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SUGGESTED MODIFICATIONS OF MERCHANT FORMULA FOR THE
2 CALCULATION OF FAILURE LOAD OF FRAMES

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ABSTRACT

The problem of calculating the theoretical failure load (W_F) of a structure with good accuracy compared with its actual failure load value is a very important task of study. Rankine and Merchant suggested a formula for obtaining the failure load of a complete structure by analogy from isolated strut case and this in fact is an approximate method. An isolated strut of an elastic critical load (W_c), yield load (W_L) and failure load (W_F) is similar to complete structure behaviour.

The application of Merchant equation for portal and pitched-roof frames leads to an overage error equal to ± 29% when the failure load calculated compared with its experimental value.

Two suggested equations may be estimated after elastic-plastic analysis have been made on three different series of frames tested before till failure in Cambridge University laboratories by Dr. J.C. Heyman. It should be noticed that for frames having the value of $(\frac{W_L}{W_c})$ less than 0.3, the failure load of frames may be obtained using suggested equation (1), and for frames having the value of $(\frac{W_L}{W_c})$ bigger than 0.3 , the failure load obtained using suggested equation (2) .

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These two suggested equations were estimated after the interpretation of the theoretical and experimental results of the different groups of frames used in that study.

Tables and curves were given for the comparison of using these two suggested equations with the experimental failure load. To shows how the calculated value is so better compared with Merchant formula results.

INTRODUCTION

Rankine (1866) suggested an empirical formula to calculate the failure load of isolated strut, as follow:

$$\frac{1}{P_R} = \frac{1}{P_F} = \frac{1}{P_E} + \frac{1}{P_p}$$

$$P_E = \pi^2 EA\left(\frac{r}{\ell}\right)^2 \quad \text{and} \quad P_p = A\,\sigma_y$$

where P_F = failure load of structure

P_p = load obtained from plastic theory

P_E = Euler's load σ_y = yield for strut material

ℓ = Length r = sylinderness ratio

E = Young's modulus A = cross section area of strut.

Merchant suggested a new approach for obtaining the failure load of a complete structure by analogy from the isolated strut case and he suggested that

$$\frac{1}{P_R} = \frac{1}{P_F} = \frac{1}{P_{cr}} + \frac{1}{P_L}$$

where

P_{cr} = elastic critical load for the structure.

P_L = plastic failure load for the structure.

CHECK OF MERCHANT EQUATION FOR APPLICATION OF PITCHED-ROOF PORTALS:

Table (1) contains the values of the elastic critical load (W_{cr}) calculated using the elastic stability analysis for series (C) tested by Dr. Heyman at cambridge University.

The plastic failure loads(W_L) were calculated and tabulated in the same table no.1 beside the experimental load obtained by Dr. Heyman.

Merchant equation was applied to obtain the theoretical value of the failure load (W_F)

Where $$\frac{1}{W_F} = \frac{1}{W_c} + \frac{1}{W_L}$$

$$\therefore \quad W_F = W_L \left(\frac{1}{1 + \frac{W_L}{W_c}} \right) \qquad (*)$$

From table (1) it is clear that Merchant equation should be modified to cover the error where the value in some frames such as frame c_7 equals to + 39.3 % in case of pitched-roof portal frames.

Table (1)

Frame NO.	(W_L) (lbs)	(W_{cr}) (lbs)	(W_F) (Merchant) lbs	Exper. failure load (Heyman)	Error %
C_4	116.2	800	102	112	-8.95 %
C_6	96.8	737	85.5	84	+1.79 %
C_8	60.1	660	55.08	52	+5.92 %
C_{10}	37.3	496	34.9	34	+3.75 %
C_5	116.2	332	86	92	-6.5 %
C_7	72.1	275	57.12	41	+39.3 %
C_9	39.5	236	33.89	26.5	+27.8 %
C_{11}	21.8	176	19.4	16	+21.2 %

SUGGESTED EQUATIONS FOR OBTAINING THE FAILURE LOAD OF DIFFERENT STRUCTURAL FRAMES:

Two suggested equations may be used to obtain the value of (W_F) regarding fig(1) in which the curve plotted to cover the relation between $(\frac{W_F}{W_L})$ value and $(\frac{W_L}{W_C})$ value for the frames tested by Dr. J. Heyman and frames tested by Dr. Merchant. The latter frames contain a group of portal frames and another group of traingulated frames and warren trusses.

After the interpretation of the experimental results and the theoretical calculations for the elastic critical load and the plastic failure load it should be noted that :

a for frames having the value of $(\frac{W_L}{W_C})$ less than (0.3), the failure load of the frames may be obtained from equation (1) which is the equation of the suggested curve.

b For Frames having the value of $(\frac{W_L}{W_C})$ bigger than 0.3 the failure load may be obtained from eqn (2) which is the equation of the suggested curve.

$$\text{If} \quad (\frac{W_L}{W_{cr}}) < 0.3 \quad \therefore \quad W_F = W_L \left[1 - 1.67 \, (\frac{W_L}{W_{cr}}) \right] \qquad (1)$$

$$\text{If} \quad (\frac{W_L}{W_{cr}}) > 0.3 \quad \therefore \quad W_F = W_L \left[\frac{1}{1 + (\frac{W_L}{W_{cr}})} + \frac{1}{4} \, (\frac{W_L}{W_{cr}})^2 \right] \qquad (2)$$

Tables (2), (3), (4) and (5) show the values of the failure load obtained using the suggested equations compared with the experimental value and also compared with the value of the failure load obtained by Merchants equation fig (1).

Table (2) shows the error for obtaining the failure load of frames series (c) using the two suggested equations, the maximum value of error was (20.6 %)(see table (1)) for comparison between Merchant equation and the author suggested results.

Table (2)

Frame No.	(W_L/W_{cr})	(W_F) lbs suggested	Test load	Error %
C_4	0.145	88.9	112.0	-20.6 %
C_6	0.132	76.0	84.0	- 9.5 %
C_8	0.091	50.0	52.0	-3.85 %
C_{10}	0.075	32.7	34.0	-3.83 %
C_5	0.350	90.0	92.0	-2.17 %
C_7	0.263	40.6	41.0	-0.975%
C_9	0.168	28.5	26.5	+7.58 %
C_{11}	0.124	17.4	16.0	+8.75 %

Note:

For values of $\left(\frac{W_L}{W_{cr}}\right) < 0.3$ equation (1) should be used .

For values of $\left(\frac{W_L}{W_{cr}}\right) > 0.3$ equation (2) should be used.

It was clear from table (1) and table (2) that the error for frame (C_7) using Merchant's equation is+ 39.3 % reduced to - 0.975 % using the suggested equation (1) (see also the error for frames C_9 & C_{11} in the above two tables)

Table (3) shows the values of W_F using the suggested equations compared with the test load and Merchant failure load for the portal frames tested by Merchant and A. Salem, i.e. the suggested two equations may be used also in case of protal frames.

Table (3)

Frame No.	W_L(lbs)	W_{cr}(lbs)	(W_L/W_{cr})	Merchant(W_F)	Suggested (W_F)	Test load
M_2	109	885	0.123	96.4	86.8	88
M_3	79	356	0.221	64.9	50.0	69
M_4	110	353	0.312	84.5	87.3	88
M_5	19.6	347	0.0564	18.6	17.8	18
M_6	37.3	885	0.0424	35.7	34.7	37
M_9	13	94	0.139	11.44	10.0	12
M_{13}	40	90	0.445	27.6	27.8	30
M_{14}	158	847	0.187	133.9	109.0	122
M_{15}	106	850	0.1245	94.5	84.4	98
M_{16}	30.4	87.5	0.348	22.15	23.5	27
M_{17}	106	890	0.119	95.2	85.0	96

Table (4) shows the values of (W_F) using the suggested equations compared with the test load and Merchant. failure load for the Warren girders tested by Merchant and A. Salem.

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Table (4)

Frame No.	W_L (lbs)	W_{cr} (lbs)	$(\frac{W_L}{W_{cr}})$	Merchant (W_F)	Suggested (W_F)	Test load
T_1	394	420	0.935	205	293	374
T_2	255	275	0.925	132.4	188	232
T_3	188	210	0.895	100	137.8	180
T_4	1720	2000	0.860	930	1420	1470
T_5	5400	9450	0.570	3440	3890	4590
T_6	6150	10400	0.590	3870	4400	4850
T_7	3700	6750	0.550	2390	2670	3330
T_8	3210	3500	0.915	1670	2375	2520
T_9	2350	3500	0.67	1410	1678	2280

. It was clear from table (4) that all the results obtained for (W_F) value using the suggested equation (2) (where $\frac{W_L}{W_{cr}} > 0.3$) have a value for (W_F) far better than the value obtained by Merchant equation, in girder (T_3) the error clear from using Merchant equation is- 44.5 % in obtaining the failure load, that error is reduced to a value - 23.5 % f we used the suggested equation (2). Also for girder (T_8) the error by using Merchant equation - 33.6 % is reduced to a value - 5.75 % if we used the suggested equation (2).

. It was clear thatlarge difference between Merchant equation and the suggested one when $(\frac{W_L}{W_{cr}})$ is higher than 0.3.

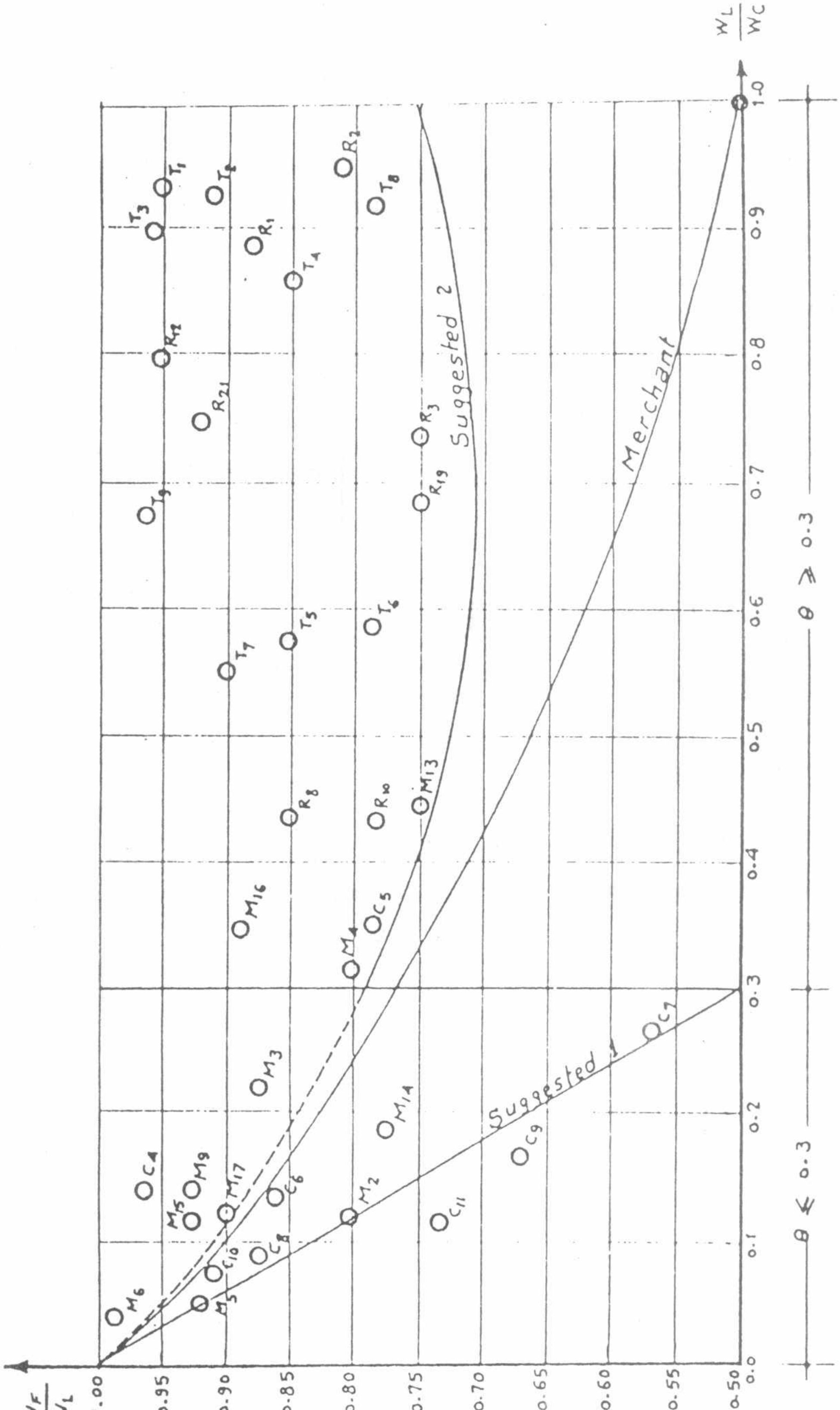
Table (5) shows the values of (W_F) using the suggested equation (2) compared with the test load[1] and Merchant failure load for the triangulated frames[3] tested by Merchant and A. Salem.

Table (5)

Frame No.	W_L (lbs)	W_{cr} (lbs)	(W_L/W_{cr})	Merchant (W_F)	Suggested (W_F)	Test load
R_1	23000	25960	0.890	12200	16800	20280
R_2	10700	11240	0.95	5500	7900	8650
R_3	32700	44570	0.735	19000	23400	24900
R_8	41000	94400	0.435	28700	30500	35130
R_{10}	26800	61300	0.435	18800	20000	21000
R_{12}	13000	16250	0.800	7200	9300	12380
R_{19}	36250	52300	0.690	21500	25800	27200
R_{21}	9860	13200	0 .750	5630	7020	9070

It was clear from table (5) that most results obtained for (W_F) value using the suggested equation (2) have a value of (W_F) far better than the value obtained by Merchant's equation, for triangulated frame (R_{12}) the error clear from using Merchant equation is- 41.7 % in obtaining the failure load, this error was reduced to the value -25% if we used the suggested equation (2).

Also for frame (R_2) the error by using Merchants equation is- 36.4 % if we used the suggested equation (2) the error would be reduced to a value equal to - 8.7 %.



Suggested equations & merchant equation

FIG. 1

CONCLUSION

- The use of Merchant formula for obtaining the failure load without modification in different cases is not sufficient.

- Two suggested formulas may be used after the modification of Merchant equations for the estimating of the theoretical failure load.

$$W_F = W_L \left[\frac{1}{1+\left(\frac{W_L}{W_{cr}}\right)} + \frac{1}{4}\left(\frac{W_L}{W_{cr}}\right)^2 \right] \text{. for } \frac{W_L}{W_{cr}} > 0.3$$

$$W_F = W_L \left[1 - 0.67 \left(\frac{W_L}{W_c}\right) \right] \text{.... for } \frac{W_L}{W_{cr}} < 0.3$$

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