PREDICTION OF THE TRANSMISSION LOSS CHARACTERISTICS
OF THE MUFFLER REVERSING SECTION

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ABSTRACT

A one-dimensional mathematical model that predicts the attenuation characteristics of the muffler reversing section has been developed. The model is based on the generalized four-pole-analogy of the propagation of acoustic plane waves in two-in-parallel tubing configuration¹. Transmission loss measurements using the two microphone random excitation technique² were carried out on four different samples of the muffler reversing section. Comparison between measured values of the transmission loss and those predicted by the mathematical model demonstrate a reasonably good overall agreement. However, shifts of the calculated transmission loss peaks at some frequencies reflect slight deviation between theory and experiment. Closer agreement between theoretical and experimental results can be obtained by developing a more complicated three dimensional model.

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INTRODUCTION:

The complex problem of predicting acoustic behaviour of mufflers has been tackled by several researches. An understanding of the parameters influencing the acoustic characteristics of the reversing section and its coupling with the other muffler parts can lead to a better control of the noise levels transmitted by the muffler. This study is an attempt to use a simple model to solve this complex problem. The generalized four-poleparameter technique applied to the in-parallel tubing configurations has been adopted. The application of this simple analogy to the muffler reversing section is shown to be practically acceptable and technically sound.

Systems under study

Two systems with different tubing configurations and bend radius requirements are under study. The first system has its termination centerlines spacing at 5.50 inches (140 mm) while the second has it at 2.75 inches (70 mm). The tubing used to form the described systems is of 2.00 inches (51 mm) outside diameter and of nominally 0.036 inches (.9 mm) wall thickness. Two reversing chamber samples are provided for each system. One such section is elliptic and the other is of rectangular cross section with the dimensions shown in Figs.(1) and (2).

Systems simplification and the application of the two in-parallel tubing generalized model:

The acoustic characteristics of the system can be predicted provided that the following assumptions are fulfilled:

- 1- Linear plane theory is adopted.
- 2- The mathematical model is one-dimensional. Only wave propagation in the axial (x) direction of pipe (a) and (y) direction of the reversing chamber (b) is considered.
- 3- The louvered pipes - theoretically - shrink to zero diameter with no considerable variation in the acoustic pressure across the pipe radius.

4- Conservative system with no losses.

5- Effects of higher order modes are neglected.

The system has been simplified to fit the two in-parallel tubing configuration in the steps demonstrated in Fig.(3). An expression of transmission loss characteristics of such configuration has been achieved¹ and proven to be:

$$T.L.=20 \log_{10} \left[\frac{(A_1 B_2 + A_2 B_1) + \frac{s}{\rho c} (A_2 B_2) + \frac{\rho c}{s} (A_2 B_3 + A_3 B_2 + A_1 B_4 + A_4 B_1 - 2) + (A_2 B_4 + A_4 B_2)}{2(A_2 + B_2)} \right] \dots\dots\dots (1)$$

where: (A_1, A_2, A_3, A_4) and (B_1, B_2, B_3, B_4) are - respectively - the four pole parameters relating acoustic pressure and volume velocity at the inlet and outlet of the pipe (a) and the reversing chamber (b).

1- The four pole parameters between points (1a) and (2a) of pipe (a) are:

$$\begin{aligned} A_1 &= \cos k \ell_a \\ A_2 &= i \frac{\rho c}{s_a} \sin k \ell_a \\ A_3 &= i \frac{s_a}{\rho c} \sin k \ell_a \\ A_4 &= \cos k \ell_a \end{aligned} \dots\dots\dots (2)$$

2- The four pole parameters between points (1b) and (2b) of the reversing chamber (b) could be modeled as a four pole network as shown in Figs.(4) and (5). The four-pole-network of the reversing chamber (b) was found to be:

(1) For Model I*

$$\begin{bmatrix} P_{1b} \\ U_{1b} \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z_L}{Z_{c(1)}} & Z_L \\ \frac{1}{Z_{c(1)}} & 1 \end{bmatrix} \times \begin{bmatrix} \cos K \ell_{d_1} & i \frac{\rho c}{S_1} \sin K \ell_{d_1} \\ i \frac{S_1}{\rho c} \sin K \ell_{d_1} & \cos K \ell_{d_1} \end{bmatrix} \times \begin{bmatrix} \cos K \ell_{d_2} & i \frac{c}{S_2} \sin K \ell_{d_2} \\ i \frac{S_2}{\rho c} \sin K \ell_{d_2} & \cos K \ell_{d_2} \end{bmatrix} \\
 \times \begin{bmatrix} \cos K \ell_{d_3} & i \frac{\rho c}{S_1} \sin K \ell_{d_3} \\ i \frac{S_1}{\rho c} \sin K \ell_{d_3} & \cos K \ell_{d_3} \end{bmatrix} \times \begin{bmatrix} 1 & Z_L \\ \frac{1}{Z_{e(2)}} & 1 + \frac{Z_L}{Z_{e(2)}} \end{bmatrix} \begin{bmatrix} P_{2b} \\ U_{2b} \end{bmatrix} \dots (3.1)$$

(1) For Model II*

$$\begin{bmatrix} P_{1b} \\ U_{1b} \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z_L}{Z_{c(1)}} & Z_L \\ \frac{Z_L}{Z_{c(1)}} & 1 \end{bmatrix} \times \begin{bmatrix} \cos K \ell_d & i \frac{\rho c}{S_1} \sin K \ell_d \\ i \frac{S_1}{\rho c} \sin K \ell_d & \cos K \ell_d \end{bmatrix} \\
 \times \begin{bmatrix} 1 & Z_L \\ \frac{1}{Z_{e(2)}} & 1 + \frac{Z_L}{Z_{e(2)}} \end{bmatrix} \begin{bmatrix} P_{2b} \\ U_{2b} \end{bmatrix} \dots (3.2)$$

By simple matrix multiplication, a direct relation between P_{1b} , U_{1b} , P_{2b} , U_{2b} is found and the four-pole-parameters (B_1 , B_2 , B_3 , B_4) are calculated. Substituting for the values of (A_1 , A_2 , A_3 , A_4) and (B_1 , B_2 , B_3 , B_4) obtained from equation (2), (3.1) or (3.2) into equation (1), an expression of the transmission loss characteristics of the system is obtained.

Comparison between predicted and measured transmission loss characteristics and discussion of results:

Transmission loss characteristics were measured by applying the two-microphone random excitation technique². The experiment set-up and instrumentation is shown in Fig.(6).

The predicted and measured transmission loss characteristics are presented in Figs.(7) through (10). The results show:

- (a) For Model I, a reasonably good agreement is obtained between the predicted and measured transmission loss characteristics of the rectangular sample [Fig.(7)]. However, the results of the elliptic sample demonstrate a downward shift in the predicted attenuation peaks with an increase of their magnitude for frequencies between 700 and 1400 Hz [Fig.(8)].
- (b) For Model II, the predicted transmission loss peaks of the rectangular sample are shifted upward for frequencies between 1200 and 1500 Hz [Fig.(9)]. The elliptic sample results demonstrate a much better agreement [Fig.(10)].

The slight variations between predicted and measured values of transmission loss characteristics are attributed to:

- 1- Model over-simplification and the assumption that the perforated tubes can be shrunk to zero radius.
- 2- Neglecting higher order modes of wave propagation in the reversing chamber.
- 3- Neglecting three dimensional effects.
- 4- Orientation of the lower patch samples in the perforated tubes.
- 5- The long straight pipes used in the experimental setup are largely resonance-dependent and might have affected the measured characteristics.

Conclusions:

In the light of the relatively close agreement between the proposed theory and experimental results, the following conclusions may be drawn:

- (1) The application of the four-pole-analogy of the two-in-parallel tubing configuration to the muffler reversing section is technically sound.
- (2) From the practical point of view, the simplification of wave propagation in the muffler reversing section by a one-dimensional model may well be accepted. Closer agreement between predicted and measured acoustic characteristics could only be obtained with the development of more complicated three dimensional model.

Nomenclature:

A_i = Four pole parameters of the bend $i = 1, 2, 3, 4$.

B_j = Four pole parameters of the reversing chamber $j = 1, 2, 3, 4$.

c = Velocity of sound.

f = Frequency.

K = Wave number.

ℓ = Pipe or branch length.

P = Acoustic pressure.

S = Cross sectional area.

U = Volume velocity.

Z = Acoustic impedance.

ρ = Average density of sound-conducting medium.

ρc = Specific acoustic impedance.

Appendix:(a) Development of the four-pole-network
of the reversing chamber

Boundary conditions at sections (1) and (2)

$$\begin{aligned} U_{1b} &= U_{c(1)} + U_{d(1)} \\ U_{2b} &= U_{d(2)} - U_{e(2)} \end{aligned} \quad \left. \vphantom{\begin{aligned} U_{1b} &= U_{c(1)} + U_{d(1)} \\ U_{2b} &= U_{d(2)} - U_{e(2)} \end{aligned}} \right\} \text{Volume velocity}$$

$$\begin{aligned} P_{c(1)} &= P_{d(1)} = P_{(1)} \\ P_{d(2)} &= P_{e(2)} = P_{(2)} \end{aligned} \quad \left. \vphantom{\begin{aligned} P_{c(1)} &= P_{d(1)} = P_{(1)} \\ P_{d(2)} &= P_{e(2)} = P_{(2)} \end{aligned}} \right\} \text{Pressure}$$

(1) Four pole parameters between pipe (1b) centerline and
section (1)

$$Z_L = \frac{P_{1b} - P_{c(1)}}{U_{1b}}$$

$$P_{1b} = P_{c(1)} + U_{1b} Z_L$$

$$P_{1b} = P_{d(1)} + U_{1b} Z_L$$

But:

$$U_{1b} = U_{c(1)} + U_{d(1)}$$

$$U_{1b} = \frac{P_{c(1)}}{Z_{c(1)}} + U_{d1}$$

$$P_{1b} = \left(1 + \frac{Z_L}{Z_{c(1)}}\right) P_{d(1)} + (Z_L) U_{d(1)} \quad (A-1)$$

$$U_{1b} = \left(\frac{1}{Z_{c(1)}}\right) P_{d(1)} + U_{d(1)} \quad (A-2)$$

Where:

Z_L = Acoustic impedance of the perforated pipe

$Z_{c(1)}$ = Acoustic impedance of the side branch (c) at section (1)

$$= -i \frac{\rho c}{S_1} \cot K \ell_c$$

(2) Four pole parameters between sections (1) and (2)

(i) For Model I:

$$\begin{bmatrix} P_{d(2)} \\ U_{d(2)} \end{bmatrix} = \begin{bmatrix} \cos K \ell_{d1} & i \frac{\rho c}{S_1} \sin K \ell_{d1} \\ i \frac{S_1}{\rho c} \sin K \ell_{d1} & \cos K \ell_{d1} \end{bmatrix} \times \begin{bmatrix} \cos K \ell_{d2} & i \frac{\rho c}{S_2} \sin K \ell_{d2} \\ i \frac{S_2}{\rho c} \sin K \ell_{d2} & \cos K \ell_{d2} \end{bmatrix} \\ \times \begin{bmatrix} \cos K \ell_{d3} & i \frac{\rho c}{S_1} \sin K \ell_{d3} \\ i \frac{S_1}{\rho c} \sin K \ell_{d3} & \cos K \ell_{d3} \end{bmatrix} \begin{bmatrix} P_{d(1)} \\ U_{d(1)} \end{bmatrix} \dots\dots\dots (A-3)$$

(ii) For Model II:

$$\begin{bmatrix} P_{d(2)} \\ U_{d(2)} \end{bmatrix} = \begin{bmatrix} \cos K \ell_d & i \frac{c}{S_1} \sin K \ell_d \\ i \frac{S_1}{\rho c} \sin K \ell_d & \cos K \ell_d \end{bmatrix} \begin{bmatrix} P_{d(1)} \\ U_{d(1)} \end{bmatrix} \dots\dots\dots (A-4)$$

(3) Four-pole-parameters between section(2) and pipe (2b) centerline:

$$Z_L = \frac{P_{d(2)} - P_{2b}}{U_{2b}}$$

$$P_{d(2)} = P_{2b} + Z_L U_{2b}$$

But:

$$U_{2b} = -U_{e(2)} + U_{d(2)}$$

$$U_{d(2)} = U_{e(2)} + U_{2b}$$

$$U_{d(2)} = \frac{P_{e(2)}}{Z_{e(2)}} + U_{2b}$$

But $P_{e(2)} = P_{d(2)}$

$$U_{d(2)} = \frac{P_{d(2)}}{Z_{e(2)}} + U_{2b}$$

$$= \frac{1}{Z_{e(2)}} (P_{2b} + Z_L U_{2b}) + U_{2b}$$

$$P_{d(2)} = P_{2b} + (Z_L) U_{2b} \quad \dots\dots\dots (A-5)$$

$$U_{d(2)} = \left(\frac{1}{Z_{e(2)}}\right) P_{2b} + \left(1 + \frac{Z_L}{Z_{e(2)}}\right) U_{2b} \quad \dots\dots\dots (A-6)$$

Where:

$Z_{e(2)}$ = The acoustic impedance of the side branch (e)
at section(2)

(i) For Model I:

$$Z_{e(2)} = -i \frac{\rho c}{S1} \cot K \ell_e$$

(ii) For Model II:

$$Z_{e(2)} = \frac{P_e(2)}{U_{e(2)}}$$

But:

$$\begin{bmatrix} P_{e(2)} \\ U_{e(2)} \end{bmatrix} = \begin{bmatrix} \cos K \ell_{e1} & i \frac{\rho c}{S1} \sin K \ell_{e1} \\ i \frac{S1}{\rho c} \sin K \ell_{e1} & \cos K \ell_{e1} \end{bmatrix} \times \begin{bmatrix} \cos K \ell_{e2} & i \frac{\rho c}{S2} \sin K \ell_{e2} \\ i \frac{S2}{\rho c} \sin K \ell_{e2} & \cos K \ell_{e2} \end{bmatrix} \\ \times \begin{bmatrix} \cos K \ell_{e3} & i \frac{\rho c}{S1} \sin K \ell_{e3} \\ i \frac{S1}{\rho c} \sin K \ell_{e3} & \cos K \ell_{e3} \end{bmatrix} \begin{bmatrix} P_{e3} \\ U_{e3} \end{bmatrix} \quad \dots\dots\dots (A-7)$$

$$\begin{matrix} P_{e(2)} & & E_1 & E_2 & P_{e3} \\ U_{e(2)} & = & E_1 & E_4 & U_{e3} \end{matrix}$$

$$P_{e(2)} = E_1 P_{e3} + E_2 U_{e3}$$

$$U_{e(2)} = E_3 P_{e3} + E_4 U_{e3}$$

where: (E_1, E_2, E_3, E_4) are obtained from equation (A-7) by simple matrix multiplication

$$\text{But } U_{e3} = 0$$

$$P_{e(2)} = E_1 P_{e3}$$

$$U_{e(2)} = E_3 P_{e3}$$

$$Z_{e(2)} = \frac{P_{e(2)}}{U_{e(2)}} = \frac{E_1}{E_3}$$

The four-pole-parameters of the reversing chamber are obtained by simple matrix multiplication of the parameters calculated for the three zones [equations 1, 2, and 3].

(b) Acoustic impedance of the perforated tubes:

The acoustic impedance (Z_L) of the perforated tubes was estimated experimentally by Sullivan⁴ as follows:

If ζ_s = Specific acoustic impedance of the louver patch sample

θ_s = Specific resistance ratio of the louver patch sample [0.16]

X_s = Specific reactance ratio of the louver patch sample [1.29×10^{-3}]

N_s = Number of louvers in the sample [6]

N_t = Number of louvers in the tube [40]

$$S_s = \text{Louver sample area [2.49 in (61.0 mm)]}$$

$$Z_L = \left(\frac{\rho c}{S_s} \right) \left(\frac{N_s}{N_t} \right) (\zeta_s)$$

$$Z_L = \left(\frac{\rho c}{S_s} \right) \left(\frac{N_s}{N_t} \right) (\theta_s + i X_s)$$

For the tubes used, this equation becomes:

$$Z_L = \left(\frac{\rho c}{2.49} \right) \left(\frac{6}{40} \right) (0.16 + i \times 1.29 \times 10^{-3} f)$$

Acknowledgement:

The author acknowledge with gratitude the assistance and supportive supervision of Professors M.J Crocker and J.W. Sullivan of the Ray W. Herrick Laboratories at Purdue University.

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FIG. (1) Dimensions of Model (II) for the samples under study

() mm

FIG.(2) Dimensions of model (III) for the samples under study

() mm

FIG.(3) Simplification of the system to fit the two-in-parallel tubing configuration

FIG. (4) Simplification of the reversing chamber.

FIG. (5) Four pole network representing the reversing chamber.

FIG. (6) Experimental set-up for the measurement of transmission loss using the two-micropone random excitation technique.

FIG. (7) Comparison between predicted and measured transmission loss characteristics for the rectangular sample of model I

FIG. (8) Comparison between predicted and measured transmission loss characteristics for the elliptic sample of model I

FIG. (9) Comparison between predicted and measured transmission loss characteristics for the rectangular sample of model II

FIG. (10) Comparison between predicted and measured transmission loss characteristics for the elliptic sample of model II