TURNOVER STIFFNESS OF BOLTED JOINT ASSEMBLY

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ABSTRACT

Bolted joint assembly is commonly used in various design problems. An optimized joint design is best suitable for one set of design parameters. This paper is concerned with the bolted joint assembly subject to a turnover load. The objective is mainly to approach as closely as possible to some appropriate mathematical analysis to the most important design parameters. These parameters are; turnover load, quality, geometry and dimensions of the two mating surfaces. Several assumptions have been taken into consideration to simplify the proposed mathematical model. Calculation of the joint stiffness and some basic optimum values would then be within the reach of designer in a minimum time and less effort.

INTRODUCTION

Machine tool rigidity may be defined by both rigidity of the parts of machine tool structure and joint stiffness. The stiffness of joints, defined by the deflection of their elements, is of great importance for precision machine tools due to the following reasons: [4]**

1. In machine tools a large number of joints are present.
2. The contact areas are of large sizes, therefore the deviations from ideal shape are inevitable.
3. Many elements of machine tool structure are of low rigidity as a result of which there is variable stress distribution over the contact area under load.

Fig. 1. shows the geometric model of a flat joint of two machined rigid elements (as for milling machines). The complex loads are transmitted across the joint interfaces, therefore one could expect that the overall static and dynamic characteristics of the machines are influenced by compliance at these joints.

The principal design criteria of joints are, therefore, the stiffness properties which must be taken into consideration by the designer, since these can influence both the short-and long-term accuracy of the machine.

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There are many publications for both fundamental and empirical nature which are concerned with the factors affecting the characteristics of the bolted joints [1, 2, 3, 4, 5, 8, 10].

Recently, the author [9] presented a mathematical model to compute the turnover stiffness. This model is based on the multi-bolted joint. Determination of joint stiffness using this model gives results related mainly to the number of bolts in the joint. These calculated values are only related to the bolt-dimensions. Also, the influence of the overall-dimensions of the joint (a_f and b_f), and the joint recess ratios (R_a and R_b) have not been yet analysed.

It is apparent that there is no comprehensive study to compute the turnover joint stiffness. Therefore, this paper is mainly concerned with the bolted joint subjected to a static turnover moment (M_y). The objective is mainly to approach as closely as possible to some appropriate mathematical analysis to the most important parameters. These parameters are; static turnover moment, geometry and dimensions of the joint, and their effect on the opening position and the turnover stiffness of the joint developed, thus presenting the basic optimum design criteria which follows in this analysis.

2. MATHEMATICAL MODEL

To determine the turnover stiffness of the connected members, the following assumptions have been taken into consideration:

- The contact stiffness between the two mating surfaces has not been taken into consideration. i.e. the members to be jointed are rigid.
- The stiffness of the jointed parts is greater than the bolt stiffness, which may be neglected [6,7].
- The normal forces in the joint are produced with the fastening bolts.

Therefore, it is clear that only the following design parameters can be analysed by using the previous assumptions:

- The position and dimensions of the joint.
- The geometry and dimensions of the joint recess.
- The magnitude of the initial load.
- The surface quality of the elements to be jointed.

Fig. 2. represent the mathematical model of the turnover loaded joint and the geometric parameters of the contact surface of the members to be jointed. These surfaces may be finished with hand scraped as usually used in the machine tool joints [8]. In this case, the normal deformation (V) due to the normal stress (σ_n) may be computed by the following relation after LEVINA. [4].

$$V = \alpha \sqrt{\sigma_n} \quad (1)$$

The following relative relations have been used to derive the mathematical model.

$$\begin{aligned} R_a &= a_{f1}/a_f & R_b &= b_{f1}/b_f \\ \gamma &= 2x_a/a_f & \theta &= 2x/a_f \end{aligned} \quad (2)$$

$$b_f(\theta) = \begin{cases} b_f & \text{for } R_a \leq |\theta| \leq 1 \\ b_f(1-R_b) & \text{for } 0 \leq |\theta| \leq R_a \end{cases} \quad (3)$$

$$v(\theta) = \frac{a_f}{2} \varphi_f(\gamma + \theta), \text{ and} \quad (4)$$

$$\sigma_n(\theta) = \frac{v^2(\theta)}{\alpha^2} = \frac{a_f^2}{4\alpha^2} \cdot \varphi_f^2(\gamma + \theta)^2 \quad (5)$$

Thus, where a rectangular joint loaded by normal force (F_1) and turnover moment (M_y), the conditions for balanced joint are (Fig.2):

$$\sum F=0 = - \int_{-1}^1 \sigma_n(\theta) \cdot b_f(\theta) \cdot \frac{a_f}{2} d\theta + F_1 \quad (6)$$

and,

$$\sum M=0 = - \int_{-1}^1 \sigma_n(\theta) \cdot b_f(\theta) \cdot \frac{a_f}{4} \cdot \theta d\theta + M_y \quad (7)$$

To simplify the mathematical model, the contact surface between the two mating elements can be distinguished by the place of the disjoin point (γ) into four cases, namely

1. The point γ lies outside the joint interface.
2. The point γ lies inside the contact surface (left-side).
3. The point γ lies within the joint recess.
4. The point γ lies inside the contact surface (right side)

It is clear that, case one has the highest rigidity disjoin point with respect to in cases 2, 3 and 4. Therefore, these cases may be neglected because it is of less interest to the designer. The following are the mathematical models when the disjoin point lies outside the joint interface, the load is subject to a completely contact surface between the two mating elements. Thus, the relation between the external moment and the co-ordinate of the disjoin point γ may be derived by the integration of equations 6 and 7 with respect to the complete joint interface.

$$\frac{M_y}{F_1 \cdot a_f} = \frac{\gamma (1 - R_a^3 R_b)}{1 - R_a^3 R_b + 3 \gamma^2 (1 - R_a R_b)} \quad (8-a)$$

Before considering effecting of the turnover moment (M_y), the normal stress (σ_n) may be considered uniformly distributed on the joint area (A_f). Thus;

$$\sigma_n = F_1 / a_f \cdot b_f (1 - R_a \cdot R_b) \quad (9)$$

Thereof,

$$\frac{M_y}{\sigma_n \cdot a_f^2 \cdot b_f} = \frac{\gamma (1 - R_a^2 \cdot R_b) (1 - R_a \cdot R_b)}{1 - R_a^3 \cdot R_b + 3 \gamma^2 (1 - R_a \cdot R_b)} \quad (8-b)$$

The angular deflection (φ_f) may be also determined using equations 7 and 8 as follows:

$$\varphi_f^2 \frac{a_f^3 \cdot b_f}{12 F_1 \cdot \alpha^2} = \frac{1}{1 - R_a^3 \cdot R_b + 3 \gamma^2 (1 - R_a \cdot R_b)} \quad (10-a)$$

$$\varphi_f^2 \frac{a_f^2}{12 \sigma_n \cdot \alpha^2} = \frac{1 - R_a \cdot R_b}{1 - R_a^3 \cdot R_b + 3 \gamma^2 (1 - R_a \cdot R_b)} \quad (10-b)$$

The relationship between φ_f and the angular deflection of the joint without recess (φ_{fo}) (putting $R_a = R_b = 0$) may be defined as:

$$\varphi_f / \varphi_{fo} = \sqrt{\frac{\gamma_o}{\gamma (1 - R_a^3 \cdot R_b)}} \quad (11)$$

$$\frac{M_y}{F_1 \cdot a_f} = \left(\frac{M_y}{F_1 \cdot a_f} \right)_o = \frac{\gamma_o}{1 + 3 \gamma_o^2} \quad (12-a)$$

and,

$$\frac{M_y}{\sigma_n \cdot a_f^2 \cdot b_f} = \left(\frac{M_y}{\sigma_n \cdot a_f^2 \cdot b_f} \right)_o = \frac{\gamma_o}{1 + 3 \gamma_o^2} \quad (12-b)$$

The rotation point of the joint interface is the point which has no normal deflection under the effect of the moment (M_y).

$$\text{i.e., } \partial v(\theta) / \partial M_y = 0 \quad (13)$$

Therefore, the deviation of the rotation point with respect to the joint centre point (0) may be generally determined from equation (6).

$$e_d = - \gamma - \varphi_f \cdot \partial \gamma / \partial \varphi_f \quad (14)$$

Lastly, the rotation point may be determined by the use of equations (8 and 10).

$$\theta_d = \frac{1 - R_a^3 \cdot R_b}{3 \gamma (1 - R_a \cdot R_b)} \quad (15)$$

From equations (8 and 10), it is clear that the relation between φ_f and M_y is non-linear. Thus, the turnover stiffness may be derived by the following partial differential.

$$C_y = \partial M_y / \partial \varphi_f \quad (16)$$

Finally, the relation between the turnover stiffness (C_y) and the angular deflection (φ_f) is given by the following equation:

$$C_y \cdot \sqrt{\frac{12 \alpha^2}{a_f^5 \cdot b_f \cdot F_1}} = \varphi_f \cdot \sqrt{\frac{a_f^3 \cdot b_f}{12 F_1 \alpha^2}} (1 - R_a^3 \cdot R_b) \left[\gamma - \frac{1 - R_a^3 \cdot R_b}{3 \gamma (1 - R_a \cdot R_b)} \right] \quad (17-a)$$

or;

$$C_y \cdot \sqrt{\frac{12 \alpha^2}{a_f^6 \cdot b_f^2 \cdot \sigma_n}} = \varphi_f \cdot \sqrt{\frac{a_f^2}{12 \alpha^2 \sigma_n}} \left[\gamma - \frac{1 - R_a^3 \cdot R_b}{3 \gamma (1 - R_a \cdot R_b)} \right] (1 - R_a^3 \cdot R_b) \quad \dots (17-b)$$

RESULTS AND DISCUSSION

The derived mathematical models have been numerically analysed and graphically represented. The following are the discussion of the design parameters of the joint.

1- The Relationship between The External Turnover Moment and the Angular Deflection:

The relationship between the turnover moment and the angular deflection is represented in Fig.3 for a constant initial load (F_1), and in Fig.4 for a constant normal stress (σ_n). Both figures are calculated for the same recess ratios ($R_a = R_b$) as dependant parameters. It shows that, the tendency of both curves is progressively increased. The non-linear relation is related to the value of the recess ratios. It shows also that the angular deflection (φ_f) increases progressively with the increase of the loading magnitude ($M_y/F_1 \cdot a_f$ or $M_y/\sigma_n \cdot a_f^2 \cdot b_f$). At a light load, it is clear that the relation between the external load and φ_f is approximately linear and is valid untill the disjoin point $\gamma = 1$. After that the angular deflection increased suddenly with the gradual increase of the external load. Also, at a constant initial load (F_1), the effect of joint recess on the

angular deflection is lighter than that at a constant normal stress.

This behaviour can be explained as follows; during the increasing of the joint recess, the normal stress and also the contact stiffness increase, but on the other hand the inertial moment of the joint area increases. This counter tendency decreases the effect of the joint recess so that it may be neglected.

2- The Relationship between the Turnover Moment and the Turnover Stiffness.

Figs.5. and 6. Show the relationship between the turnover moment (M_y) and the turnover stiffness (C_y). It is clear for all joint recess ratios that, the stiffness decreases when the moment increases. Also, it can be said that the stiffness value firstly decreases progressively untill the disjoin point $\gamma = 1$, after that it decreases degressively.

Practically, it is needed to determine the optimum dimensions of the joint surface at a constanst initial load (F_1). The reasons for this trend may be due to the simplification of the mathematical model, which is based on the following parameters:

- a- Dimensions and number of fastening bolts to be used.
- b- The max. initial tightening force of every bolt.
- c- The needed area of every used flange construction.
- d- The weakening of the elements to be jointed due to the bolt-hole.
- e- The permitted compression stress in the contact surface between the elements to be fastened.
- f- Production procedure and its economical cost.

From Fig.5, it can also be seen that, at the light loading values ($M_y/F_1 \cdot a_f \leq 0.2$), an optimum condition of the stiffness occures at $R_a = R_b \approx 0.6$, while at the heavy loading values ($M_y/F_1 \cdot a_f \geq 0.2$) the joint stiffness increases with the decreasing of the joint surface. The highest joint stiffness in this case occures at $R_a = R_b = 0$ (i.e., without joint recess). But, the designer can better use $R_a = R_b = 0.4$ to reduce the defficulties in the joint production and minimize the production cost.

3- Effect of the Joint Recess Ratio on the Turnover Stiffness:

For more comprehensive study of the effect of the joint recess on the stiffness value; it can be analysed the relation between the stiffness and their angular deflection at a constant initial load (F_1). To simplify the analysis, it is assumed that, in the region of a light external load the relation is linear (S.Fig.3.). Accordingly, substituting by the following limits $\gamma = \infty$ and $\varphi_f = 0$ in equation (8), we have;

$$C_y \sqrt{\frac{12 \alpha^2}{a_f^5 \cdot b_f \cdot F_1}} = \frac{1 - R_a^3 R_b}{\sqrt{3(1 - R_a \cdot R_b)}} \quad (18-a)$$

$$C_y \sqrt{\frac{12 \alpha^2}{a_f^6 \cdot b_f^2 \sigma_n}} = \frac{1 - R_a^3 R_b}{\sqrt{3}} \quad (18-b)$$

When $M_y/F_1 \cdot a_f = M_y/\sigma_n \cdot a_f \cdot b_f = 0$, the relation between the external moment and its angular deflection is also linear. So, using

$$C_y = M_y / \varphi_f \quad (19)$$

the linear relation between the (M_y) and the (φ_f) becomes;

$$\varphi_f \sqrt{\frac{a_f^3 \cdot b_f}{12 \alpha^2 F_1}} = \frac{M_y}{a_f \cdot F_1} \frac{\sqrt{3(1 - R_a \cdot R_b)}}{1 - R_a^3 R_b} \quad (20-a)$$

also,

$$\varphi_f \sqrt{\frac{a_f^2}{12 \alpha^2 \sigma_n}} = \frac{M_y}{a_f^2 \cdot b_f \sigma_n} \frac{\sqrt{3}}{1 - R_a^3 R_b} \quad (20-b)$$

By substituting the moment of inertia of the joint surface,

$$I_f = a_f^3 \cdot b_f (1 - R_a \cdot R_b) / 12 \quad (21)$$

in equation (20), we have

$$\varphi_f = \frac{M_y}{2 I_f} \frac{\alpha}{\sqrt{\sigma_n}} \quad (22)$$

$$\text{Thus, } C_y = 2 I_f \sqrt{\sigma_n} / \alpha \quad (23)$$

The effect of the joint recess ratio on the turnover stiffness may be evaluated by the equation of the relative stiffness (C_R). This equation can be derived from equation (18-a). Where the joint surface without recess have the ratio $R_a = R_b = 0$.

$$(C_R) f_1 = \left(\frac{C_y}{C_{y0}} \right) f_1 = \frac{1 - R_a^3 R_b}{\sqrt{1 - R_a \cdot R_b}} \quad (24)$$

This equation has been presented for the parameters R_a and R_b in Figs. 7 and 8. It is clear that, the highest relative stiffness (C_R) occurs at the ratio $R_a = 0.6$ and $R_b = 1$. The excess in stiffness with respect to the joint surface without recess reaches about 24% (Fig.7). On the other hand, at the ratio $R_a = 1$ and $R_b = 0.6$ the decrease in the stiffness reaches about 37% (Fig.8). Therefore, the position of the joint with respect to

the external moment has an effect. So, the designer must choose the convenient joint position and the joint recess ratios for his joint. Simply, for the recess ratio $R_a = R_b = 0.6$ the stiffness has clearly an optimum condition (fig.8). It exceeds by approximately 9% with respect to the closed joint surface.

4- Effect of the Turnover Moment on the Position of the Centre Point of the Joint Surface:

The relation between the displacement of the rotation point (θ_d) and the external load may be analysed by using equation (15). Fig.9. represent the limits of this displacement by neglecting the effect of the joint recess ratios (R_a, R_b). It is clear that the relation may be approximately linear in the region $0 \leq M_y/F_1.a_f \leq 0.2$. Therefore, for this region, the following linear relation can be derived:

$$\theta_d = M_y/F_1.a_f \quad (25)$$

Also, it can be seen that the displacement θ_d increases progressively after $M_y/F_1.a_f \approx 0.2$. The normal deflection of the joint centre point (at $\theta = 0$) can be derived from equation (6).

$$V(0) = 0,5 a_f \cdot \varphi_f \cdot \gamma \quad (26)$$

Fig. 10 shows the relation between $V(0)$ and the external load ($M_y/F_1.a_f$). The deflection decreases slowly untill the start of the disjoining (at $\gamma = 1$), after that it decreases quickly. In other words, in the region $0 \leq M_y/F_1.a_f \leq 0.25$ the vertical deflection of the joint centre point is mainly affected by the joint recess ratio only.

CONCLUSION

The turnover stiffness of the bolted joint assembly is greatly affected by the turnover moment and the dimensions of the joint surface. The joint recess has also a pronounced effect on the joint stiffness. Based on the results and discussion in this paper, the following optimum conditions can be recommended:

- 1- The angular joint deflection increases progressively with the increase of the turnover moment. It can also be seen that, the relation may be considered linear untill the start of the disjoining and in the load region $M_y/F_1.a_f \leq 0.2$.
- 2- For all joint recess ratios, the stiffness decreases with the moment increases. An optimum condition of the stiffness occurs at $R_a = R_b \approx 0.6$ for the loading value $M_y/F_1.a_f \leq 0.2$. Where, the stiffness increases by about 9% than the closed joint surface.

- 3- In the region $0 \leq M_y/F_1.a_f \leq 0.2$, the relation between the displacement of the joint rotation point and the moment can be considered linear.
- 4- The normal deflection of the joint center point is mainly affected by the joint recess ratio within the load region $0 \leq M_y/F_1.a_f \leq 0.25$.

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NOMENCLATURES

- A_f Joint area (mm^2).
- a_f Length of the joint (mm).
- a_{f1} Length of the joint recess (mm)
- b_f Width of the joint (mm).
- b_{f1} Width of the joint recess (mm)
- R_a Recess length ratio.
- R_b Recess width ratio.
- x_a Coordinate of the disjoin point
- δ Relative coordinate of the disjoin point.
- θ Relative coordinate to x-direction (-)
- α Factor, depending on the mechanical properties of material and surface quality of the two mating parts.

0.1	0.1
0.2	0.2
0.3	0.3
0.4	0.4
0.5	0.5
0.6	0.6
0.7	0.7
0.8	0.8
0.9	0.9
1.0	1.0
1.1	1.1
1.2	1.2
1.3	1.3
1.4	1.4
1.5	1.5
1.6	1.6
1.7	1.7
1.8	1.8
1.9	1.9
2.0	2.0
2.1	2.1
2.2	2.2
2.3	2.3
2.4	2.4
2.5	2.5
2.6	2.6
2.7	2.7
2.8	2.8
2.9	2.9
3.0	3.0
3.1	3.1
3.2	3.2
3.3	3.3
3.4	3.4
3.5	3.5
3.6	3.6
3.7	3.7
3.8	3.8
3.9	3.9
4.0	4.0
4.1	4.1
4.2	4.2
4.3	4.3
4.4	4.4
4.5	4.5
4.6	4.6
4.7	4.7
4.8	4.8
4.9	4.9
5.0	5.0
5.1	5.1
5.2	5.2
5.3	5.3
5.4	5.4
5.5	5.5
5.6	5.6
5.7	5.7
5.8	5.8
5.9	5.9
6.0	6.0
6.1	6.1
6.2	6.2
6.3	6.3
6.4	6.4
6.5	6.5
6.6	6.6
6.7	6.7
6.8	6.8
6.9	6.9
7.0	7.0
7.1	7.1
7.2	7.2
7.3	7.3
7.4	7.4
7.5	7.5
7.6	7.6
7.7	7.7
7.8	7.8
7.9	7.9
8.0	8.0
8.1	8.1
8.2	8.2
8.3	8.3
8.4	8.4
8.5	8.5
8.6	8.6
8.7	8.7
8.8	8.8
8.9	8.9
9.0	9.0
9.1	9.1
9.2	9.2
9.3	9.3
9.4	9.4
9.5	9.5
9.6	9.6
9.7	9.7
9.8	9.8
9.9	9.9
10.0	10.0

$\frac{1}{\sqrt{2}}$

(1) $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = \frac{\partial L}{\partial x}$

Fig. 2. Mathematical model of a
three-year logging cycle.

Angular Deflection $\varphi_f \sqrt{\frac{a_f^3 \cdot b_f}{12} \propto F_f$

Angular Deflection $\varphi_f \sqrt{\frac{a_f^2}{12} \propto \sigma_n$

Fig. 6. The Dependence Between the Turnover Ratio and the Stresses at a Constant Temperature.

Fig. 7. The Dependence Between the Turnover Ratio and the Stresses at a Constant Temperature.

Fig. 7. The Relation between the Joint Recess Ratio and the Stiffness.

Fig. 8. The Relation between the Joint Recess Ratio and the Stiffness.

Fig. 10. The Relation between the Deflection of the Joint Centre Point and the External Load.

Fig. 9. The Relation between the Rotation Point and the Load.