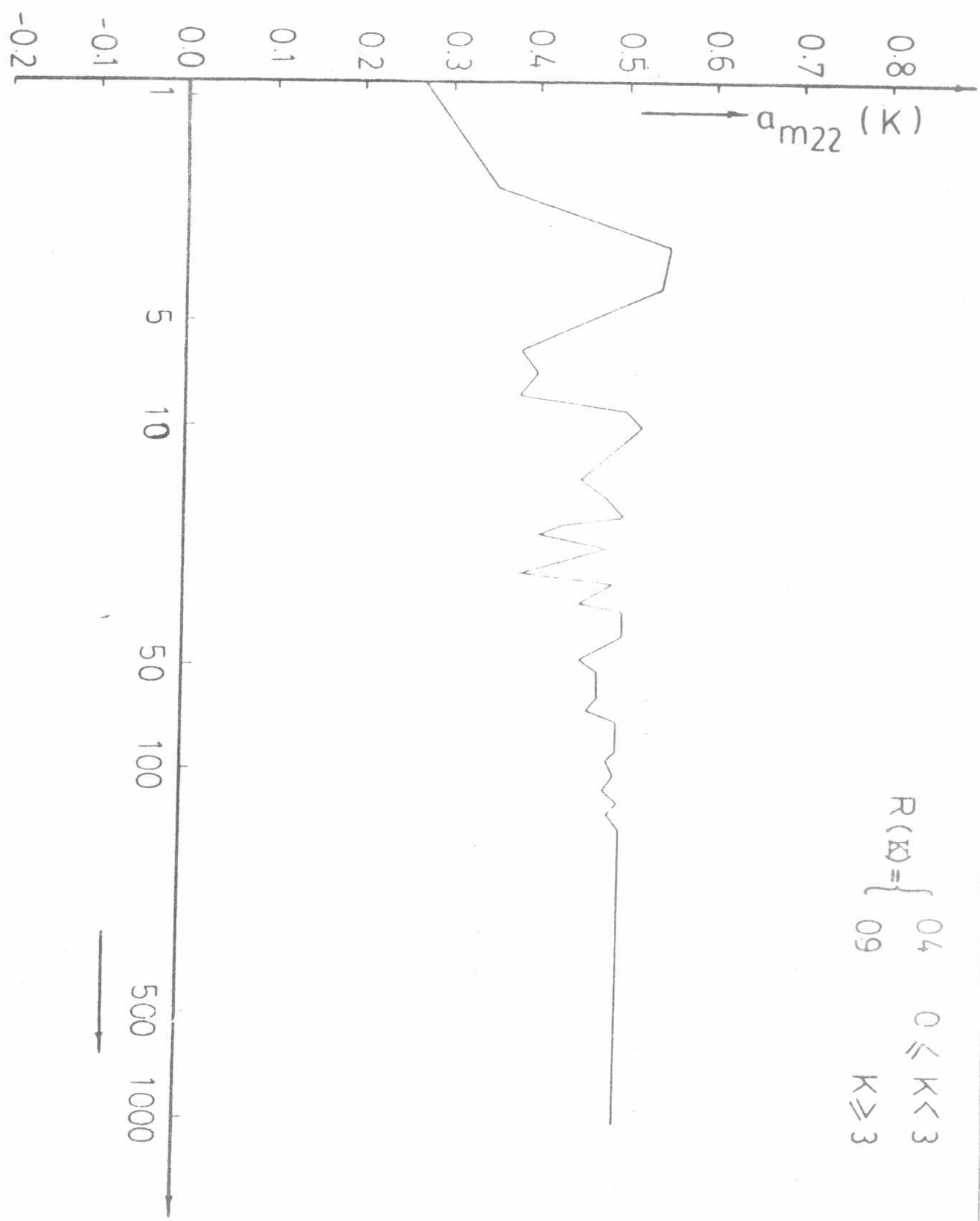

Fig. (1) Response of model parameter $a_{m22}(K)$

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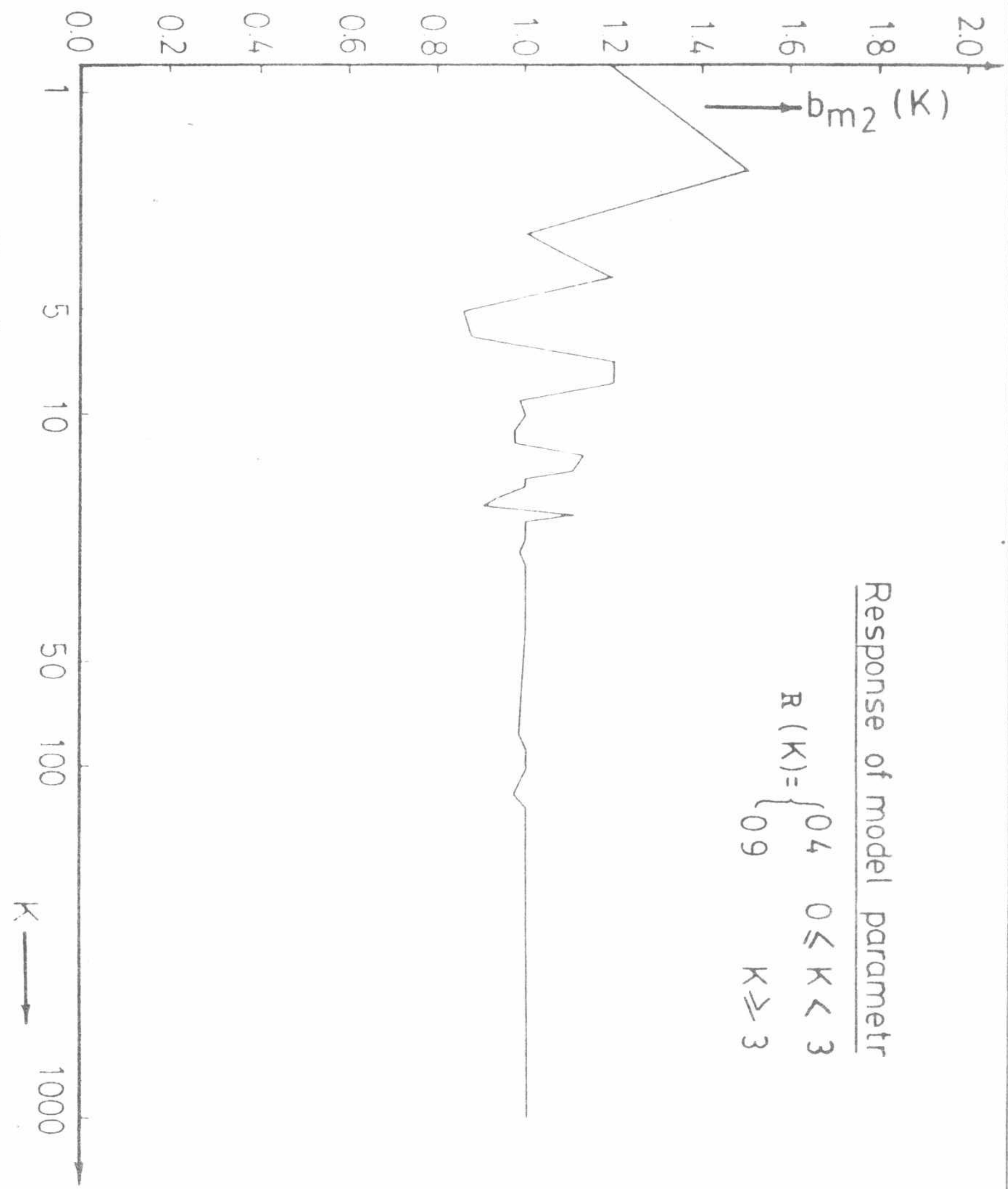

Fig(2) Response of model parameter $b_{m2}(K)$

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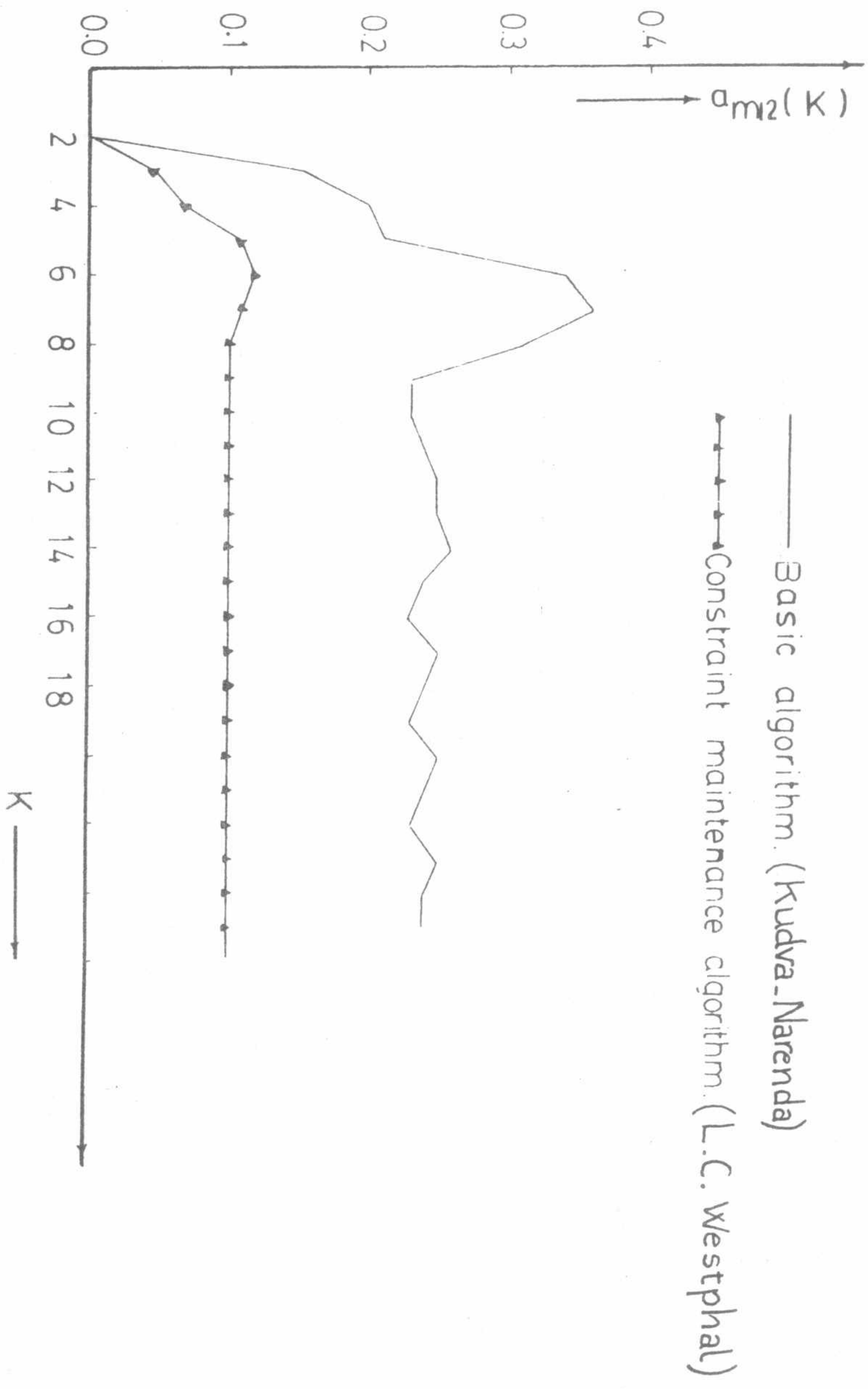

Fig.(3) Response of the model parameter $a_{m12}(K)$

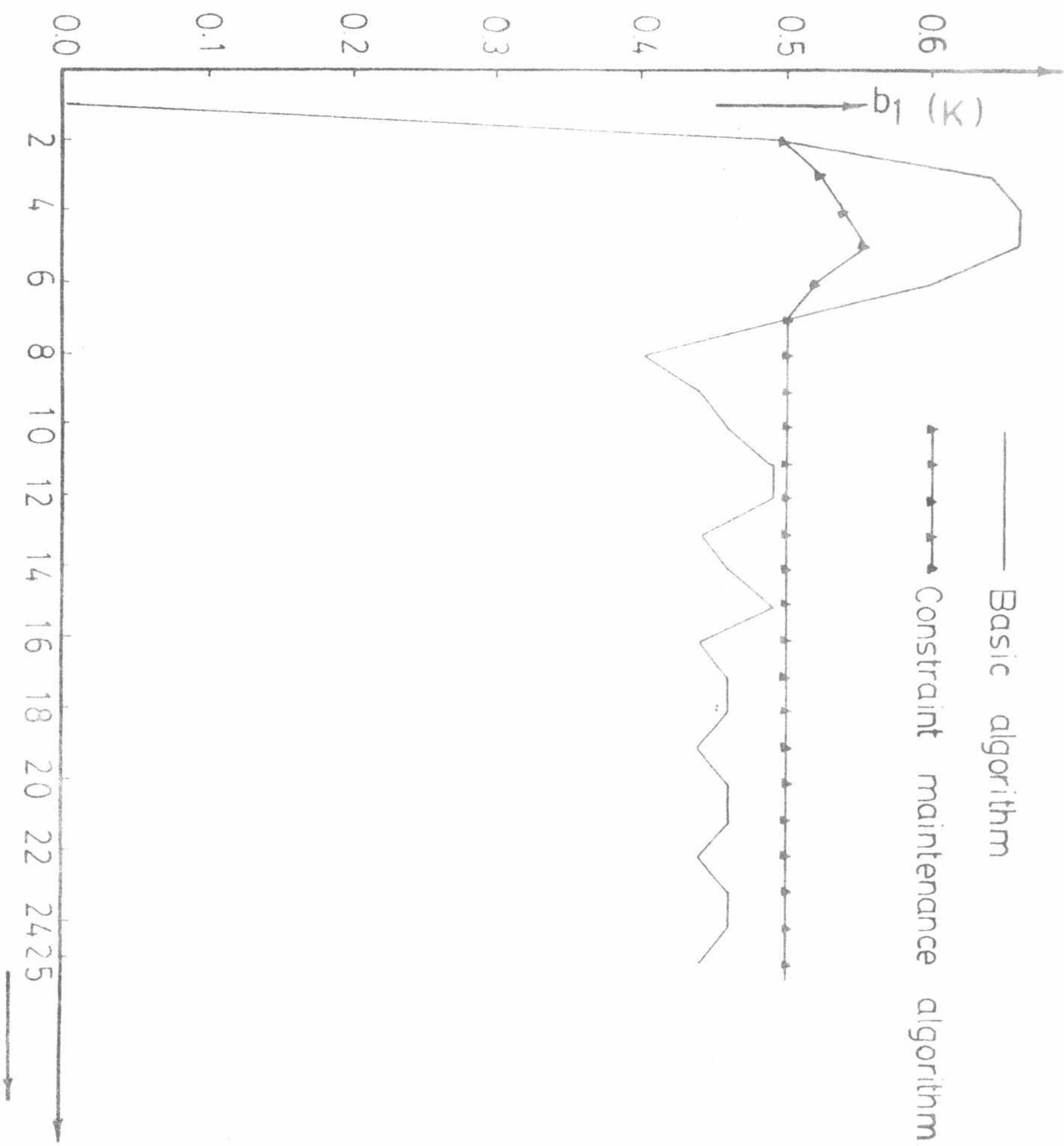

Fig.(4) Response of the model parameter $b_1(K)$

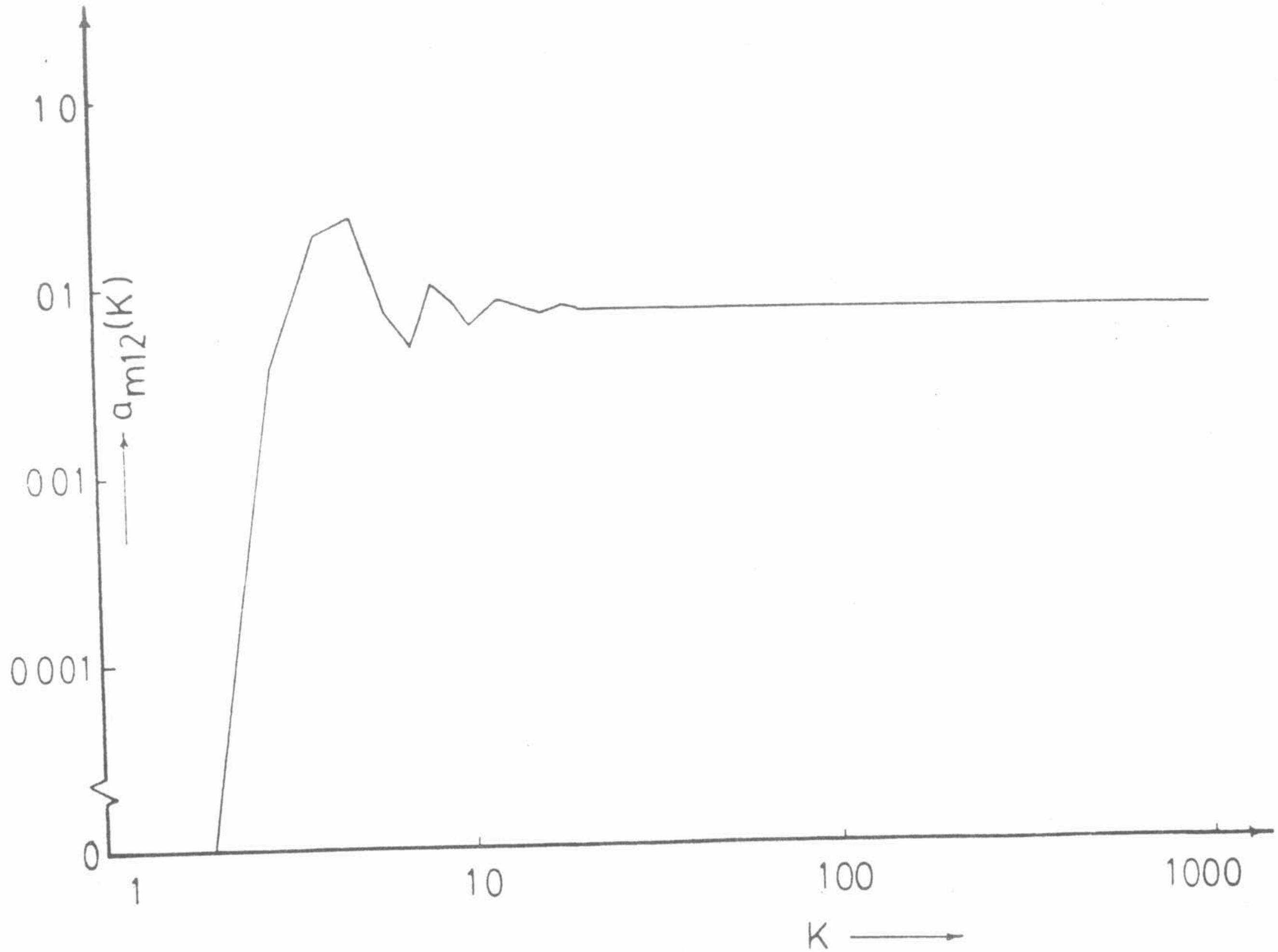

Fig.(5) Response of model parameter $a_{m12}(K)$

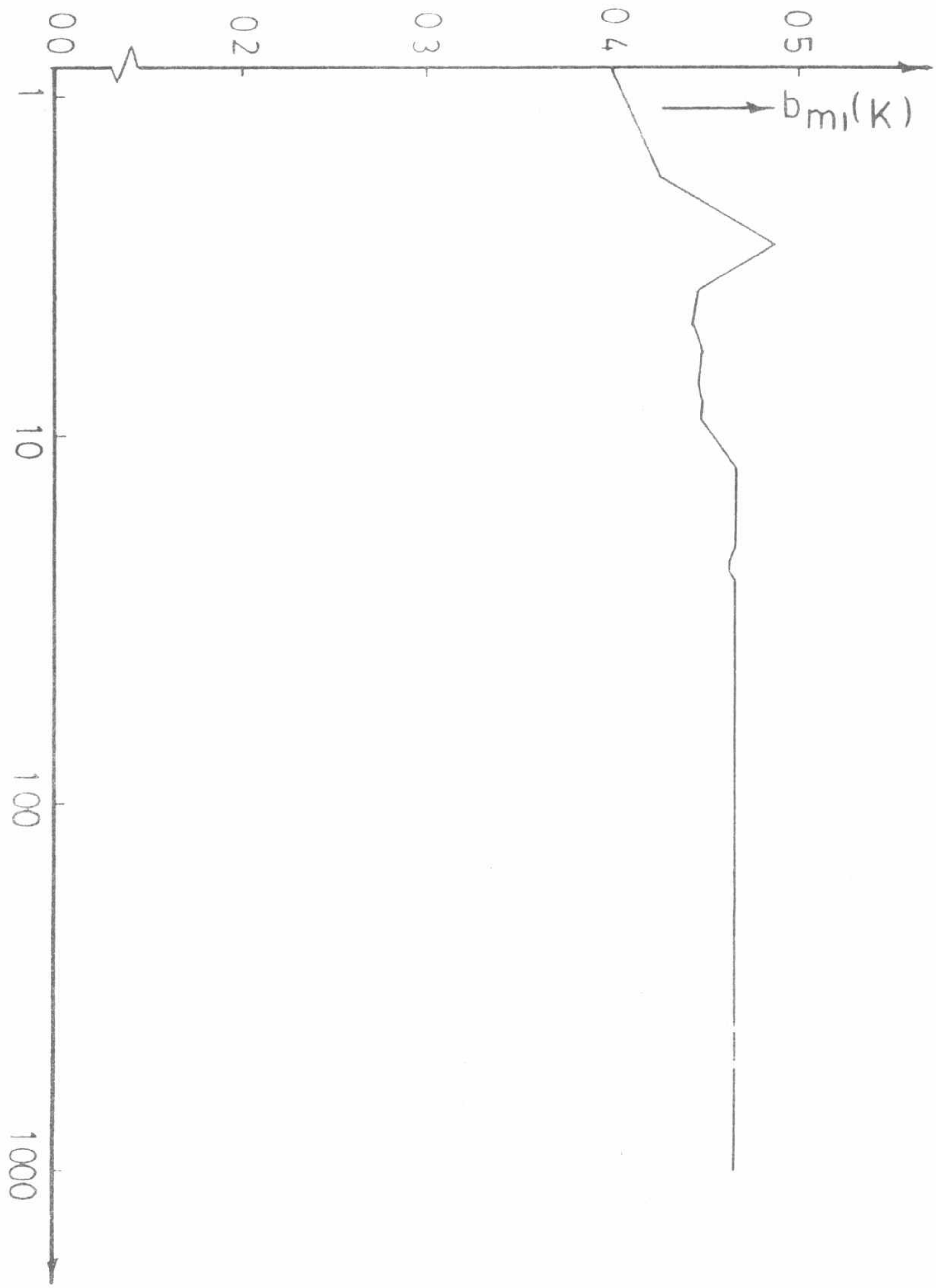

Fig.(6) Response of model parameter b_{ml} (K)