



**ELECTROCHEMICAL CUTTING-PREDICTING EQUATIONS  
BY RESPONSE SURFACE METHODOLOGY (RSM) TO THE  
SELECTION OF OPTIMAL MACHINING VARIABLES.**

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**ABSTRACT**

The use of tubular cathodes considerably widens the technological possibilities in ECM of large-size components specially if the allowance will be removed in the form of a solid body.

The present paper reports experimental investigations on this technique in the electrochemical cutting operation. Experiments were carried out to study the influence of machining parameters such as feed rate, applied voltage, electrolyte conductivity and electrolyte flow-rate on the width of cut, electrolysing current and volumetric metal removal rate.

Experiments were planned on the basis of response surface methodology (RSM) technique. Three mathematical models correlating process parameters and their interactions with response parameters have been achieved. These models can be used in selecting optimum process parameters for obtaining the desired controlled width of cut with restricted consumed current.

**INTRODUCTION**

Electrochemical machining (ECM) has been applied to a variety of machining operations, viz turning, drilling grinding, deburring and cavity sinking. However, little work has been done towards the application of electrochemical dissolution to the cutting process using tubular cathode shape tool [1-3].

This technique is preferred over ECM specially when the amount of metal removal from the shaped surface is large, as is usual, on heavy engineering workpieces.

Such a method of machining involves definite difficulties caused by the need of a high powered current source. However, an optimum selection of operating parameters for this technique has not been reported so far.

To perform the present investigation in the classical method, one variable approach being followed at a time, the number of required experiments would be very large. For four factors at

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five levels as in the present case, the number of experiments would be equal to  $4^5$  (=1024). Furthermore, this gives only the independent effect of the process parameters on the objective function, while giving no idea of their interaction and higher order effects.

To reduce this size of experimental program without substantially reducing accuracy, the present paper has reported the results of an experimental program based on statistically designed experiments to study the influence of process parameters on the response parameters.

In order to improve the reliability of experimental data and also to get a clear insight into the nature of interaction between the parameters studied, the experimental program was planned in accordance with the principles of experimental design suggested by Box and Hunter [4].

In this study, interest is focussed on the degree to which the feed rate ( $f$ ), applied voltage ( $V$ ), conductivity ( $k$ ), electrolyte flow-rate ( $v$ ) and their interactions affect the width of cut, electrolysing current and metal removal rate. A 4-variable 5-level experimental design containing 31 test runs was performed.

The experiments were planned using half replicate of 2 factorial central composite rotatable design ( $k$  = number of variables).

Mathematical models fitted to the experimental data will contribute towards selection of optimum process conditions. Lastly, these mathematical functions could then be adapted and generalized to select the optimum machining condition, for various ECM process at the minimum number of required tests.

#### EXPERIMENTAL SET-UP

In the present investigation, a tubular cathode made from a copper tube with external diameter 5 mm and wall thickness 1 mm has been used to perform this work. The tubular cathode tool was provided with three rows of holes arranged at  $90^\circ$  along its working gap portion to supply the electrolyte in the interelectrode gap (Fig.1).

The diameters of these holes were 2 mm spaced at 2 mm intervals. The body of the cathode was covered with an insulating material to minimize current leakages and stray machining (Fig.2).

The specimens were made of ground steel 37 with rectangular cross-section (40 X 25 X 10 mm). The electrolyte was an aqueous solution of sodium chloride. The initial gap of 1 mm was set with feeler and also by the use of reverse motion of the spindle after the contact between tool and workpiece. Figure 3 shows schematic diagram of the designed electrochemical machine.

In this process the electrolyte purity has a large influence on the accuracy of the resultant workpiece profile. For the nature of this operation, it was considered essential to incorporate an efficient electrolyte cleaning and under a fine filtration system to avoid severe sparks and tool damage. This phenomenon is attributed to the multi entrances of the electrolyte into the interelectrode gap.



### PLANNING OF EXPERIMENTS

To improve the reliability of results, experiments were planned on the basis of response surface methodology (RSM) technique for statistical design of experiments. Central composite rotatable design for various variables was chosen (5). For 4 - variable and 5- level the experimental program consists of 16 corner points at  $\pm 1$  levels, 8 axial points at  $\pm 2$  levels and a center points at zero level repeated 7 times. This involves a total of 31 experimental observations. The scheme of experimentation adopted in the present study was listed in Table 1 and the levels for the variables  $X_1, X_2, X_3$  and  $X_4$  were given in Table 2. The values of the operating levels for the variables fall within practical limits of machining conditions and provide a range which enables possible effects to be detected. The coded numbers for the variables used in Tables 1 & 2 were obtained from the following transformation equations:

$$X_1 = \frac{2 (f - f_{+1})}{f_{+1} - f_{-1}} + 1 \quad \dots\dots\dots (1)$$

where  $X_1$  is the coded value of the variable  $f$  and  $f_{+1}$  and  $f_{-1}$  are values of the variable  $f$  at  $+1$  and  $-1$  levels respectively.

$$X_2 = \frac{2 (V - V_{+1})}{V_{+1} - V_{-1}} + 1 \quad \dots\dots\dots (2)$$

$$X_3 = \frac{2 (K - K_{+1})}{K_{+1} - K_{-1}} + 1 \quad \dots\dots\dots (3)$$

$$X_4 = \frac{2 (v - v_{+1})}{v_{+1} - v_{-1}} + 1 \quad \dots\dots\dots (4)$$

### Mathematical Modelling

According to Table 1, a total of 31 experiments were conducted, each having the combination of various values of process variables  $X_1, X_2, X_3,$  and  $X_4$ . Width of cut  $W$ , electrolysing consumed current  $I$ , and volumetric metal removal rate (VMRR) were chosen to be response variables for the experiments. Each of these responses  $W, I,$  and (VMRR) was fitted into a polynomial response surface equation of second order.

A polynomial response surface of second order can be represented by:

$$\frac{k}{k} \dots \frac{k}{k} \dots \sum \dots \dots \dots (5)$$



Where  $Y$  is the response and  $X_i$  ( $i = 1, 2, \dots, K = 4$ ) are coded levels of  $k$  quantitative variables or factors. The coefficients  $b_0, b_1$  ect. in Eqn. (5) can be calculated by solving the following equations system:

$$[Y]_{31 \times 1} = [X]_{31 \times 15} \cdot [B]_{15 \times 1}$$

where  $X$  is the matrix of independent variables in coded form,  $Y$  is the column matrix of the observed response, and  $B$  is the column matrix of the regression coefficients. The following is the solution of the above equations system:

$$\begin{aligned} b_0 &= 0.1428 (Oy) - 0.0357 (iiy) \\ b_i &= 0.0416 (iy) \\ b_{ii} &= 0.0312 (iiy) + 0.0037 (iiy) - 0.0357 (Oy) \\ b_{ij} &= 0.0625 (ijy) \end{aligned} \quad (6)$$

For each model, the coefficients which are less significant will be deleted from the regression Eqn. (6) by using student's  $t$  test [6]. In order to test the adequacy of the proposed models, a variance analysis must be performed [7].

### RESULTS AND DISCUSSION

Table 3 shows the experimental conditions and the observed values for the three responses  $W, I$  and  $VMRR$ . The final forms of the mathematical models are as follows:

$$W = 9.46 + 0.13 X_1 + 0.3 X_2 + 0.5 X_3 + 0.3 X_1^2 - 0.1 X_2^2 - 0.11 X_3^2 - 0.1 X_4^2 - 0.13 X_1 X_3$$

$$I = 79.15 + 12.5 X_1 + 0.87 X_2 + 3.03 X_3 + 2.75 X_1^2 + 1.54 X_2^2 - 0.94 X_4^2 + 1.8 X_1 X_3$$

$$VMRR = 15.06 + 3.22 X_1 + 0.4 X_2 + 0.66 X_3 - 0.4 X_1^2 - 0.33 X_2^2 - 0.33 X_3^2 - 0.3 X_4^2$$

The previous three models are in coded form. In term of actual machining variables, these models must be transformed using eqns. (1-4):

$$W = -14.83 - 2.04 f + 1.15 V + 497.5 K + 0.8 v + 1.2 f^2 - 0.025 V^2 - 6875 K^2 - 0.025 v^2 - 65 f K \quad (7)$$

$$I = 163.4 - 26 f - 15 V - 592.5 K + 7.52 v + 11 f^2 + 0.385 V^2 - 0.235 v^2 + 900 fK \quad (8)$$

$$VMRR = -69.25 + 11.24 f + 3.5 V + 990 K + 2.4 v - 0.16 f^2 - 0.0825 V^2 - 20625 K^2 - 0.075 v^2 \quad (9)$$



It should be noted that the Computed values obtained from the above equations may deviated from the experimental results by the standard divation.

The analysis of the mathematical models represented by the Eqns.(1-9) has emphasised that each factor of the machining parameters has a sensible effect on the response parameters starting at a certain value. Added to that, the final value of each response parameter is the summation of all machining parameters and their interactions.

As example, the effect of feed squared  $f^2$  starts to be more sensible than feed rate  $f$  after the value of 1.7 mm./min. The working parameters  $f, K^2, V^2, v^2$  and the interaction  $fk$  are the main factors in decreasing the width of cut as shown in Eqn.(7).

The analysis of different terms in Eqn. (8) indicated that  $f^2, V^2$ , and the interaction  $fK$  are the main factors affecting the increase in the value of the consumed current. The effect of  $f$  and  $f^2$  nullifying each other at feed rate 2.36 mm./min.

To increase the metal removal rate, the parameters of  $f, V, K$  and  $v$  should be increased. At the same time, Eqn. (9) indicated that applied voltage squared  $V^2$  and electrolyte flow rate squared  $v^2$  have a more sensible effect on decreasing the consumed current than that of  $f^2$  and  $K^2$ .

The analysis of the response parameters using RSM technique has the advantage of explaining the effect and the limits of each working parameter on the value of the resultant response parameter. So, tool designer in ECM can use this technique to predict the performance of the electrochemical process, through the study of the effect of each working parameter and their interaction. This advantage was missing in the classical methods.

The previous three models are in coded form. In term of actual machining variables, these models must be transformed using Eqns. (1-4).

The mathematical models obtained earlier were plotted to study the influence of process parameters on the three responses ( $W, I$ , and  $VMRR$ ). The parameters not mentioned in the figures were kept at zero level. These results are discussed under various response parameters.

As shown in Fig. 4 increase in feed rate leads to a decrease in the width of cut. When the gap decreases beyond a certain limit, at feed rate more than 2 mm./min., the value of the width of cut has been increased. This result has been attributed to the high increase in both electrolyte temperature and void fraction at the high feed rates [8]. According to this result, an increase in the electrolyte conductivity has been observed and has led to high metal removal at the workpiece sides. This increase has been done in spite of the expected decrease in width of cut induced at the high feed rate values, provided that the temperature rise is negligible, since the effect of feed rate is comparatively small with respect to the increase in the temperature rise.

Figure 5 shows that as feed rate increases, the consumed current increases. This result is due to the decrease in the value of the electrical resistance at the high feed rates. The interactions between the various parameters



have appeared below level of feed rate 1 mm./min. Experimental investigations have revealed that the increase in the applied voltage has a limited effect on the consumed current. This result has been attributed to the small variations between the different applied voltages. Further, the value of the over potential is not stable along the experimental tests [9].

Figure 5 shows that as the electrolyte velocity increases, the consumed current decreases as a result to the decrease in the electrolyte conductivity at the low temperature. This decrease was insignificant. Figure 6 shows the significant effect of the increase in the feed rate on the volumetric metal removal rate. The weighable value of this increase was in the frontal side. The variations in the perpendicular side (width) were relatively limited. In this study, the surface of the workpieces that were machined by this technique contained separate macro-irregularities in the form of lines due to non-uniform flow of electrolyte in the machining.

### CONCLUSIONS

Cutting with tubular cathode can be considered a simple, low costing operation, valid to cut the hard metals with a reasonable dimensional accuracy (semi-finishing operation). The response surface methodology (RSM), used in this paper, has proved to be an effective tool for analysis of the electrochemical cutting process. The amount of experimental tests performed by this technique was 3% of the required efforts in the classical methods.

### REFERENCES

- [1] Kargin, G.V., "Electrochemical Machining of Shaped Surfaces on Round Components by Tubular Shaped Cathode Tools". Russian Engineering Journal, Vol. 55, Issue 11, P. 64 (1975).
- [2] Hewidy, M. S., Fattouh, m., and Elkhabeery M. "Some Economical Aspects of ECM Processes", 1th. A.M.E. Conf., Millitary Technical College, Cairo, Egypt. P. 87, (1984).
- [3] Streeniya, V.S. "Electrochemical. Shaping Using Tubular Section Cathode Tools". Russian Engineering Journal, Vol. 55, Issue 4, P. 73, (1975).
- [4] Box, G.E.P. and Hunter, J.S. "Multifactor Experimental Designs". Ann. Math. Stat., 28 (1957) 195.
- [5] Fedorov, V.V. "Theory of Optimal Experiments", Academic Press, New York, (1972).
- [6] Lipson, C. and Sheth, N.J. "Statistical Design and Analysis of Engineering Experiments". McGraw Hill, New York, (1973).
- [7] Biles, W.E. and Swain, J.J. "Optimization and Industrial Experimental". John Wiley and Sons, New York, (1980).
- [8] Jain, V.K., Jain, Vinod Kumar and Pandey, P.C. "Computer Aided Analysis of ECBD Process". 23 MTDR Conf., P. 257, (1982).

- [9] Köing, W. and Humb, H.J "Mathematical Model for the Calculation of the Anode in ECM". Annals CIRP, Vol. 25, P. 83, (1977).

**NOMENCLATURE**

- B** Coefficient matrix
- $b_i$**  Coefficients in response equation
- f** Feed rate.
- I** Electrolysing current.
- K** Electrolyte conductivity.
- k** No. of variables.
- V** Applied voltage.
- v** Electrolyte flow-rate.
- X** Coded variable in RSM Design.
- Y** Process respons variable, Viz., width of cut, electrolysing current and volumetric metal removal rate (VMRR).

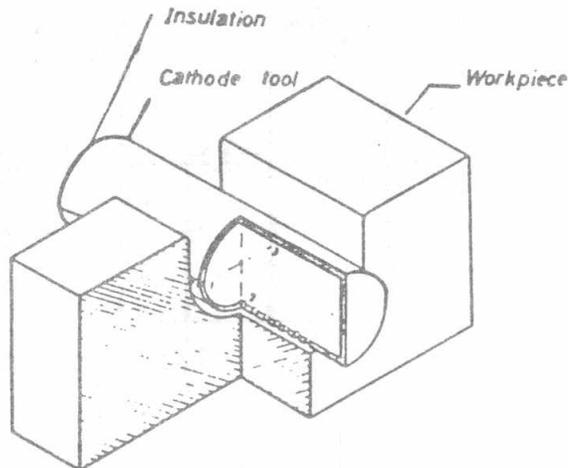


Fig.(1) Tubular cathode and electrolyte supply system  
1,2,3 three row of holes for electrolyte supply.

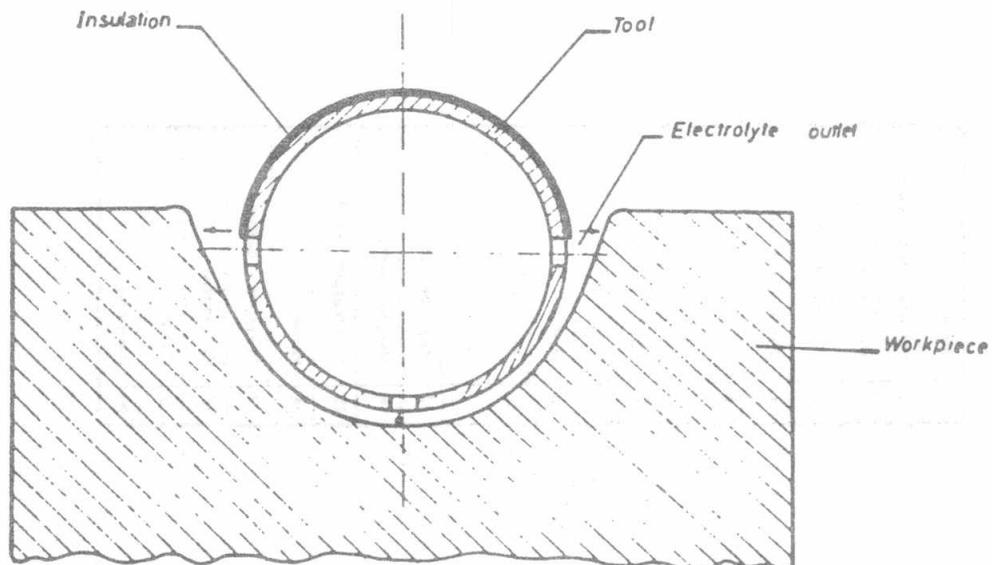


Fig.(2) Shaping with a tubular shaped cathode.

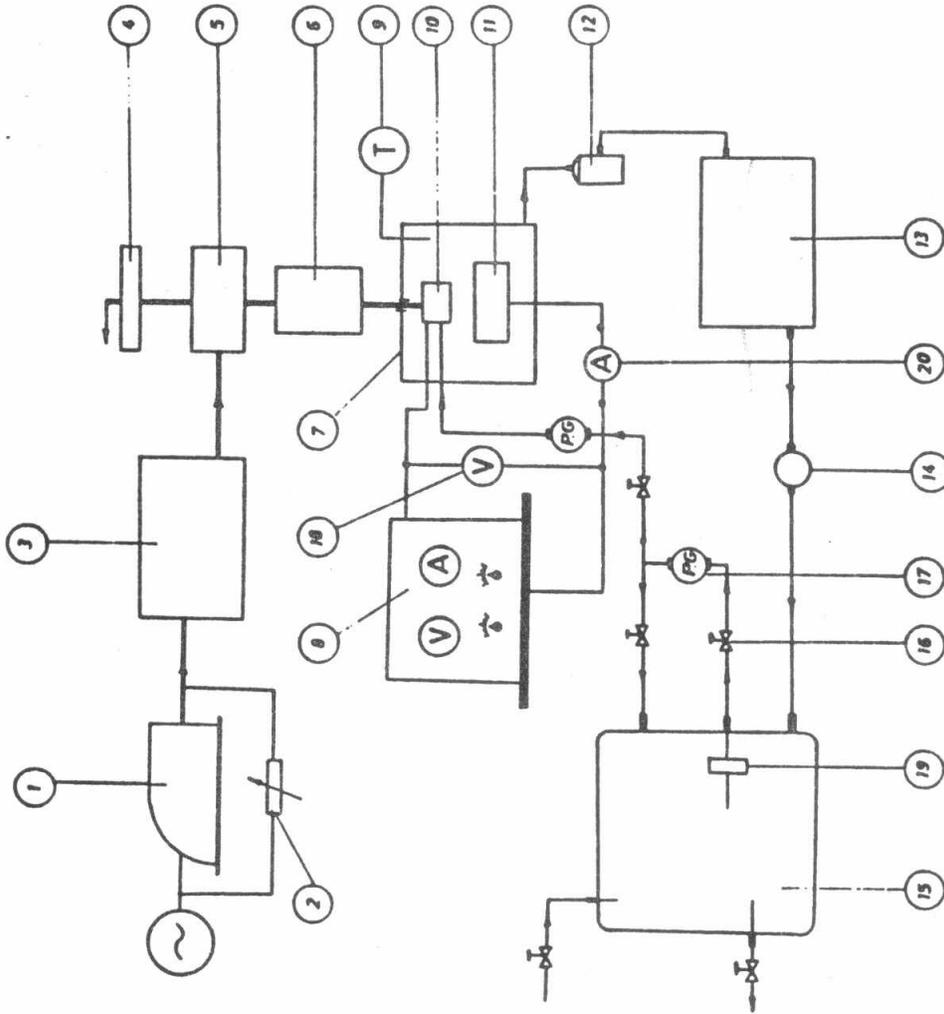


Fig.(3) Block diagram of electrochemical cell.

1	Motor
2	Variable resistance
3	Reduction units
4	Calibrated disc
5	Warm gear unit
6	Spindle
7	Machining cell
8	Power supply
9	Thermometer
10	Tool
11	Workpiece
12	Filter
13	Reservoir tank
14	Return pump
15	Electrolyte tank
16	Valve
17	Pressure gauge
18	Voltmeter
19	Filter
20	Ammeter



Table (1): Experimental Design Matrix

Exp. No.	Feed		Conductivity $\Omega^{-1} \text{mm}^{-1}$	Electrolyte flow-rate $\frac{\text{X}_4}{\text{lit./min.}}$
	$\text{X}_1$ mm/min.	$\text{X}_2$ Volt		
1	-1	-1	-1	-1
2	+1	-1	-1	-1
3	-1	+1	-1	-1
4	+1	+1	-1	-1
5	+1	-1	+1	-1
6	-1	+1	+1	-1
7	+1	+1	+1	-1
8	-1	-1	+1	-1
9	+1	-1	+1	+1
10	-1	+1	+1	+1
11	-1	-1	+1	+1
12	+1	+1	+1	+1
13	-1	-1	-1	+1
14	-1	+1	-1	+1
15	+1	-1	-1	+1
16	+1	+1	-1	+1
17	-2	0	0	0
18	+2	0	0	0
19	0	-2	0	0
20	0	+2	0	0
21	0	0	-2	0
22	0	0	+2	0
23	0	0	0	-2
24	0	0	0	+2
25	0	0	0	0
26	0	0	0	0
27	0	0	0	0
28	0	0	0	0
29	0	0	0	0
30	0	0	0	0
31	0	0	0	0

Table (2): Coding of Process Parameters

Level	Feed		Conductivity $\Omega^{-1} \text{mm}^{-1}$	Electrolyte flow-rate $\frac{\text{v}}{\text{lit./min.}}$
	$\text{f}$ mm/min.	$\text{v}$ volts		
-2	0.5	16	0.012	12
-1	1.0	18	0.016	14
0	1.5	20	0.020	16
+1	2.0	22	0.024	18
+2	2.5	24	0.028	20

Table (3): Response parameters

Exp. No.	Width of cut $\text{W}$ mm.	Electrolysing current $\text{I}$ A	Volumetric metal removal rate (VMRR) $\text{mm}^3/\text{min}/\text{mm}$ .
2	7.61	83	15.20
3	0.61	64	9.50
4	7.96	89	15.90
5	8.20	93	16.60
6	10.92	67	10.90
7	8.77	98	17.50
8	10.00	62	10.12
9	8.28	93	16.60
10	10.92	67	10.90
11	10.07	65	10.00
12	8.70	100	17.50
13	8.91	61	8.90
14	9.50	65	9.50
15	7.60	84	15.20
16	7.97	88	16.00
17	12.95	68	6.52
18	7.74	110	19.32
19	8.24	88	12.50
20	9.25	80	14.00
21	7.76	70	12.00
22	9.66	81	14.50
23	8.74	75	13.00
24	8.86	73	13.80
25	8.81	71	14.00
26	8.92	78	15.00
27	8.87	76	14.30
28	8.85	74	13.90
29	8.81	74	14.40
30	8.77	71	13.80
31	8.71	70	13.44

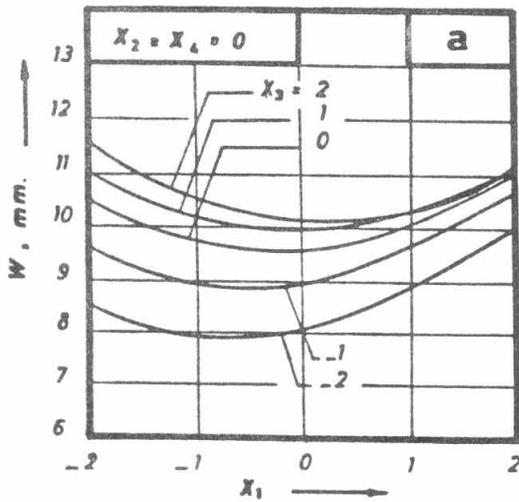


Fig.(4.a)

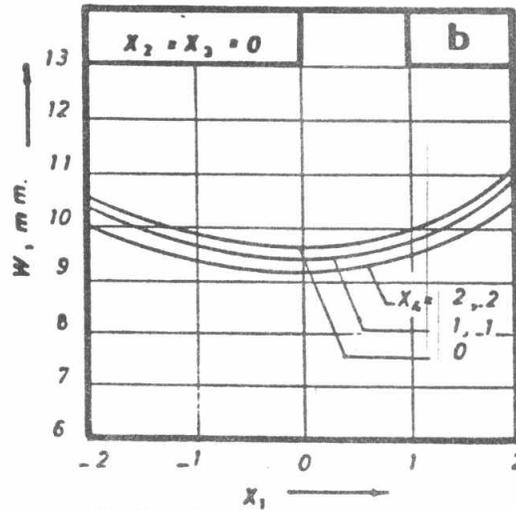


Fig.(4.b)

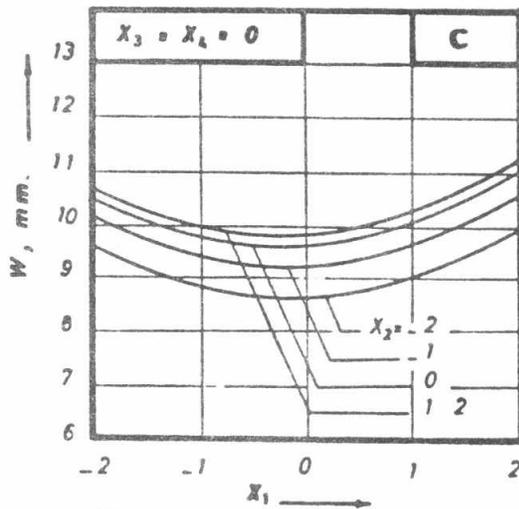


Fig.(4.c)

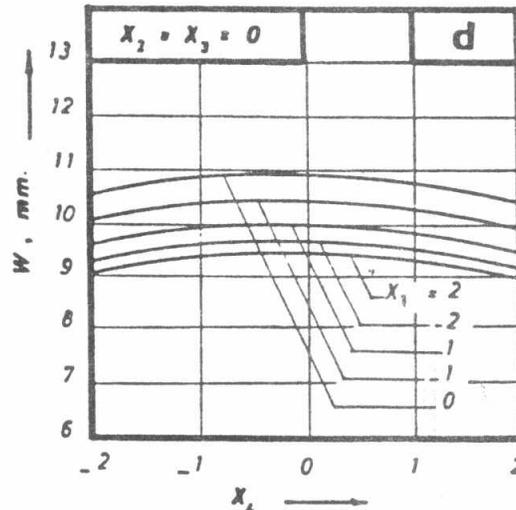


Fig.(4.d)

Fig.(4) Effect of (a-c) feed rate and (d) electrolyte flow rate on the width of cut.

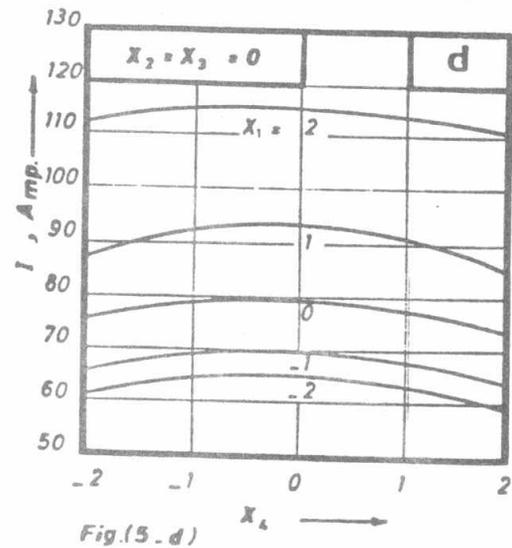
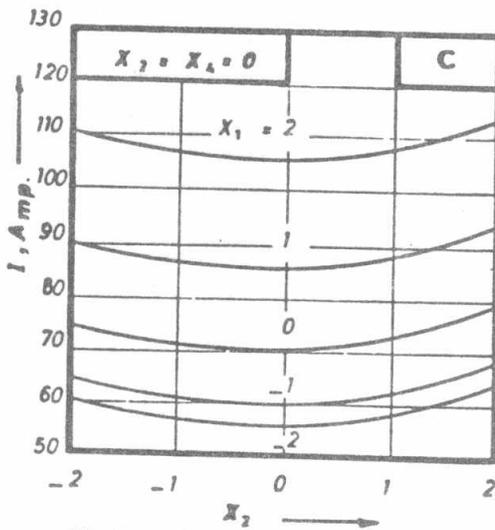
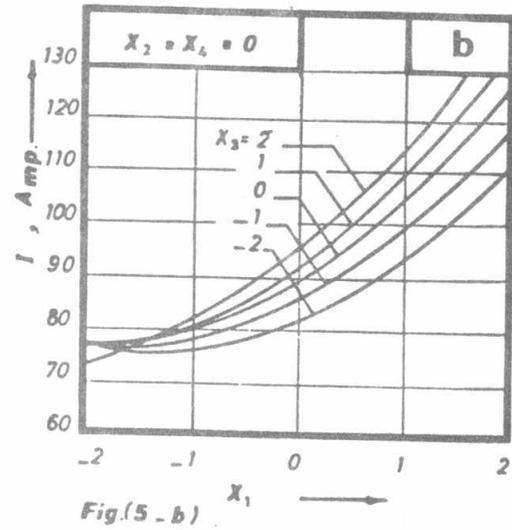
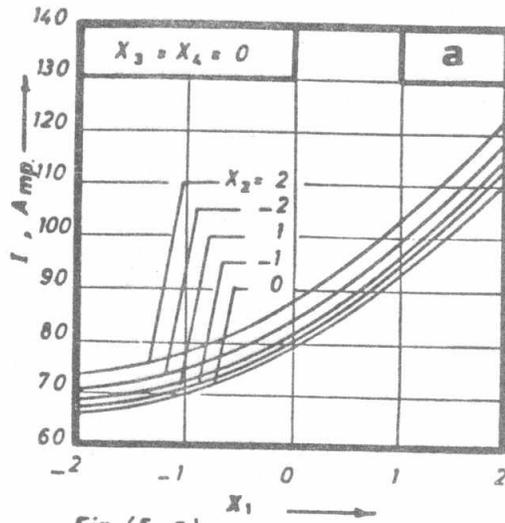


Fig.(5) Effect of (a & b) feed rate , (c) applied voltage , (d) electrolyte flow rate on the consumed current.

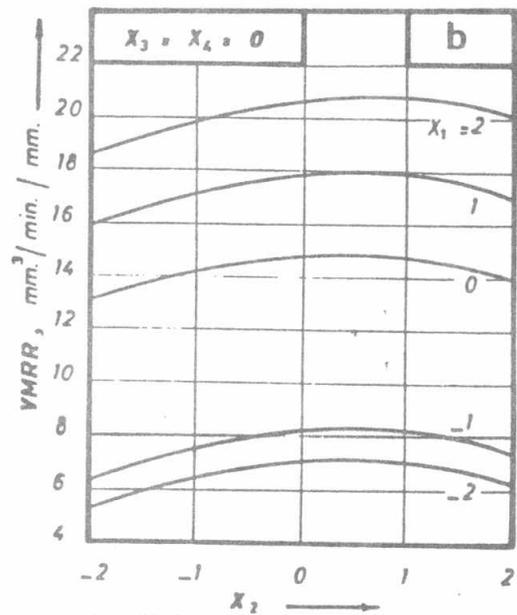
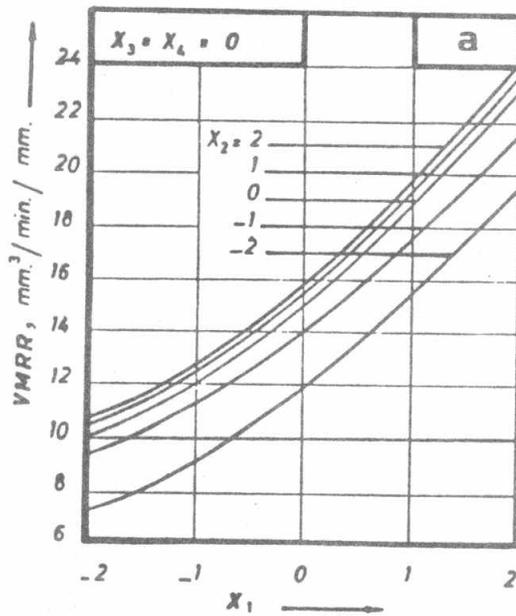


Fig.(6) Effect of (a) feed rate and (b) applied voltage on the

