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MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

OPTIMAL DESIGN OF HYDROSTATIC BEARINGS

By

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ABSTRACT

Optimum hydrostatic circular rotating bearing design is the objective of the designer. Most of the previous designs concerned with the optimization of single objective function such as minimization of power loss or maximization of static and dynamic stiffness. An alogorithm for solving bicriteria programming problems has been constructed and used successfully for solving the problem of optimal design of hydrostatic thrust bearing under rotation. A significant results has been obtained.

INTRODUCTION

Hydrostatic bearing systems are finding wide acceptance in a variety of applications. These include the support of massive slow antenna structures, frictionless space-vehicle simulators, machine tool slideways, measuring and inspection instruments, test equipments and medical equipments. Since externally pressurized bearing "Hydrostatic" have been used successfully in various applications where precise frictionless motions are necessary, and because they have the inherent advantages of virtually zero static and dynamic friction, absance of wear, negligible maintenance, presence of damping in the fluid film.

Against these advantages, it is only right to mention the drawbacks to the use of hydrostatic bearings. An inherent one is relatively poor behaviour in the presence of large dynamic forces owing to small damping action of the fluid film.

The ideal in design is to arrange that the bearing will operate under working load conditions with a clearance corresponding to the maximum stiffness portion of the curve, so that extra load will have a minimum effect on the film thickness. This procedure of design methods is of a limited accuracy and only one goal can

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be obtained. Actual, real problems in variably involves non-deterministic systems for which a variety of conflecting objectives exist. Therefore the vector optimization problem appears when decision maker must take a decision satisfying the maximization (minimization) of more than one conflecting objectives. The aim of optimization is the achievement of maximum load support for minimum power dissipation, maximum dynamic and static stiffness.

There are a number of variables of the design process and the designer has to decide the best value for each one.

One of the multiobjective techniques for solving vector optimization problems is used to obtain a subset of the efficient solutions for the concerned problem.

Several significant results has been obtained from which any one of them can be selected depending on the relative weights which can be imposed on the different conflecting objectives.

THEORY

The performance characteristics of externally pressurized bearings can be obtained by applying the general lubrication assumptions on the Navier-Stokes equations[1]. The performance characteristics of the externally pressurized bearings are the load carrying capacity, volume flow rate, static and dynamic stiffness. The inlet pressure and flow rate are related by the following expression

$$P_o = \frac{6\mu Q}{\pi h_o^3} \ln \frac{R}{R_o} , \qquad \text{........(1)}$$

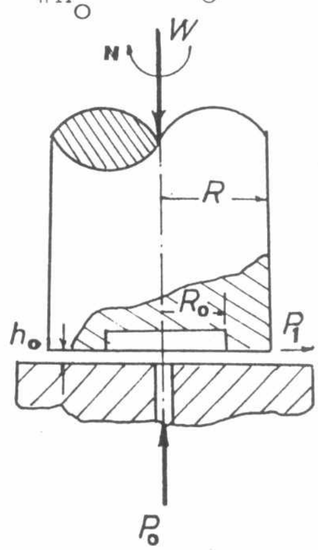

FIG (1) Hydrostatic thrust bearing

The outlet pressure P_1 is assumed to be atmospheric. The load-carrying capacity is given by:

$$W = \frac{P_o \pi}{2} \frac{(R^2 - R_o^2)}{\ell n(R/R_o)} \quad lb \qquad \text{.........(2)}$$

The pumping energy is:

$$E_p = \frac{Q}{\eta_p} (P_o - P_1), \text{ in.lb/sec} \qquad \text{.........(3)}$$

The friction loss is:

$$E_f = \left(\frac{2\pi N}{60}\right)^2 \frac{2\mu\pi}{h_o} \left(\frac{R^4}{4} - \frac{R_o^4}{4}\right) \quad \text{in.lb/sec} \qquad \text{.........(4)}$$

The temperature rise of the oil in passing through the bearing [2] is estimated by

$$\Delta T = \frac{E_f}{12 \times 778\ Q\gamma C} \quad {}^oF, \qquad \text{.........(5)}$$

The oil viscosity is a function of temperature, given by

$$\log_{10} \log_{10} (\nu + 0.8) = n \log T + C_1, \qquad \text{.........(6)}$$

The static stiffness of externally pressurized bearings is depends mainly on the method of the bearing compensation [3,4].
The static stiffness for a bearing compensated by capillary tube is

$$\lambda = \frac{3W}{h_o} \left(1 - \frac{P_o}{P_s}\right) \qquad \text{.........(7)}$$

for sharp edged orifice

$$\lambda = \frac{3W}{h_o} \left[\frac{2(1 - P_o/P_s)}{2 - P_o/P_s}\right] \qquad \text{.........(8)}$$

and for flow control value is:

$$\lambda = \frac{3W}{h_o} \qquad \text{.........(9)}$$

PROBLEM FORMULATION OPTIMIZATION CRITERIONS:

The criterions are to minimize total power loss (F_1) and to maximize the stiffness (F_2) to the bearing. The above expression must be set up in sequence so that the computer begins with known values of the design variables R, R_o, μ, Q, h_o. [5]

1. $\min F_1 = E_f + E_p$ ....(10)

2. $\max F_2 = \frac{3W}{h_o}$ ....(11)

Constraints

The sepecified load (W_s) must be less than W

$$W - W_s \geq o \qquad \text{.........(12)}$$

The inlet oil pressure must be less than the specified maximum designated P_{max}

$$P_{max} - P_o \geq o \quad \text{.........(13)}$$

The oil temperature rise must be less than specification, designated ΔT_{max}

$$\Delta T_{max} - \Delta T \geq o \quad \text{.........(14)}$$

The oil film thickness is not to be less than a specified amount h_{min}

$$h_o - h_{min} \geq o \quad \text{.........(15)}$$

The computer must be told that R must be greater than R_o

$$R - R_o \geq o \quad \text{.........(16)}$$

to avoid the problem of unlogic convergence results obtained, which appears when recess radius approaching the value of bearing outer radius.

So the value of $R - R_o \gg h_o$ to make sure that the flow is laminar, and the values of enterance and exist loss are negligible. The exist loss will be $\frac{V^2}{2g}$.

If the exist loss to be very mall fraction of the pressure drop

$$L_{ie} - \frac{\gamma}{g} \left(\frac{Q}{2\pi R h_o}\right)^2 \geq o \quad \text{.........(17)}$$

Also it must be observed that when the value $(R - R_o)$ becomes very small, the bearing here, is subject to a very hight pressure that may cause a failure of the bearing or makes a surface damage

$$P_a - \frac{W}{(R^2 - R_o^2)} \geq o \quad \text{.........(18)}$$

PROGRAM FOR CHARACTERIZING THE SET OF EFFICIENT SOLUTIONS IN BICRITERIA CONVEX PROGRAMMING PROBLEMS.

A. Purpose.

This program solve the bicriteria convex programming problem which has the following form:

BCPP $\quad \min \quad [f_1(x) \; , \; f_2(x)]$

subject to

$$M = \{x \in R^N / g_k(x) \leq o, \; k = 1,2,\ldots,M\},$$

The efficient solutions of BCPP can be characterized in terms of optimal solution of the following nonnegative weighted sum problem:-

$P(\lambda)$ $\quad \min_{x \in M} \lambda f_1(x) + (1 - \lambda) f_2(x)$

B. Method.

The algorithm proceeds as follows:

1. start with $\bar{\lambda} = o$
2. solve $P(\bar{\lambda})$ using the sequential unconstrained minimization technique (SUMT) to obtain $\bar{x}$ which is efficient solution of BCPP.

3. determine the stability set of the first kind corresponding to $\bar{x}$., [6]

4. select a new value of $\bar{\lambda}$ and repeat the procedure until the parametric space of $\bar{\lambda}$ through $[0,1]$ is fully determined.

A logic diagram of this method is given in Fig. (2)

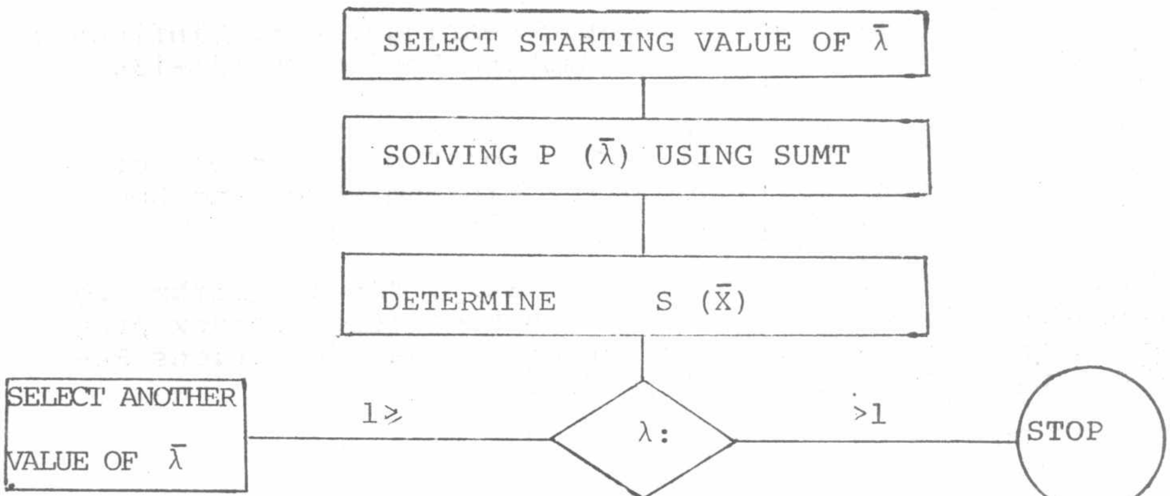

Fig. (2) Conceptable flow chart

RESULTS

The results obtained through using the algorithm, gives a subset of all efficient solutions. The designer have many options through this method of solution. The decision maker can takes the decision satisfing the optimization of more than one conflecting objectives.

CONCLUSION

1. The method of design gives a subset of all efficient solutions
2. The choise of the optimum design depends on the decision maker "designer".

3. The conflecting objectives can easly solved and gives optimum options.

ACKNOWLEDGEMENT

The authors express their gratitude to Prof. Dr. M.S.A. Osman and Prof. Dr. A.M. Sarhan for sponsoring development of the computer program and preparation of this paper.

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6 - 8 May 1986 , Cairo

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APPENDIX (1)

It is required to obtain the optimal design of a hydrostatic bearing which illustrated in Fig. (1). The primary specifications are:

Speed (N), 750 rpm.
Weight (W_s), 101070 lb
Lubricant used, SAE 20 oil (n=-3.55, C_1 = 10.04)
Oil pump efficiency (η_p), 70%
Maximum oil pressure (P_{max}), 1000 psi
Maximum temperature rise of oil (ΔT_{max}), 50°F
Minimum oil film thickness ($h_{min.}$), 0.001 in.

The above problem can be reformulate to take the following equivalent forms

$$\min F_1 = 38698.928 \frac{\mu}{h_o}\left(\frac{R^4}{4} - \frac{R_o^4}{4}\right) + 2.7297542 \frac{\mu Q^2}{h_o^3} \ell n \frac{R}{R_o}$$

$$\max F_2 = \frac{3W}{h_o}$$

subject to

$$\frac{3\ Q}{h_o^3} \ell n \frac{R}{R_o} \left(\frac{R^2 - R_o^2}{\ell n\ R/R_o^2}\right) - 101070.0 \geq o$$

$$1.910828 \frac{3\mu Q}{h_o^3} \quad n \frac{R}{R_o} - 1000.0 \geq o$$

$$-48413.719 \frac{\mu}{h_o Q} \left(\frac{R^4}{4} - \frac{R_o^4}{4}\right) + 50 \geq o$$

$$h_o - 0.001 \geq o$$

$$R - R_o \geq o$$

$$0.001 - 0.000024 \frac{Q^2}{R^2 h_o^2} \geq o$$

$$5000.0 - 0.955414 \frac{\mu Q}{h_o^3} \frac{\ell n \frac{R}{R_o}}{\ell n \frac{R}{R_o^2}} \geq o$$

Table (1)

λ	R in	R_o in	μ $lb.Sec/in^2$	Q $in^3/sec.$	h_o in	F_1 in.lb/sec.	F_2 lb/in
0	8.1550330	7.0213320	0.0000025	10.0213861	0.0022206	0.21704021E7	0.3566456E9
0.1	8.2066371	7.0987354	0.0000026	9.7807432	0.0022354	0.21488074E7	0.34784974E9
0.2	8.0190836	7.1385401	0.0000028	9.3410532	0.0028709	0.21157801E7	0.34109875E9
0.3	8.0021035	7.2503481	0.0000025	9.2998312	0.0028732	0.2110085E7	0.34082651E9
0.4	8.0102576	7.4039012	0.0000031	9.5970243	0.0029561	0.20987304E7	0.33852906E9
0.5	8.0880432	7.4098347	0.0000031	9.4321604	0.002979	0.2087035E7	0.33665789E9
0.6	8.001396	7.5104380	0.0000030	9.4938105	0.002734	0.20770703E7	0.33434891E9
0.7	7.9680604	7.3261104	0.0000028	9.4919064	0.0026815	0.2069437E7	0.33395004E9
0.8	7.9583118	7.1980318	0.0000026	9.4914064	0.0026815	0.20610947E7	0.33299247E9
0.9	7.8359397	7.0516871	0.0000028	10.1867260	0.002584	0.20487918E7	0.33187649E9
1	7.6421939	6.9531898	0.0000024	10.159078	0.0023478	0.2030356167	0.33108691E9

NOMENCLATURE:

Q	:	volume flow rate, in^3/sec.
P_o	:	inlet recess pressure.
h_o	:	film thickness, in.
μ_o	:	oil viscosity, $lb.sec/in^2$.
R	:	outer bearing radius, in.
R_o	:	recess radius, in.
W_o	:	load carrying capacity, lb.
E_p	:	pumping Energy, in.lb/sec.
E_f	:	friction loss.
P_s	:	Supply pressure.
N_s	:	rotational speed r.p.m.
ΔT	:	temperature rise.
W_s	:	specified load, lb.
P_{max}	:	maximum designated pressure.
ΔT_{max}	:	maximum designated temperature rise.
h_{min}	:	minimum film thickness.
L_{ie}	:	exist loss, lb/in^2.
P_a	:	surface damage endurace stress, lb/in^2.
η_p	:	pump effeciency.
F_1	:	first objective function.
F_2	:	second objective function.
λ	:	nonnegative weight, $o \leq \lambda \leq 1$
γ	:	weight density of oil, lb/in^3.
C	:	specific heat of oil $Btu/lb.^oF$.
n, C_1	:	constants for a given oil
υ	:	kinetic viscosity, centistokes.
T	:	absolute temperature, Rankine.