DESIGN OF HYDROSTATIC SCREW AND NUT

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ABSTRACT

This article is concerned with a proposed simple method to design an externally pressurized trapezoidal and rectangular screw and nut mechanisms. The general lubrication assumptions are applied to the governing equations and continuity equation of the lubricant. The reduced equations are then used in an assumed double film annular bearing which is considered to be equivalent to one turn of the hydrostatic screw. It is found that the performance characteristics of a hydrostatic screw such as power consumed, volume flow rate, and static stiffness can be estimated in an easy manner. Furthermore, the optimum operations conditions can easily be determined.

INTRODUCTION

A power screw is a mechanism that is used in machinery to change the angular motion into linear motion. It is usually utilized in power transmission applications. When using conventional lead screw and nut, many problems such as power losses due to friction, wear, and stick-slip phenomena were found. It has been realized that using of externally pressurized oil lead screw would overcome most of those problems.

In this investigation, an attempt is made to introduce a proposed method to design a hydrostatic screw. A system of double film annular bearings is used and considered to be equivalent to one screw turn.

This approach will enable a designer to predict the performance characteristics of a hydrostatic screw such as power, volume flow rate of the lubricant, and static stiffness of the fluid films. The optimum operating conditions are also presented.

ANALYSIS

Figure (1) shows a section of one side of a hydrostatic screw. This screw may be represented by a system of double film annular bearing which is shown in Fig. (2).

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For clarity of the analysis, one of two opposed pads of this system is presented in Fig. (3), as a single pad annular bearing. From Figs (1 and 3), the following relationships can be drawn:

\[ d_1 = d_{\text{inn}} \] 
\[ d_o = \sqrt{d_{\text{inn}}^2 + 2(d_{\text{out}} \sec \varphi + d_{\text{inn}} \sec \varphi)b \sec \alpha} \] 
\[ d_2 = \frac{(d_{\text{inn}} + \sqrt{d_{\text{inn}}^2 + 2(d_{\text{out}} \sec \varphi + d_{\text{inn}} \sec \varphi)b \sec \alpha})^2}{2\left[d_{\text{inn}} + 2(d_{\text{out}} \sec \varphi + d_{\text{inn}} \sec \varphi)b \sec \alpha\right]} - \frac{2\left[d_{\text{out}} \sec \varphi + d_{\text{inn}} \sec \varphi\right]t \sec \alpha}{2\left[d_{\text{inn}} + 2(d_{\text{out}} \sec \varphi + d_{\text{inn}} \sec \varphi)b \sec \alpha\right]} \] 
\[ d_3 = (d_{\text{inn}} + \sqrt{d_{\text{inn}}^2 + 2(d_{\text{out}} \sec \varphi + d_{\text{inn}} \sec \varphi)b \sec \alpha}) \] 
\[ + \frac{(d_{\text{inn}} + \sqrt{d_{\text{inn}}^2 + 2(d_{\text{out}} \sec \varphi + d_{\text{inn}} \sec \varphi)b \sec \alpha})^2}{2\left[d_{\text{inn}} + 2(d_{\text{out}} \sec \varphi + d_{\text{inn}} \sec \varphi)b \sec \alpha\right]} - \frac{2\left[d_{\text{out}} \sec \varphi + d_{\text{inn}} \sec \varphi\right]t \sec \alpha}{2\left[d_{\text{inn}} + 2(d_{\text{out}} \sec \varphi + d_{\text{inn}} \sec \varphi)b \sec \alpha\right]} \]
The performance characteristics of the equivalent annular bearing (for the top and lower pad of the equivalent system that shown in Fig. (2) can be obtained by applying the following assumptions to the Navier-Stoke's equations:
1- The flow is incompressible.
2- The flow is one dimensional "radial flow"
3- The inertia forces of the lubricant are neglected.
4- The body forces of the lubricant are neglected.
The reduced Navier-Stoke's equation will be:
\[
\frac{\partial p}{\partial r} = \frac{1}{\mu} \frac{\partial^2 v_r}{\partial z^2}
\]

The load carrying capacity of the bearing is
\[
W = \int p \, dA
\]
\[
W = p_1 \pi (R_o^2 - R_1^2) \cdot \frac{1}{2} \left[ \frac{R_o^2 - R_3^2}{2(R_o^2 - R_1^2)} \log_e \frac{R_o}{R_3} - \log_e \frac{R_2}{R_3} \right]
\]

The volume flow rate can be determined as
\[
Q = \int v_r \, dA = \int v_r \pi \, dz
\]
\[
Q = p_1 \cdot a_f \cdot \frac{h^3}{\mu} \cdot \pi \left[ \frac{1}{a_f} \log_e \frac{R_o}{R_3} + \frac{1}{\log_e \frac{R_2}{R_1}} \right]
\]

The static stiffness of one side of the double film annular bearing, may be written as
\[
\lambda = \frac{3 p_s \cdot a_f \cdot A}{h} k_s
\]

Where \(k_s\) is the stiffness coefficient and depends mainly on the method of compensation.

i) For orifice plates compensation
\[
k_s = \left( \frac{p_1}{p_s} \right) \cdot 2 \left( 1 - \frac{p_1}{p_s} \right) / \left( 2 - \frac{p_1}{p_s} \right)
\]

ii) For capillary tube compensation:
\[
k_s = \frac{p_1}{p_s} (1 - \frac{p_1}{p_s})
\]

iii) For flow control valve compensation:
\[
k_s = \frac{p_1}{p_s}
\]

SCREW PERFORMANCE

The performance of a screw is similar to that for a set of double film opposed annular bearings. The number of these bearings in that set is equal to the number of screw turns.
The volume flow rate required to separate lead screw from nut can be given by:

$$Q_s = n \left( Q_{tf} + Q_{ff} \right)$$  \hspace{1cm} (14)

The static stiffness is

$$\lambda_s = n \left( \lambda_{tf} + \lambda_{ff} \right) \cos^2 \alpha$$  \hspace{1cm} (15)

Since both the top and lower flanks take the same geometry, equation (14) can take the following form:

$$\lambda_s = \frac{3 \rho_s \cdot a_f \cdot A}{h} \left( \frac{k_{ss}}{k_{sf} + k_{sp}} \right) \cos^2 \alpha$$  \hspace{1cm} (16)

where

$$k_{ss} = \left( k_{sf} + k_{sp} \right)$$

The power losses are occurred due to pumping action and friction, i.e.

$$E_{losses} = E_{pumping} + E_{friction}$$  \hspace{1cm} (17)

The pumping loss is given by:

$$E_{pumping} = P_1 \cdot Q_s$$  \hspace{1cm} (18)

whereas the frictional loss is given by

$$E_{frictional} = \frac{n \cdot N^2}{65.9688} \left( \frac{h_{tf} + h_{ff}}{h_{tf} \cdot h_{ff}} \right)$$ \hspace{1cm} (19)

Equations (14 through 19) are valid in the analysis of trapezoidal hydrostatic screw, while in the case of rectangular hydrostatic screw the value of \((\alpha)\) is equal to zero. However, in the latter case a possibility of radial friction may take place.

RESULTS

Figure (4) shows the effect of the pressure ratio \(P_1 \cdot P_2\) on the coefficient \(k_s\) at different compensation methods. It can be seen that the use of flow control valve, as a compensating element, makes the static stiffness coefficient of the externally pressurized bearing increases linearly as the pressure ratio increases. In case of using of sharp-edged orifice, the maximum stiffness coefficient occurs at pressure ratio of 0.61. While using the capillary tube restrictors, the maximum stiffness coefficient takes place at a pressure ratio equals to 0.5.

These results are valid for a constant supply pressure.

Figure (5) presents the relation between the stiffness coefficient \(k_s\) and the fluid film thickness \(h_{tf} + h_{ff} = 100 \mu m\) using the three different restrictors for various pressure ratios. It clear from this figure that the minumum value of \(k_s\) occurs at \(h_{tf} = h_{ff}\). Also, it can be seen for a certain film thickness that the stiffness coefficient increases as pressure ratio increases, when using flow control valve as a restrictor. Moreover, for a constant film thickness, the maxima stiffness coefficients occur at pressure ratios 0.5 and 0.61 when utilizing capillary tube and sharp-edged orifice restrictors, respectively. Figure (6) shows the relation between
the friction loss and the film thickenss for $n N_\mu \frac{2}{65.9688} \frac{(R^4 - R_i^4)}{1} + \frac{(R_j^4 - R^4)}{1} = 1$. The same figure presents the relation between volume flow rate and the film thickness for

\[
\frac{n \mu P_1}{6} \left[ \frac{1}{\gamma_n (R_0/R_3)} + \frac{1}{\gamma_n (R_j/R_i)} \right] = 1.
\]

It can be seen that at $(h_{tf} = h_{ff}^*)$, both the volume flow rate and friction loss are minimum.

**CONCLUSIONS**

For hydrostatic screw that has the optimum operating condition (minimum power losses), for a certain pressure and geometry, both the top and lower flank film thicknesses must be equal. This case, however, gives the minimum static stiffness.

To obtain maximum screw stiffness, it is preferred to use flow control valves as restrictors, using pressure ratio equals to unity.

**REFERENCES**


**NOMENCLATURE**

- $W$: load carrying capacity.
- $Q$: volume flow rate of the equivalent bearing.
- $N$: r.p.m.
- $Q_s$: required volume flow rate of the screw.
- $E_{loss}$: energy losses.
- $d_{in}$: internal screw diameter.
- $d_{in}$: internal recess diameter.
- $d_{out}$: outer recess diameter.
- $d_{out}$: outer screw diameter.
- $R_i$: internal radius of equivalent annular bearing.
- $b = \frac{(d_{out} - d_{in})}{2}$
- $d_{in}$: inner diameter of the nut.
- $R_2$: internal recess radius of equivalent annular bearing.
- $R_3$: outer recess radius of equivalent annular bearing.
- $R_0$: outer radius of equivalent annular bearing.
- $L$: screw pitch.
- $V_r$: radial velocity of lubricant flow.
- $h_{ff}$: fluid film thickness.
- $P_s$: supply pressure.
- $P_{af}$: inlet recess pressure.
- $\phi$: shape factor.
number of screw turns.
n
lower flank.

\( \xi_f \)
top flank.

\( \xi_t \)
screw flank angle.

\( \varphi \)
screw angle \( \pi d/L \)

\( \mu \)
lubricant viscosity.

\( \lambda \)
static stiffness of the equivalent bearing.

\( \lambda_s \)
static stiffness of the screw.

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**Fig. (4) Stiffness Coefficient For Single Pad Annular Pad Bearing.**
Fig. (5) Stiffness Coefficient of Double Film Bearing
Fig (6) Hydrostatic Screw Performance

\[ \frac{n N \nu}{65.9688} \left[ \left( R_2 - R_i \right)^4 + \left( R_o - R_3 \right)^4 \right] = 1 \]

\[ \frac{n \pi \bar{P}_t}{6 \mu} \left[ \frac{1}{\ln (R_o / R_i)} + \frac{1}{\ln (R_2 / R_1)} \right] = 1 \]