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A NUMERICAL STUDY OF LAMINAR FLOW HEAT TRANSFER IN A POROUS ANNULI

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ABSTRACT

The laminar flow of both fully developed and uniform entry velocity profiles in a permeable annuli was studied. The set of continuity, momentum and energy equations with its boundary conditions were solved numerically by computer program using finite difference method for a wide range of parameters. It was found from the numerical results that for the inlet parabolic velocity profile the axial velocity profile near the wall along the annuli increases due to suction and decreases due to injection. The axial pressure drop along the annuli with suction is smaller than it with injection. The local Nusselt number at the entry region increases due to suction and decreases due to injection through the inner wall of annuli.

INTRODUCTION

In recent years, the heat transfer parameters of the flow through an annulus with a porous walls is an interesting aspect for its multiple applications. For example, transpiration cooling, boundary layer control, heat pipes, film condensation, film evaporation and the phenomena of osmosis. Due to suction or injection a transversal velocity appears in the radial direction and its value depends upon the intension of mass transfer through the walls. Various methods have been proposed by several authors. First, Investigators[1-2]obtained a closed form solution for flows in a porous annulus with the same amount of fluid entering the outer wall as of fluid leaving the inner wall. Terrill[3-4] gave a first order perturbation solution for the flow in a porous annulus with small values of transversal velocities. Later, Authors [5-7] examined the flow in annulus with impermeable walls. The purpose of the present study is to examine the laminar flow heat transfer at the entry region of annuli with a porous walls for a wide range of parameters.

THE GOVERNING EQUATIONS

A steady flow of fluid through an annulus with a porous walls is considered. Let the inner and outer walls of the annulus with uniform height and

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circumference and given by r=b and r=a respectively. The thermal boundary conditions are isothermal inner wall and adiabatic outer wall. A cylinderical coordinate system is chosen with the axis oz lying along the centre of the annulus and v,u are taken to be the velocity components in the directions of r and z increasing respectively. Geometry and coordinate system are shown in Fig.1. The formulation of the governing continuity, momentum and energy equations can be simplified by the following assumptions:

a- The flow is laminar
b- The physical properties of the fluid are constant
c- Axisymmetry conditions
d- The transversal velocity at the wall is constant
e- The molecular transport in axial direction is negligible. Hornbeck and others [8] showed that the molecular transport in axial direction can be neglected when RePr > 400
f- The pressure drop in radial direction is neglected. Schmidt and Zeldin [9] confirmed that the radial pressure drop for the flow in a porous pipe can be neglected.

Under the assumed conditions, the continuity, momentum and energy equations are;

$$\frac{\partial u}{\partial z} + \frac{\partial (rv)}{\partial r} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{2}$$

$$u\frac{\partial t}{\partial z} + v\frac{\partial t}{\partial r} = \frac{k}{\rho c}\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right) \tag{3}$$

With the following initial and boundary conditions;

$$\text{For } z = 0 \text{ and } b < r < a, \quad u = u_0(r), \quad v = 0 \text{ and } t = t_0 \tag{4}$$

$$\text{For } z > 0 \text{ and } r = a, \quad u = 0, \quad v = v_a \text{ and } \frac{\partial t}{\partial r} = 0 \tag{5}$$

$$\text{For } z > 0 \text{ and } r = b, \quad u = 0, \quad v = v_b \text{ and } t = t_b \tag{6}$$

In boundary conditions Eq.(4), the axial velocity function u at the inlet not defined but two variants (uniform and parabolic velocity profiles) are applied in calculations.

With reference to the outside radius of annuli a and the average axial velocity at the inlet $\bar{u}_0$, define the following nondimensional variables;

$$R = r/a, \quad Z = z\nu/a^2\bar{u}_0, \quad U = u/\bar{u}_0, \quad V = va/\nu, \quad P = (p-p_0)/\rho\bar{u}_0^2,$$

$$T = (t-t_0)/(t_b-t_0), \quad Pr = \mu c/k, \quad K = b/a \tag{7}$$

By introducing these dimensionless variables into Eqs. (1-6), then the resulting nondimensional forms of the continuity, momentum and energy equations are as follows;

$$\frac{\partial U}{\partial Z} + \frac{V}{R} + \frac{\partial V}{\partial R} = 0 \tag{8}$$

$$U\frac{\partial U}{\partial Z} + V\frac{\partial U}{\partial R} = -\frac{\partial P}{\partial Z} + \frac{\partial^2 U}{\partial R^2} + \frac{1}{R}\frac{\partial U}{\partial R} \tag{9}$$

$$U\frac{\partial T}{\partial Z} + V\frac{\partial T}{\partial R} = \frac{1}{Pr}\left[\frac{\partial^2 T}{\partial R^2} + \frac{1}{R}\frac{\partial T}{\partial R}\right] \tag{10}$$

With the following dimensionless initial and boundary conditions;

For $Z = 0$ and $K < R < 1$, $U = U_o(R)$, $V = 0$ and $T = 0$ (11)

For $Z > 0$ and $R = 1$, $U = 0$, $V = V_a$ and $\frac{\partial T}{\partial R} = 0$ (12)

For $Z > 0$ and $R = K$, $U = 0$, $V = V_b$ and $T = 1$ (13)

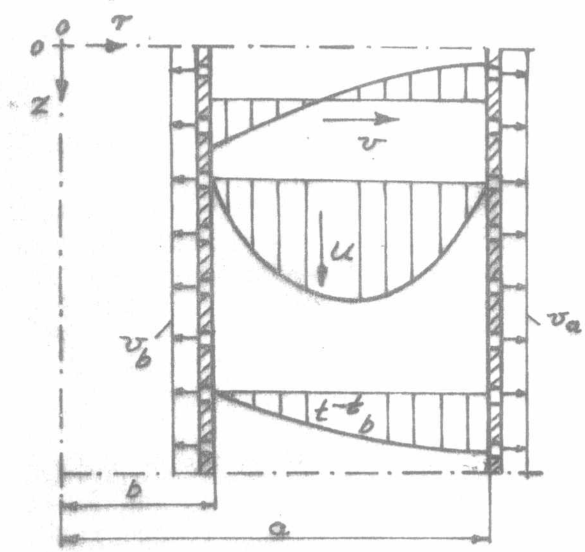

Fig. 1: Geometry and coordinate system

NUMERICAL SOLUTION

Descretion the area between the annulus walls into a sets of equal rectangles with no equal sides closely at the entry and the walls as shown in Fig.2. The finite difference forms of Eqs. (8-10) can be written as follows:

$$\frac{U_{i+1,j+1}+U_{i,j+1}-U_{i+1,j}-U_{i,j}}{2\Delta Z}+\frac{V_{i+1,j+1}-V_{i,j+1}}{\Delta R}$$

$$+\frac{V_{i+1,j+1}+V_{i,j+1}}{2\left[K+(i-1/2)\Delta R\right]}=0 \tag{14}$$

$$U_{i,j}\frac{U_{i,j+1}-U_{i,j}}{\Delta Z}+V_{i,j}\frac{U_{i+1,j+1}-U_{i-1,j+1}}{2\Delta R}=\frac{P_{i,j}-P_{i,j+1}}{\Delta Z}$$

$$+\frac{U_{i+1,j+1}-2U_{i,j+1}+U_{i-1,j+1}}{(\Delta R)^2}+\frac{1}{K+(i-1)\Delta R}\,\frac{U_{i+1+1,j+1}-U_{i-1,j+1}}{2\Delta R} \tag{15}$$

$$U_{i,j}\frac{T_{i,j+1}-T_{i,j}}{Z}+V_{i,j}\frac{T_{i+1,j+1}-T_{i-1,j+1}}{2\ R}=\frac{1}{Pr}\left[\frac{T_{i+1,j+1}-2T_{i,j+1}}{(\Delta R)^2}\right.$$

$$\left.+\frac{T_{i-1,j+1}}{(\Delta R)^2}+\frac{1}{K+(i-1)\Delta R}\,\frac{T_{i+1,j+1}-T_{i-1,j+1}}{2\Delta R}\right]=0 \tag{16}$$

The variables with subscripts j and j+1 in the difference equations represent the knowns and unknowns respectively. Starting with the initial and boundary conditions Eqs.(11-13), which applied to the difference equations (14-16), with the aid of mesh points as shown in Fig.2. The resulting difference equations are a simultaneous linear algebraic equations which are solved numerically by means of a computer program for a wide range of parameters.

RESULTS AND DISCUSSION

For a wide range of parameters, uniform and parabolic axial velocity profiles at the inlet, transversal velocity V_a and V_b at the outer and inner walls of annuli respectively from 0 to $\pm$ 10 and Prandtl number Pr from 0.5 to 1. A numerical results were obtained for the axial velocity profile, the axial pressure drop and the local Nusselt number at the entry region for a different values of transversal velocity at the walls. Fig.3, shows the effect of the transversal velocity at the walls upon the dimensionless axial velocity profile at the entry region. It indicates that the axial velocity for parabolic inlet profile increase near the wall with suction and decreases near the wall with injection. The axial pressure drop at the entry region for both uniform and parabolic velocity profiles at the inlet can be presented as in Fig.4. It is clear that the axial pressure drop with suction is smaller than it with injection through the annulus wall. Fig.5, shows the effect of the uniform and parabolic velocity profiles upon the local Nusselt number for Pr=0.7 at the entry region. It was found that the local Nusselt number increases with suction and decrease with injection through the inner wall of the annulus.

CONCLUSIONS

This study indicates that, for a parabolic velocity profile at the inlet, the axial velocity profile at the entry region increases near the wall with suction and decreases by injection. For both uniform and parabolic velocity

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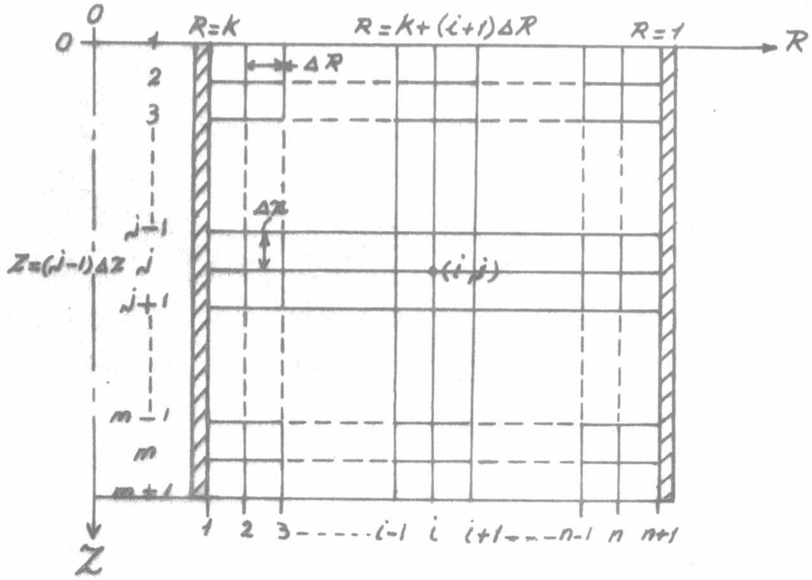

Fig.2. Finite difference representation

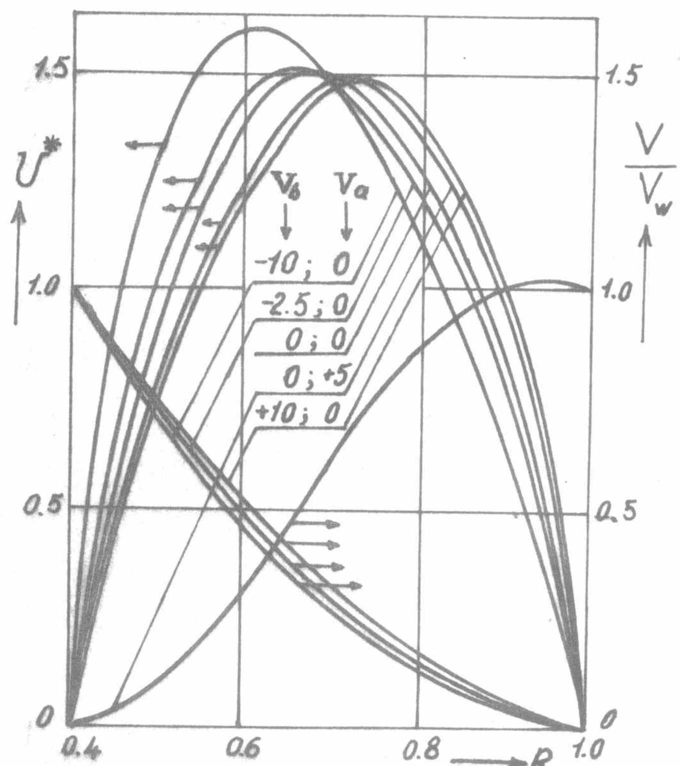

Fig.3. Axial and transversal velocity profiles at the entry region

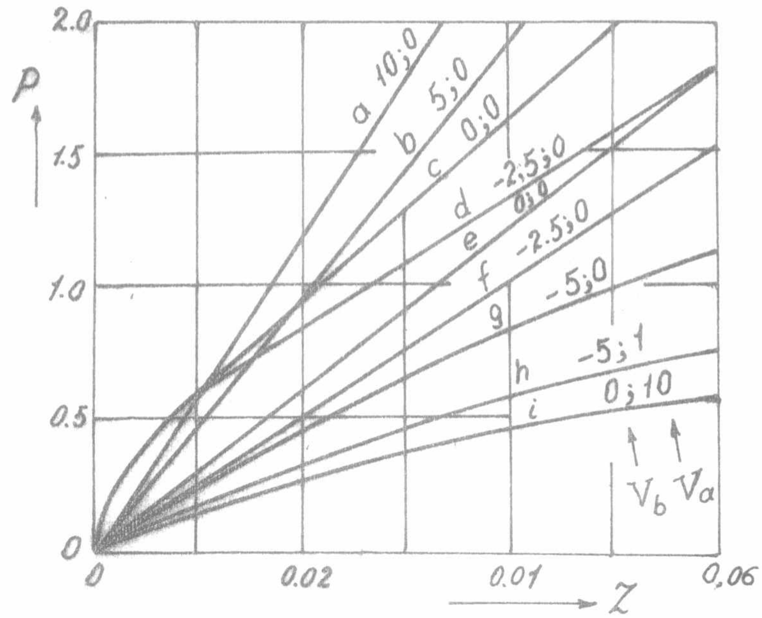

Fig.4. Axial pressure drop at the entry region
c,d- for inlet uniform velocity profile
a,b,e,f,g,h,i- for inlet parabolic velocity profile

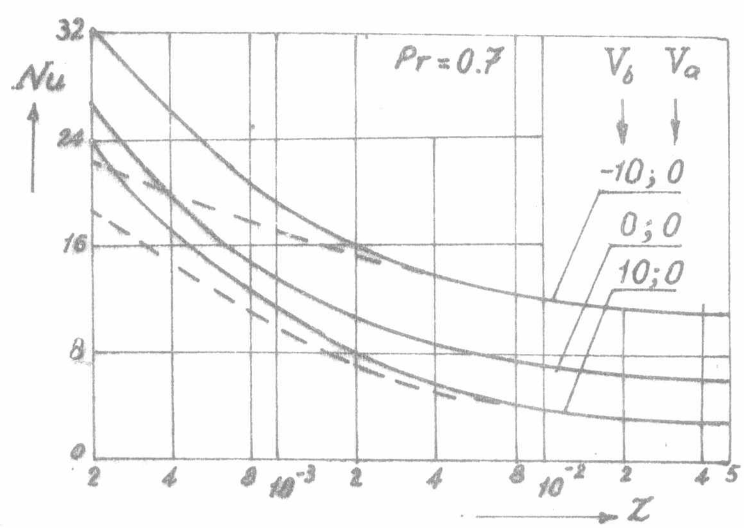

Fig.5. Local Nusselt number at the entry region
—— for inlet uniform velocity profile
---- for inlet parabolic velocity profile

profiles at the inlet, the axial pressure drop at the entry region is smaller with suction than it with injection through the porous walls. For both uniform and parabolic velocity profiles at the inlet, the local Nusselt number at the entry region increases due to suction and decreases with injection through the inner wall of annuli.

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NOMENCLATURE

Greek Letters

μ - viscosity, kg/hr.m

ν - ($\nu = \mu/\rho$)-kinematic viscosity,m^2/hr

ρ - density, kg/m^3

Alphapitic Letters

a - outside radius of the annular space,m

b - inside radius of the annular space, m

c - specific heat,kcal/kg.^oC

d - [d=2(a-b)] -hydraulic diameter,m

h -[$h=-k\frac{\partial t}{\partial r}\Big|_{r=b}/(t-t_b)$]-local heat transfer coefficient,kcal/hr.$m^2.^oC$

K - (K=b/a) -annulus radius ratio,dimensionless

k - thermal conductivity, kcal/hr.m.^oC

Nu- ($Nu=hd/k$)-local Nusselt number, dimensionless
P - [$P=(p-p_o)/\rho\bar{u}_o^2$] -pressure drop at any point, dimensionless
Pe- ($Pr=\mu c/k$)-Prandtl number, dimensionless
p - pressure at any point, kg_f/m^2
p_o- pressure at annulus inlet, kg_f/m^2
R - ($R=r/a$)-radial coordinate, dimensionless
Re- ($Re=\bar{u}_o a/\nu$) - Reynolds number, dimensionless.
T - [$T=(t-t_o)/(t_b-t_o)$] -temperature at any point, dimensionless
$\bar{T}$ - [$\bar{T}=(\bar{t}-t_o)/(t_b-t_o)$]-mixed-mean average temperature, dimensionless
$\bar{t}$ - ($\bar{t}=\int_b^a utrdr/\int_b^a urdr$)-mixed-mean temperature averaged over any cross section, ^oC
t - temperature at any point, ^oC
t_b- isothermal temperature at the inner radius of the annular space, ^oC
t_o- temperature at the annulus entrance, ^oC
U - ($U=u/\bar{u}_o$)-axial velocity, dimensionless
U^*- ($U^*=U\bar{u}_o/\bar{u}_z$)-reduced axial velocity, dimensionless
u - axial velocity, m/hr
$\bar{u}_o$- ($\bar{u}_o=\int_b^a urdr/\int_b^a rdr$)-average axial velocity at annulus entrance, m/hr
$\bar{u}_z$- average axial velocity at any z along the annuli, m/hr
V - ($V=va/\nu$)-transversal or radial velocity, dimensionless
v - transversal or radial velocity, m/hr
Z - ($Z=z/a$)-axial coordinate, dimensionless
z - axial coordinate, m