INHOMOGENEOUS DEFORMATION ZONE IN TENSILE SPECIMEN

By

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ABSTRACT

This paper analyses the relation between longitudinal- and cross sectional-strain for plastic inhomogeneous deformation at the neck of tensile test specimen. The geometry of neck profile is described according to instantaneous measurements of diameters along the axis of the test specimen. An improved neck measuring instrument (MI) is proposed for plotting the neck profile during the test. Evaluating formulae corresponding to the proposed MI are deduced. The results of the proposed evaluating formulae for describing the mechanics of neck deformation are compared to those found elsewhere. The validity of the proposed method of evaluation is verified analytically and experimentally.

INTRODUCTION

After the onset of necking, the tensile test specimen deforms no longer homogeneously. The relation between lateral and longitudinal deformation can not be obtained directly from the dimensions of the test specimen. The specimen deforms by a strain rate which differs absolutely from the nominal strain rate \( /1 - 5/\). The profile of tensile test specimen changes continuously with progress of deformation.

Since the state of deformation depends on the shape of specimen, trials have been made in order to describe it mathematically /6,7/ . The GAUSS-failure curve was applied in lit./6/ as: (see fig.1)

\[
y = y_c e^{-f^2 x^2}
\]

where: \( y \) and \( x \) = coordinates of any point on the profile,
\( y_c \) = profile depth at the neck,
\( f = f(y_c, r) \) profile sharpness,
\( r \) = radius of curvature at the smallest diameter of the neck.

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CAUCHY's Curve was applied in lit. /7/ in the following form:

\[ y = \frac{y_c}{1 + \psi(y_c, r) x^2} \]  

(2)

where: \( \psi(y_c, r) = f^2 \)

Measuring instruments with one measuring jaw are constructed for continuous registering of the necking diameter \((d_{\text{c}})\) /4,5,8/ (see fig.1). Diameter Time curves \((d-t\)-curves\) can be produced by these measuring instruments. Neither the radius of curvature nor the inclination of the profile is possible to be determined by the available instruments. They are to be mathematically deduced from the postulated functions (equus.(1) or (2) or any other equation). Moreover, by these methods, the radius of curvature depends on determination of the homogeneous strain \((\phi_c)\). The error in estimating \((\phi_c)\) may exceed 10% /9/.

Improvements of measuring instruments of the given type are proposed in this paper, so that they may be capable to determine the inclination of the profile as well as the neck diameter during the test. The certainty in evaluating the radius of curvature will be greatly improved.

FEATURES OF THE MEASURING INSTRUMENT

The proposed measuring instrument is provided with two measuring jaws \((A)\) and \((B)\) as given schematically in fig.2. The two displacement transducers \((1)\) and \((2)\) will give, at a time \(t_i\), the values of \(D_{\text{i}}\) and \(D_{\text{Bi}}\) of test specimen with profile denoted by \((i)\). The \((MI)\) can be moved longitudinally in the axial direction of the test specimen. The deformed portion will be scanned by the two jaws, so that jaw \((B)\) will always be reseted to the smallest diameter at the neck. Each of the two displacement transducers is attached to an \((x-y)\) plotter, so that \(y\) represents the diameter \(D\) and \(x\) represents the time \(t\).

The positions of the two measuring jaws with respect to two successive profiles \((o)\) and \((1)\) with incremental time difference \(\Delta t\) is illustrated in fig.3. The elongation \(Ax\) is obtained from the tensile test machine. \(\Delta D, \Delta t\) and \(dD/dt\) may be determined from the \((D-t)\) curves.
Fig. 3, Positions of the two measuring jaws with respect to profiles (o) and (1) in the general case.

By choosing proper time increments, the enveloping tangents to the profiles may be obtained. The necking stadium may be accurately determined. The final profile is illustrated in fig. 4.

Naturally the (x-y) plotter can be used to produce the whole profile by plotting the diameter D against the longitudinal movement of the measuring instrument, but in this case the tensile test must be interrupted. Since this interruption will change the test conditions, experiments of this kind are not recommended. However, the evaluation of the obtained data from the (HII) does not require neither the construction of the profile nor the determination of necking stadium as follows.

EVALUATION OF MEASUREMENTS

The evaluation method is based on the following assumptions:

1. Since the evaluation is concerned with plastic deformations, the elastic portion is neglected.

2. Applying the volume constant principle, then no deformation is possible in the cross sections beyond the point of intersection of two profiles (point K, fig. 3), where no change in diameter occurs.

3. The maximum inclination of profile (i) may be represented by the maximum tangent:

\[ 2 \tan \alpha = \frac{D_{ai} - D_{bi}}{S} = \Delta D \frac{A_i}{B_i} / S \]  

(3)

4. For small displacement \( \Delta x \), \( D_{ai} \) and \( D_{bi} \) (fig. 3) may be calculated according to the corresponding inclinations.

The change in shape of the test specimen with a time increment \( \Delta t \) is schematically represented in fig. 5. According to the volume constancy assumption, the minus change in shape \( V(-) \) is equal to that of positive sign \( V(+) \). Since not all the deformed segment is included between the two measuring jaws, it is necessarily, before equating \( V(-) \) to \( V(+) \), to reduce the positive volume-
change $V(\cdot)$ by a value $\delta^v$ which can be determined from the measurements and the constant distance $S$ according to the assumptions (1) to (4). The following relation may be deduced:

$$\varepsilon = \frac{\Delta x}{x_o} = \frac{C_0 D^2_{Bo} - C_1 D^2_{B1}}{D^2_{B1} + D^2_{B1} - \delta^*} \quad (4)$$

where, $\varepsilon = \text{local longitudinal conventional strain}$,

$\Delta x = \text{the half change in length of the specimen with a time increment } \Delta t = t_1 - t_0$,

$x_o = \text{the half initial deformable length concerned with plastic deformation at the time } t_0$,

$D_{B1}' = D_{B1} + 2 \tan \alpha_1 \cdot \Delta x$,

$C_o = 1 + \left( \frac{D_{Ao}}{D_{Bo}} \right)^2$,

$C_1 = 1 + \left( \frac{D_{A1}}{D_{B1}} \right)^2$,

$\delta^* = \delta^v / \Delta x$.

ANALYTICAL INVESTIGATIONS ON SPECIAL CASES

1) Identity of observed- and deformable- segments:

If the deformable segment is identical with the initial observed segment (fig.6), the following conditions prevail:

$A_0 x_o \neq A_1 x_1$,

$x_o = S$,

$\delta^* = 0$,

$D_{Ao} = D_{A1}$

where, $A_o$ and $A_1$ are minimum cross sectional areas at necks of profiles (o) and (1) respectively.

$x_1 = \text{final length of the observed deformable segment at time } t_1$.

Therefore, equ.(4) becomes:

$$\varepsilon = \frac{D^2_{Bo} - D^2_{B1}}{D^2_{B1} + D^2_{B1}} \quad (5)$$

By comparing eq.(5) to eq.(4), the following remarks may be obtained:

1. $D_{A1}$ does not appear in both equations (4) and (5). It is only required for evaluation of the inclination ($\tan \alpha_1$).

2. $\delta^*$, $D_{Ao}$ and $D_{A1}'$ vanish in equ. (3) when the condition ($D_{Ao} = D_{A1}'$) is fulfilled.
3. If \((D_{AO} = D'_{A1})\), then \(x_o\) must equal to \(S\).

It follows that equ. (5) may be used for evaluating the general case if the condition \((D_{AO} = D'_{A1})\) is satisfied mathematically through accurate calculation of the point of intersection \((K)\). From this analysis it is obvious that the measuring jaw \((A)\) does not require to coincide with the point of intersection of the profiles in general.

2) Homogeneous deformation:

If the deformation is homogeneous along the whole specimen length, the following conditions will prevail:

\[x_o \neq S,\]
\[\delta^* \neq 0,\]
\[A_0 x_o = A_1 x_1,\]
\[D_{AO} = D_{BO},\]
\[D'_{A1} = D'_{B1} = D_{B1},\] \(C_0 = 2\) and \(C_1 = 0\).

Substituting these conditions in equ. (4), we get:

\[
\varepsilon = \left(\frac{D^2_{Bo} - D^2_{B1}}{D_{B1}}\right) / \frac{D_{B1}}{D_{B1}}
\]

Equ. (6) may be written as:

\[
\phi = \phi_{Bo} - \phi_{B1} = -\phi_q
\]

where, \(\phi_{Bo}, \phi_{B1}\) = logarithmic cross sectional strain at points Bo and B1, respectively,

\(\phi_1\) = logarithmic longitudinal strain at the whole length of the observed segment,

\(\phi_q\) = logarithmic cross sectional strain, representative to the homogeneous deformation.

From equ. (7) it is seen that the proposed evaluation technique is also valid for the homogeneous deformation.

Similarly, the general case may be formulated as:

\[
\phi_1 = \frac{\phi_{Bo} + \phi_{AO} - \phi_{A1} + \phi_c}{\phi_{B1} + \phi_c}
\]

where, \(\phi_c = \phi_{B1}\) = logarithmic cross sectional strain at point of contraction of the neck.

From eq. (8) it is seen that the longitudinal strain \(\phi_1\) is not in simple relation with the cross sectional strain \(\phi_c\), at the narrowest cross section. But it may be expressed through different cross sectional strains which may be considered as equivalent to the homogeneous cross sectional strain.
EXPERIMENTAL RESULTS

Pilot experiments are run in order to investigate the validity of the evaluation technique. Tensile test specimens are chosen of 0.15%c mild steel with 10 mm diameter. In order to obtain the required dimensions for evaluating the deduced formulae, the test specimens are photographed during the deformation. The pictures are synchronized chronologically with respect to the displacement of cross head of the test machine.

![Graph showing reconstructed fracture profile](image)

Fig. 8, Reconstructed fracture profile of test specimen of mild steel with \( D = 10 \) mm, cross head speed \( = 2 \) mm/min

A reconstructed necking profile of fracture is shown in fig.8. The inclinations of preceding profiles are calculated according to equ.(3). The profile of fracture is produced according to the various enveloping tangents as well as the considered elongation \( (\Delta x) \). For visualizing reasons, the increments are given coarsely. The constructed profile coincided accurately with the actual profile (denoted by circles). The deformable regions concerning the coarse deformations are shown by vertical lines. Tested specimens showed that the region of deformation decreases according to a progressive function, which requires more investigations in order to determine the relationship between its various parameters. Values of \( \varepsilon \) are given beside \( (\varepsilon = \Delta x/x_o) \) at the beginning of each deformation zone. The two values are found sufficiently equal, except at the neighbourhood of fracture, whereby the relative difference may reach 36%. By choosing smaller increments, this difference will be greatly reduced.
By using equ.(1) for describing the necking profile and equating the volume of the specimen after and before deformation, the following relation may be obtained:

\[
\varepsilon - \varepsilon_g = \frac{2}{L_o} (d_g - d_c) \left[ c_1 + c_2 \varepsilon \frac{C}{c} g - c_3 \varepsilon \frac{C}{c} \right] \]

where, \( \varepsilon = \) total plastic longitudinal strain,
\( \varepsilon_g = \) homogeneous part of plastic strain,
\( d_g = \) Diameter of specimen at the end of homogeneous deformation,
\( c_1 = 1 - (1/2/2), \quad c_2 = (1/\sqrt{2}) - 1 \quad \text{and} \quad c_3 = 1/2/2. \)

Results of equ.(9) are presented in fig. 9, whereby the radius of curvature \( r \) is experimentally determined through the relation:

\[
r = \frac{1}{(y_c \varepsilon \tan^2 \alpha)}
\]

Calculations of strain according to lit./7,10/ are also given in fig.9 compared to results of equ.(9). It is obviously seen that results of equ.(9) are quite accurate and represent the experimental values.

Fig. 9, inhomogeneous longitudinal strain calculated:
according to equ.(9) --------
according to lit./7/ -- -- --
according to lit./10/ --- ---
CONCLUSIONS

Modifications to neck measuring instruments are proposed, so that the profile inclination of tensile test specimen as well as the radius of curvature at the neck can be accurately determined. The relation between longitudinal- and cross sectional- strain is analytically deduced through a proposed technique of evaluation. Analytical description for the neck stadium is derived. Theoretical investigations on the proposed technique verified its validity as well as its obedience to the assumption of volume constancy. Experimental investigations on mild steel showed that the proposed technique is quite accurate all over the inhomogeneous deformation up to the point of fracture.

REFERENCES


