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JOINT CALCULATION OF REORDER LEVEL AND REPLENISHMENT ORDER QUANTITIES IN INVENTORY CONTROL

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ABSTRACT

For true optimum operation of reorder level policy of inventory control , reorder levels and replenishment order quantities must be calculated jointly . This is because the size of one quantity directly influences the size of the other in the overall reorder level inventory situation - Assuming that demand per unit time is distributed normally and leadtime is constant . From normal distribution table, an approximate relation between percentage probability (F) of stockout and a safty stock of (k) standard deviation of demand during the leadtime is given . Using this relation in the total cost equation to calculate the stock-out term , we can get a direct value for replenishment order quantity and reorder level quantity which ensure the minimum cost required for optimum policy - An example is given to prove that this method proves the optimum solution rather than the one of separate calculation of the two parameters of this policy .

ANALYSIS

Assumptions are made that (1) demand per unit time is distributed normally , (2) leadtimes are constant - And the resultant probability of a stockout occuring, given a safty stock of (k) standard deviations of demand during the leadtime is described with a maximum error of only 3% by two linear logarithmic approximations see Fig. 1 such that :

$$\log_e F = a - bk \tag{1}$$

$$\text{or} \quad F = e^{a-bk}$$

where a=5.65 & b=2.49 for $1.3 \leq k \leq 3.2$ ($0.1\% \leq F \leq 10\%$)

a=4.08 & b=1.32 for $0 \leq k \leq 1.3$ ($10\% \leq F \leq 50\%$)

The total annual cost (including a stockout term) can then be evaluated as follows :

C = number of stockouts per year X cost of an individual stockout + number of replenishment orders placed X cost of placing an order + 1/2 average replenishment quantity X unit cost of holding + safty stock X unit cost of holding .

Thus

$$C = \frac{\bar{A}C_s\, e^{a-bk}}{100q} + \frac{\bar{A}C_o}{q} + \frac{qC_h}{2} + k\sigma_d\sqrt{\bar{L}}C_h \tag{2}$$

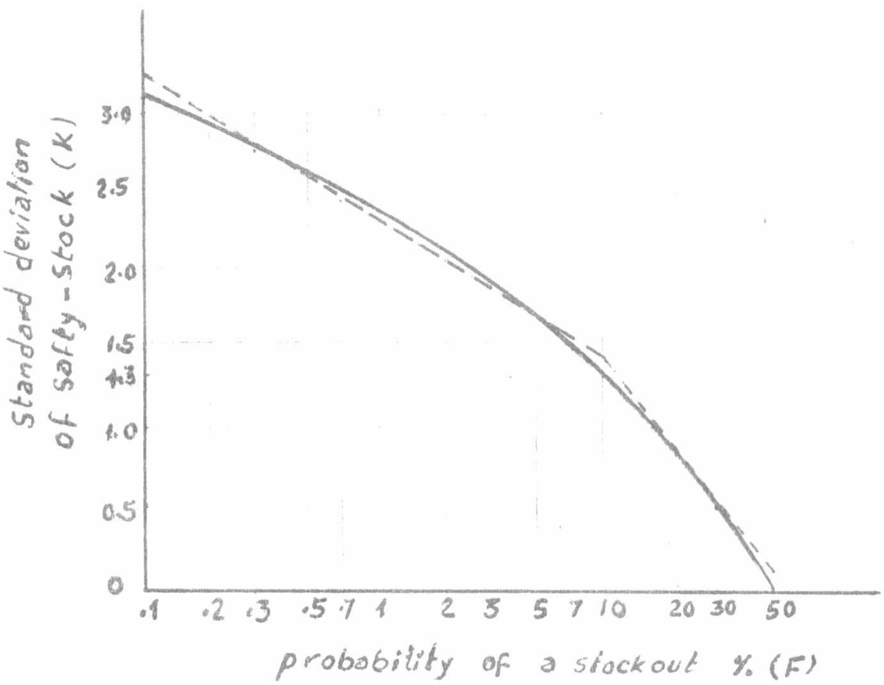

Fig. (1)

Where :

- A = the annual usage , demand or sale for inventory item .
- C_s = cost of an individual stockout (shortage) .
- k = standard normal deviate ,
- q = the replenishment order size ,
- C_o = the cost of placing a replenishment order ,
- C_h = the annual cost of holding one unit of inventory ;
- σ_d = the standard deviation of demand per unit time ,

The values of (k^*) and (q^*) that minimise C are found by differentiating C partially w.r.t. both k and q and setting these partial derivatives $\partial C/\partial k$ and $\partial C/\partial q$ simultaneously to zero .

Hence

$$\frac{-b\, A C_s\, e^{a-bk^*}}{100 q^*} + \sigma_d \sqrt{L}\, C_h = 0$$

and

$$\frac{-A C_s\, e^{a-bk^*}}{100 q^{*2}} - \frac{A C_o}{q^{*2}} + \frac{C_h}{2} = 0$$

from which :

$$q^* = \frac{\sigma_d \sqrt{L}}{b} + \sqrt{\left[\frac{\sigma_d^2 L}{b^2} + \frac{2A\, C_o}{C_h}\right]} \qquad (3)$$

The corresponding number of standard deviations of demand during the leadtime (k) required as a safty stock is given by

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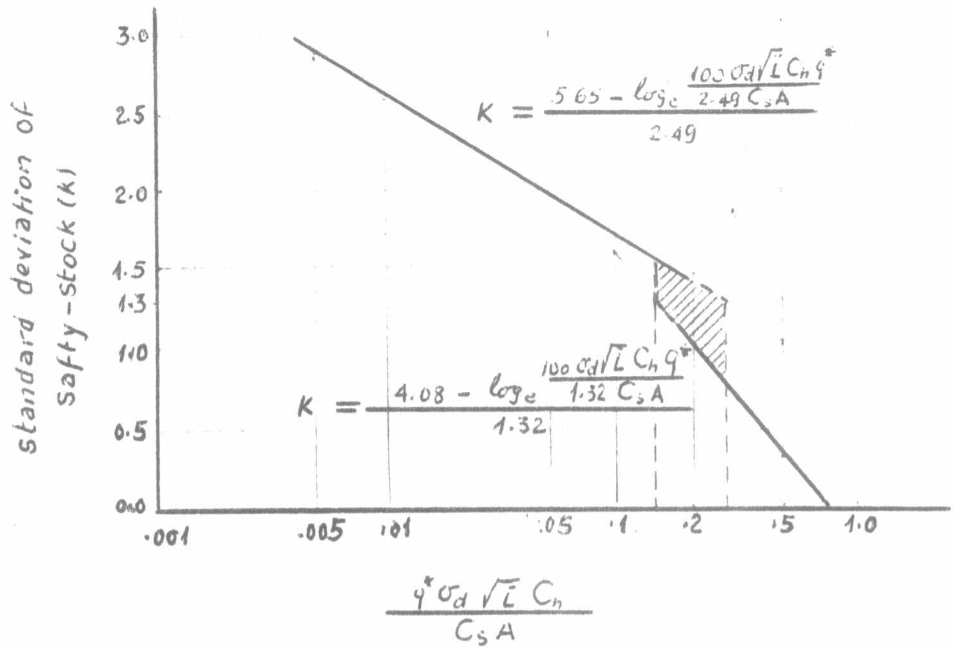

Fig. -2-

$$k^{*} = \frac{1}{b}\left[a - \frac{\log_e 100\,\sigma_d\sqrt{L}C_h\,q^{*}}{bC_s\;A}\right] \qquad (4)$$

Hence for $1.3 \leq k \leq 3.2$ $\qquad (\,0.1\% \leq F \leq 10\%\,)$

$$k^{*} = \frac{1}{2.49}\left[5.65 - \frac{\log_e 100\,\sigma_d\sqrt{L}C_h q^{*}}{2.49C_s\;A}\right]$$

and for $0 \leq k \leq 1.3$ $\qquad (\,10\% \leq F \leq 50\%\,)$

$$k^{*} = \frac{1}{1.32}\left[4.08 - \frac{\log_e 100\,\sigma_d\sqrt{L}C_h q^{*}}{1.32C_s\;A}\right]$$

These two formulae for (k^{*}) produce overlapping solutions in the region

$$0.13 < \frac{q^{*}\,\sigma_d\,\sqrt{L}\,C_h}{C_s\;A} < 0.25$$

Having found the value of (k^{*}) the reorder level required can be found using

$$M = \bar{D}\,\bar{L} + k^{*}\,\sigma_d\sqrt{L}$$

Where it cannot be assumed that the leadtime is constant, an attempt should be made to evaluate the standard deviation of demand during the leadtime as either $(\sqrt{L\sigma_d^2 + \bar{D}^2\sigma_l^2}\,)$ or from

1.1 to 1.4 times the standard deviation of demand during an average leadtime (according to Lewis ..[1]) .

Example :

Management of an enterprise consider to work with the reorder policy for its inventory control given that :-

The demand per week is distributed normally with an average value of 50 and a standard deviation of 37.5, also the lead-time is considered to be constant and with a duration of 3 weeks - Choosing a service level of 98.9 % i.e. probability of stock-out occuring to be 1.1% , from normal tables the normal standard deviate is found to be (K) = 2.3 . Taking the ordering costs to be 1.5 LE/order , the holding cost/unit/ period = 0.25 LE. and the shortage cost to be 10 LE.

Solution : (using the seperate calculation method)

The replenishment order quantity (q) = 173 units .
the reorder level quantity (M) = 300 units .
the total costs using this method = 80.80 LE.

Solution : (using the joint calculation method)

Case (A) high customer service
the replenishment order quantity (q) = 200 units (equ. 3)
stadard normal deviate (k) = 1.6 (equ. 4)
the total costs = 76.35 LE. (equ. 2)

Case (B) low customer service
the replenishment order quantity (q) = 230 units (equ. 3)
standard normal deviate (k) = 1.25 (equ. 4)
the total costs = 77.63 LE. (equ. 2) .

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