



PRELIMINARY STUDY FOR THE INFLUENCE OF
CONDENSATION DURING A PROJECTILE MOTION

By

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ABSTRACT

This paper represents an initiation for studying the influence of change of phase of a water vapour-inert gas mixture that acts upon moving projectile in a barrel experimentally sustained by theoretical model.

In this study, the equations expressing the projectile motion and the change of state of the driving gas are analysed. In addition, the influence of boundary conditions is discussed. This is carried out for the purpose of determination of the calculation procedure which can be adapted in the case of two phase flow, if the condensation may take place by theoretical expectation. At the same time, it is also to obtain the geometrical characteristics necessary for the construction of an experimental set-up for studying the condensation influences.

It was found that the theoretical model expressing the projectile motion and the change of state of gas, proved its accuracy by comparison with experimental results. It is proved also that even in these particular conditions of gas expansion, the condensation of water vapour can take place. A conclusion that an increase of projectile velocity has been proved if the water vapour is added to the propelling gas

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NOMENCLATURES

a	Local velocity of sound.
d	Internal barrel diameter.
L	Length of barrel.
M	Projectile mass.
m	Mass of gas.
p	Pressure.
S	Barrel bore area.
T	Temperature.
u	Local velocity of sound.
v	Projectile velocity.
R	Resistance to projectile motion.
X	Displacement.
	Greek Letters:
γ	Specific heat ratio.
ρ	Specific mass.
	Sufficies:
a	Atmospheric.
b	Behind the projectile.
f	In front of the projectile.
O	Initial condition.

1. INTRODUCTION

There is no doubt that the study of projectile motion is very important, specially that one resulting from the action of a compressible gas. The most important domains in this field are those of interior ballistics of firing weapons and the development of high muzzle velocity guns.

The purpose here is to introduce the bases of a study for the influence of water vapour condensation through gas mixture acting upon projectile like that shown by other different application as for example the condensation through nozzles.

This study proposes eventually the analysis of projectile motion and the change of state of gas for the following purposes:

1. Definition of an easy applicable calculation procedure in the case of two phase flow.
2. Obtaining the necessary geometrical characteristics for an experimental set-up showing the influence of condensation on the projectile motion.

The main assumptions considered in this study are:

- One dimensional unsteady isentropic flow.
- Ideal and non viscous gas.

2. FORMULATION OF THE PROBLEM

Consider a projectile at rest inside a barrel or cylinder which separates a gas with high pressure behind it and another gas with low pressure or the atmosphere in front of it. If the mechanism hindering the projectile motion is released, the projectile will be accelerated till obtaining equal pressure on both sides (if it happens).

When the projectile moves, the gas layers in contact with its faces and initially at rest will obtain the same velocity like the projectile. The projectile motion can be considered as an infinite number of infinitesimal perturbations called Mach waves propagating in the gas with the sound

velocity. At first, these perturbations form a simple expansion wave in the high pressure side and simple compression one in the low pressure side.

As the projectile continues its motion, the mentioned perturbations are changing to complex forms as shown in Fig. (1).

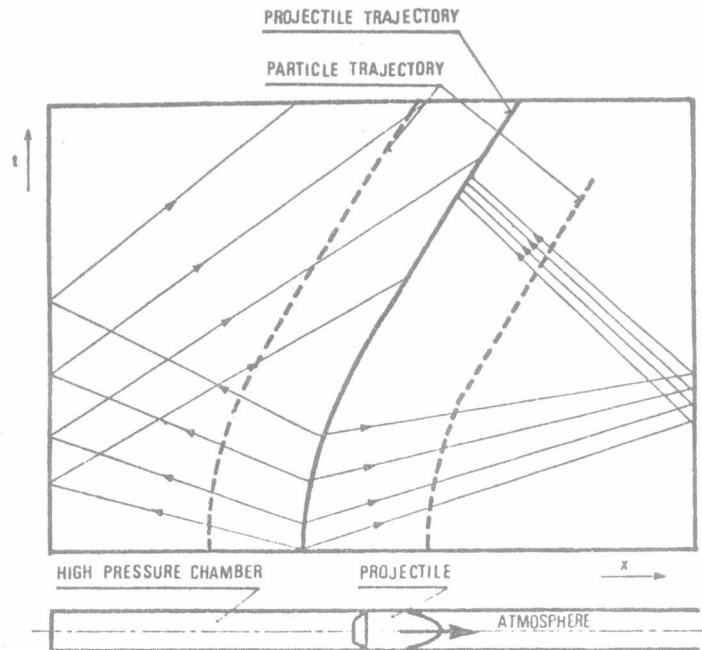


Fig.(1) Expansion and compression waves produced by projectile motion.

3. WORKING EQUATIONS OF THE PROCESS

According to the previously mentioned assumptions, the working equations are :

a) Conservation of mass.

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0 \quad (1)$$

b) Conservation of momentum

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) + \frac{\partial p}{\partial x} = 0 \quad (2)$$

c) The flow being isentropic, the thermodynamic variables are related together by the relations

$$\frac{a}{a_0} = \sqrt{\frac{T}{T_0}} = \left(\frac{p}{p_0} \right)^{\frac{\gamma-1}{2\gamma}} = \left(\frac{\rho}{\rho_0} \right)^{\frac{\gamma-1}{2}} \quad (3)$$

Choosing a and u as state parameters and x , t as independent variables the conservation equations may be written as:

$$a \frac{\partial u}{\partial x} + \frac{2}{\gamma-1} \frac{\partial u}{\partial t} + \frac{2}{\gamma-1} u \frac{\partial a}{\partial x} = 0 \quad (4)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial a}{\partial t} + a \frac{\partial a}{\partial x} = 0 \quad (5)$$

Which are valid for both gases in front and behind the projectile.

Supposing perfect sealing between the projectile and barrel, the equation of projectile motion is

$$M \frac{dv}{dt} = S (p_b - p_f) - R \quad (6)$$

The boundary conditions at both terminals of the barrel are:

$$\text{at } x = 0 \quad u = 0 \quad (7)$$

$$\text{at } x = L \quad p = p_a$$

According to the classical method of analysis [4], the system of equations (4 ÷ 6) has a corresponding family of characteristics having slope at every point given by the equation

$$\left(\frac{dx}{dt} \right)^2 - 2 \frac{dx}{dt} + (u^2 - a^2) = 0 \quad (8)$$

and the two corresponding families of characteristics α_1 and α_2 have the following relations in the (x,t) plane:

$$\frac{dx}{dt} = u \pm a$$

The state characteristics C^+ and C^- are designated by Γ^+ and Γ^- hence:

$$du \pm \frac{2}{\gamma-1} da = 0 \quad (9)$$

or

$$u + \frac{2}{\gamma-1} a = \sigma \quad (10)$$

and

$$u - \frac{2}{\gamma-1} a = \nu \quad (11)$$

where σ and ν are constants.

4. SOLUTION BY THE METHOD OF CHARACTERISTICS USING A CONSTANT STEP OF TIME

Two computational techniques are utilised Fig. (2):

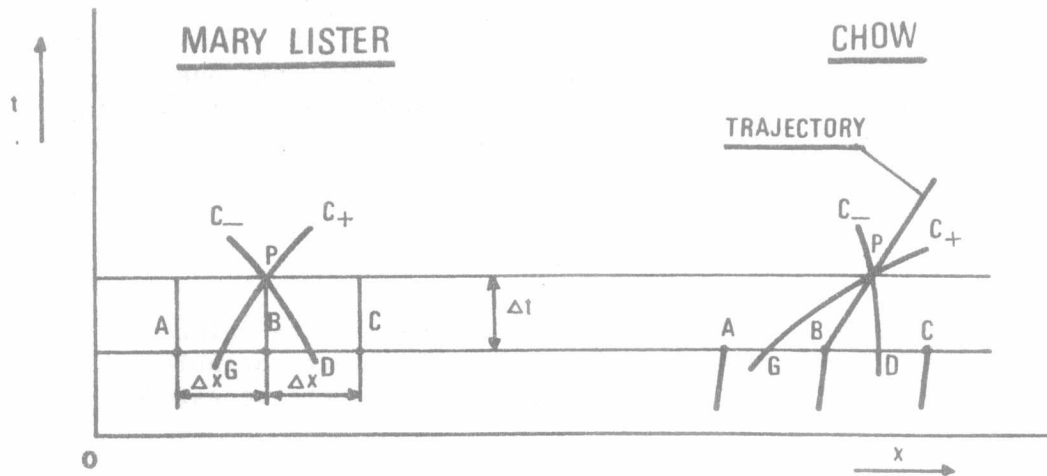
- The first made by Harter and developed by Mary Lister [3] and calculates the change of parameters at determined stations chosen in advance.
- The second is proposed by Chow [2] calculates the evolution of parameters along the particle trajectory. This method is adapted in calculations due to the absence of interpolation for determination of trajectory.

Knowing the state variables at the distributed points on the base line $t=t_0$, the step of time can be determined using the stability condition:

$$\Delta t < \Delta t^* = \max |u+a| \quad (12)$$

In the calculations, the following relation is adapted:

$$\Delta t = 0.9 \Delta t^* \quad (13)$$



4.1. Calculation of Parameters at Ordinary Point

As shown in Fig. (2) the variables are known on the base line $t=t_0$. For calculation of parameters, the following relations are used:

$$x_p - x_B = [u]_{pB} \Delta t \quad \text{along BP,} \quad (14)$$

$$x_p - x_G = [u+a]_{pG} \Delta t \quad (15)$$

and $(u_p - u_G) + \frac{2}{\gamma-1} (a_p - a_G) = 0 \quad \text{along PG} \quad (16)$

$$x_p - x_D = [u-a] \Delta t \quad (17)$$

$$(u_p - u_D) - \frac{2}{\gamma-1} (a_p - a_D) = 0 \quad \text{along PD} \quad (18)$$

The iteration procedure can be applied for the solution of these equations if the values between square brackets are known. For the first approximation, it is sufficient to replace these values by the corresponding values at point B. The iteration process is repeated using the mean values till the convergence of results.

4.2. Calculation of Projectile Trajectory

The projectile trajectory is determined using the following system of equations:

- Equation of projectile trajectory

$$x_E^* - x_E = [v]_{E^*E} \Delta t \quad (19)$$

- Equation of projectile motion

$$M \frac{\Delta v}{\Delta t} = 0.5 \{ (p_E^* - p_F^*) + (p_E - p_F) \} S - R \quad (20)$$

- Equations of characteristics as shown in Fig. (3)

$$(u_E - u_G) + \frac{2}{\gamma-1} (a_E^* - a_G) = 0 \quad (21)$$

and $x_E^* - x_G = [u + a]_{E^*G} \Delta t \quad \text{along EG} \quad (22)$

$$(u_F^* - u_D) - \frac{2}{\gamma-1} (a_F^* - a_D) = 0 \quad (23)$$

$$x_{F^*} - x_D = [u - a]_{F^* D} \Delta t \quad \text{along } F^* D \quad (24)$$

This system of equations (19 - 24) can be transformed into one equation in p_E under the form $Y(p) = 0$, for which the solution is obtained using Newton iteration method:

$$p_{n+1} = p_n - \left(\frac{Y(p)}{Y'(p)} \right)_n \quad (25)$$

where $Y'(p) = \frac{\partial(Y(p))}{\partial p}$

The first iteration is started by giving the pressure its value at point E. Thus the pressure at point E is known and the other variables are calculated at both projectile faces. The process is there repeated till the convergence of results.

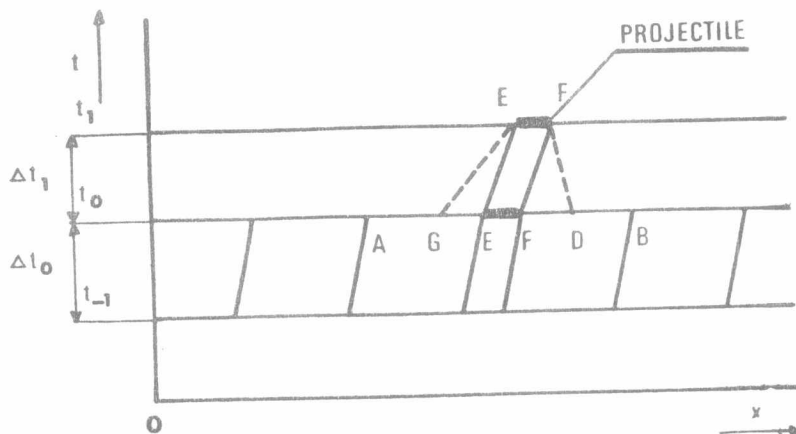


Fig. (3) Calculation of projectile trajectory.

4.3. Calculation of Parameters at Barrel Ends

In the studied case, there are two ends for the barrel influencing the reflection of compression and expansion waves. For the muzzle of the barrel where the pressure is atmospheric one, the scheme of calculation is shown in Fig. (4) and the necessary equation are:

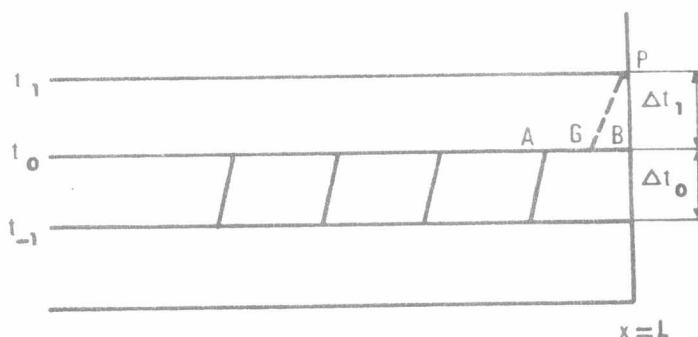


Fig. (4) Calculation of parameters at the barrel muzzle.

Along PG

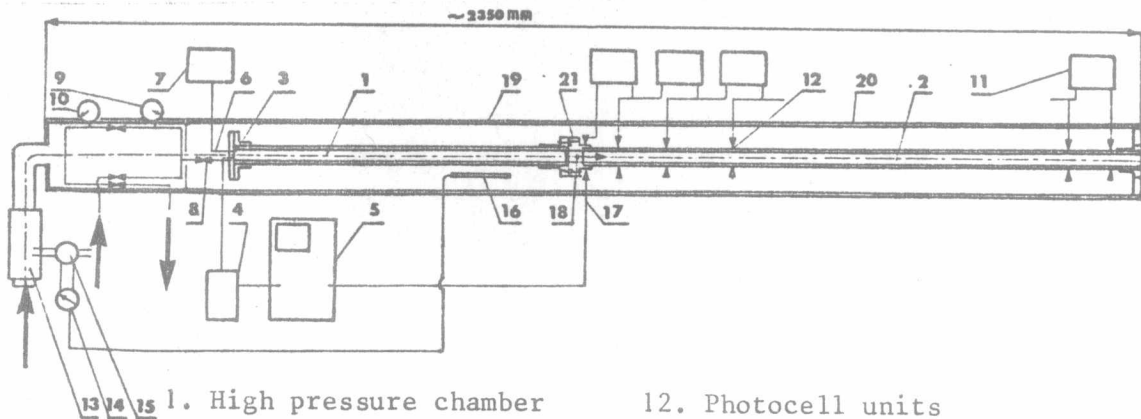
$$x_p - x_G = [u + a]_{pG} \Delta t$$

$$(u_p - u_G) + \frac{2}{\gamma - 1} (a_p - a_G) = 0$$

and the iteration procedure is started using the values at point B for obtaining the point G. The same procedure is applied for the closed end of the barrel (chamber base) but the condition at this end is zero flow velocity.

5. EXPERIMENTAL STUDY

An experimental set-up was built for studying the projectile motion. The main dimensions of this installation were obtained on the bases of theoretical results. The scheme of this set-up is shown in Fig. (5) and it comprises:



- | | | |
|------------|--------------------------|--|
| 13, 14, 15 | 1. High pressure chamber | 12. Photocell units |
| | 2. Low pressure chamber | 13-16 Air heating and control units |
| | 3. Pressure pick-up | 17. Triggering photocell |
| | 4. Amplifier | 18. Projectile with membrane |
| | 5. Oscilloscope | 19, 20 shielding covers with isolation |
| | 6. Thermocouple | 21. Mechanism controlling Projectile motion. |
| | 7. Digital voltmeter | |
| | 8. Valve | |
| | 9, 10 Manometers | |
| | 11. Time counters | |

Fig. (5) Layout of the experimental set-up.

- high pressure chamber where the high pressure gas is introduced. Before the introduction of this gas, a vacuum pump is used for the extraction of air which is necessary for the introduction of water vapour when studying the influence of condensation.
- Low pressure chamber from glass tube where the piston moves. This chamber (tube) is isolated from the high pressure one by means of a membrane attached to the projectile for securing of sealing.
- The trigger mechanism controlling the projectile motion.

The mentioned installation is accompanied with measuring devices permitting the definition of initial state and the evolution of some parameters during projectile motion.

The temperature of gas is measured by means of a system of thermocouples

distributed along the high pressure chamber before every firing. The static pressure in the high pressure chamber is measured using high precision manometer, while its change during projectile motion is realised by means of membrane type pressure pick-up with strain gauges mounted on the chamber base.

The displacement of projectile is measured by means of photocell stations distributed along the low pressure tube and connected to time counters.

An example of the obtained experimental results is shown in Fig. (6) representing the change of pressure on the bottom of high pressure chamber.

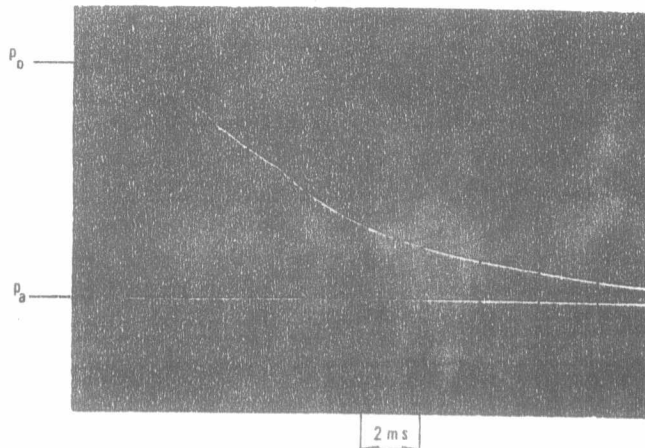


Fig. (6) Registered pressure on the bottom of chamber
gas: N_2 , $p_0 = 11 \text{ atm}$, $T_0 = 340^\circ\text{K}$, $M = 19.86 \text{ g}$, $x_0 = 0.595 \text{ m}$,
 $d = 25 \text{ mm}$.

6. THEORETICAL AND EXPERIMENTAL RESULTS

The theoretical calculations were carried out on a digital computer model IRIS-80 equipped with tracing unit Benson at INSA-Lyon France. An example of theoretical results is shown in Fig. (7). The evolution of gas pressure and temperature at the bottom of chamber and on the base of projectile has sudden rates of change. These changes could be easily explained by the diagram of wave front motion shown in the (x,t) plane of projectile displacement.

A comparison of theoretical and experimental results is shown in Fig. (8) in the case of N_2 propelling gas. Similar results were obtained using Helium and Argon. From these results we can deduce an excellent agreement between them. So; utilizing the introduced theoretical model for analysing the change of gas parameters and the determination of projectile motion parameters gives good results.

The study of projectile motion indicates that it depends mainly on the following dimensionless group:

$$\beta = \frac{S p_0 x_0}{M a^2}$$

The influences of this group on the projectile velocity and gas temperature are shown in Fig. (9,10). It is clear that the increase of the value of this group increases projectile velocity and acceleration and decreases more rapidly the gas temperature.

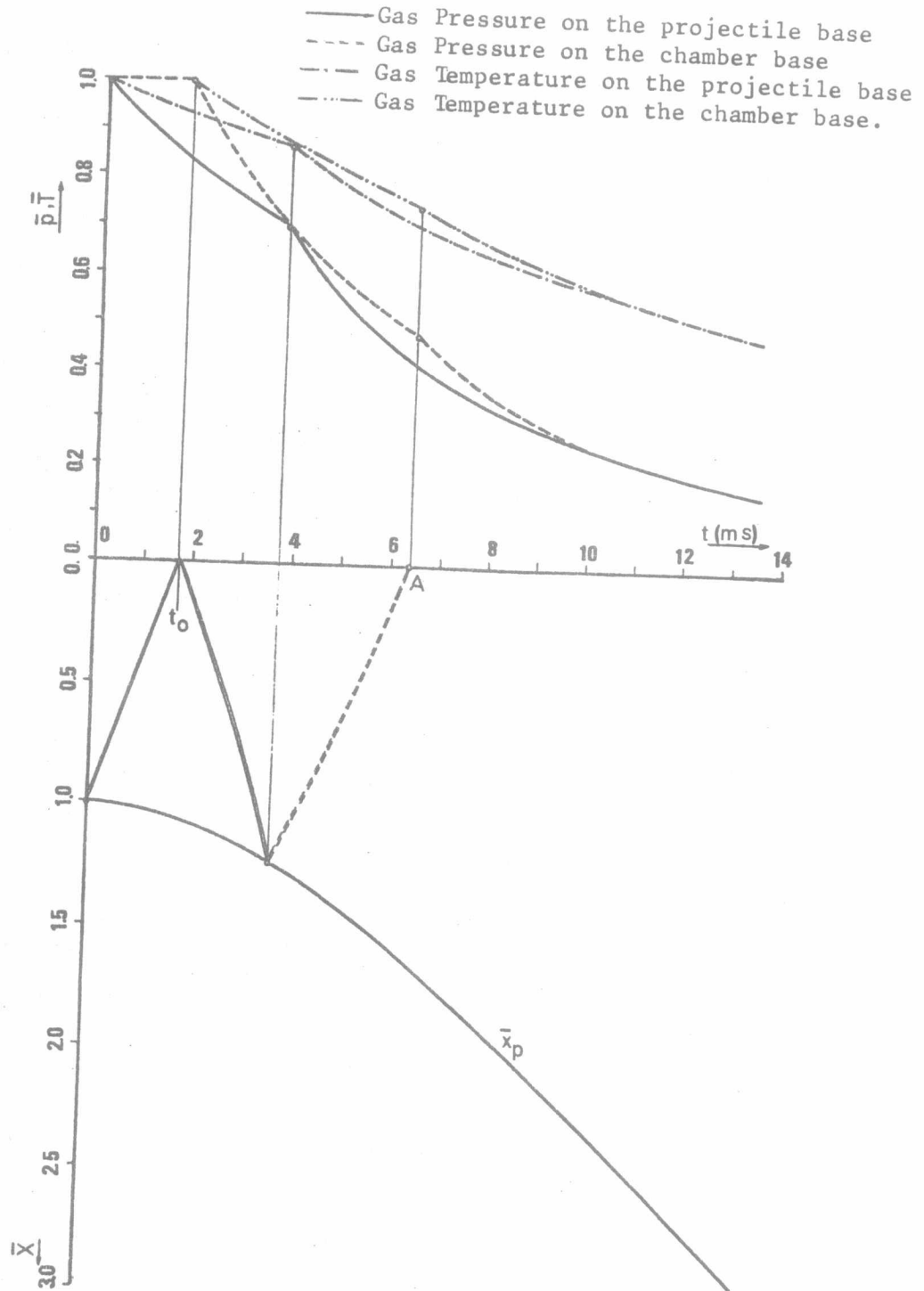


Fig. 7. Correlation between the diagram of wave front and the evolution of gas parameters.

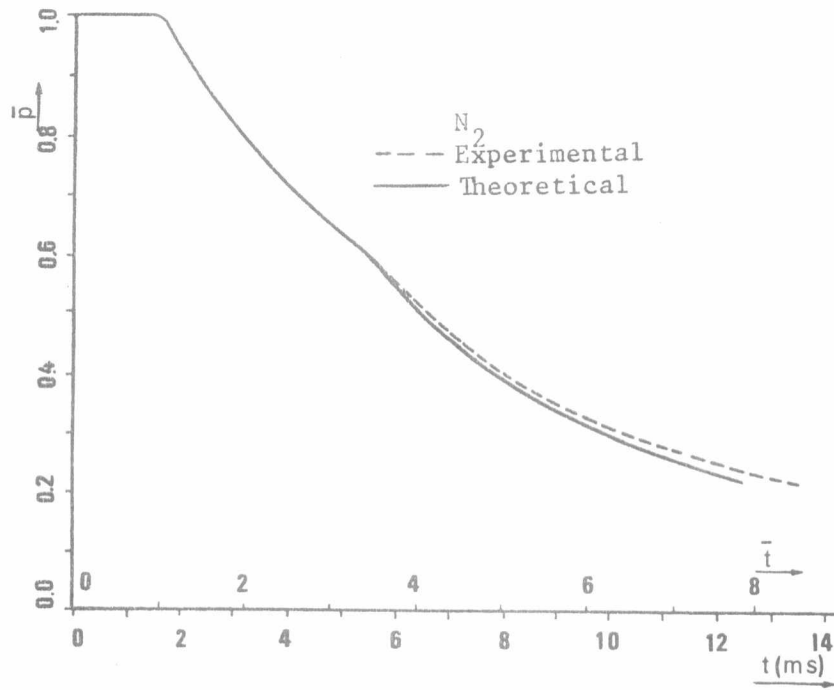


Fig. (8) Comparison of theoretical and experimental gas pressure change at the chamber bottom data written on Fig. (6).

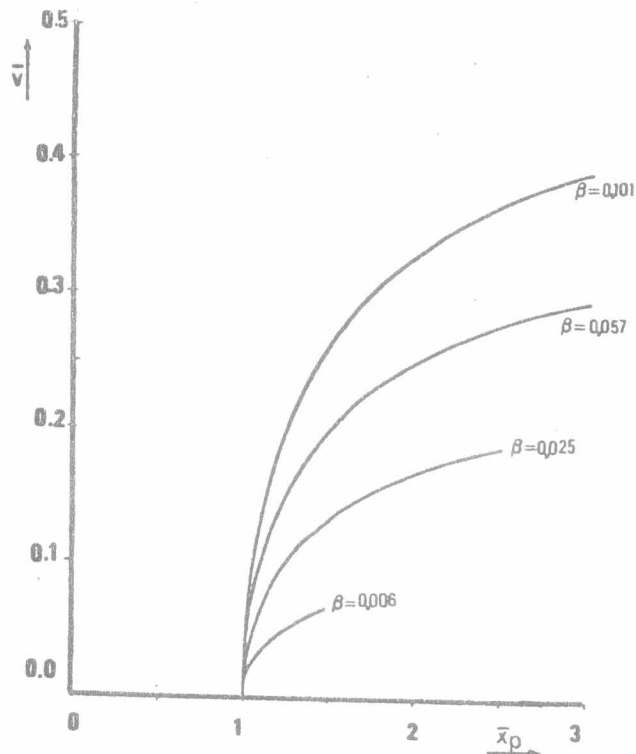


Fig. (9) Variation of projectile velocity with displacement in dependence of the dimensionless group

$$\beta = S p_0 x_0 / Ma_0^2 .$$

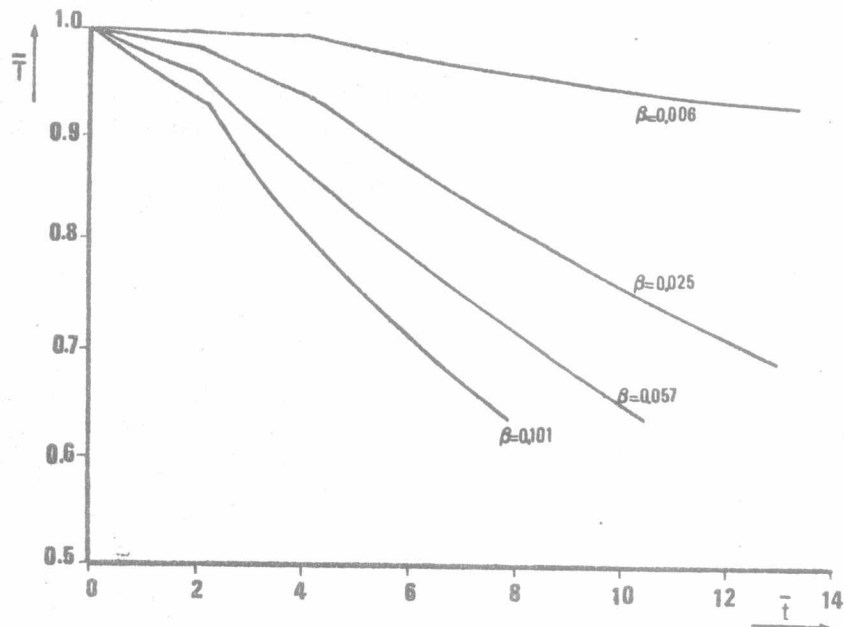


Fig. (10) Variation of gas temperature behind the projectile with time in dependence of dimensionless group

$$\beta = S p_o x_o / Ma_o^2$$

It can be concluded that for obtaining more rapid cooling of gas temperature, i.e. more favorable conditions for condensation, it is necessary to increase the value of this dimensionless group .

It is noted that there is an important decrease concerning the gas temperature which ensures the supersaturation of water vapour [1] in the gas mixture. Another criterion favorising the condensation is the time duration of the process which can be obtained by lengthening the low pressure part of the barrel.

Considering the previous studies concerning the condensation [1,6,8] it is to be mentioned that they were carried out in other condensation conditions of expansion as those through nozzles and shock tubes where the rate of cooling is controlled by the parameters appearing in the dimensionless group .

7. CONCLUSION

The homogeneous condensation takes place usually at temperatures below the saturation ones. The essential condition for occurring this phenomena is to obtain a sufficient cooling of the gas. The experimentation and computation show [1,6,8] that it depends specifically on the rate of cooling. A second condition is eventually the duration of saturated state so that the condensed phase has the time for development and releasing a significant heat.

The analysis of previous theoretical and experimental results permits the definition of the best choice satisfying the last mentioned conditions.

The temperature drop of the driving gas necessary for obtaining favorable conditions of condensation is obtained by increasing projectile acceleration. On the other hand, the prolongation of the saturation state can

be attained by permitting longer projectile motion insides the barrel.

The mentioned data gives an imagination about the main features of the experimental installation for studying the influence of water vapour condensation. These conclusions encourage us to proceed the work for theoretical and experimental investigation of the condensation phenomena in these special conditions.

Acknowledement

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References:

1. Barrand J.P. et Rieutord E., " Etude de la condensation de vapeur d'eau au cours d'une détente rapide dans un long tube", Int. J. Heat Mass Transfer, Vol. 16, p 101 - 108 , (1973).
2. Chow, T.L, "Solution by Characteristics at Fixed Time Interval of the Equations of one Dimensional Unsteady Flow", Journal of Computational physics, 12, p 153 - 170 , (1973).
3. Lister M. "The Numerical Solution of Hyperbolic Partial Differential Equations by the Method of Characteristics", in Mathematical Methods for Digital Computer, Ralston and Wilf, Wiley, New - York , (1960).
4. Owczarek J.A., "Fundamentals of Gas Dynamics". International Text-book Company , U.S.A., (1964).
5. Seigel A.E, "Theory of High muzzle Velocity Guns", in interior Ballistics of Guns, Krier and summer field, Vol. 66 Progress in Astronautics and Aeronautics, AIAA, New - York, (1979).
6. Sislian J.P. Condensation of water vapour with or without a carrier gas in a shock tube, UTIAS report no 201, CN ISSN 0082 - 5255, (1975).
7. Shapiro H.A. "The Dynamics and Thermodynamics of Compressible Fluids", Voi II, Ronald, (1954).
8. WEGENER, P.P. "Gas dynamis of Expansion Flows with condensation and Homogeneous Nucleation of water vapour" in Non equilibrium Flows led. pp. WEGEWER, 1969, Vol.1, part 1., Marcel DEKKER.