MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

STATIC STIFFNESS OF THE BOLTED CONNECTIONS SUBJECT TO THE NORMAL AND SHEAR LOADS.

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ABSTRACT

This paper discusses the normal and shear stiffness of the bolted connections. It has shown that the shear stiffness can be determined from the parameters that define the normal stiffness of the machined surfaces. The factors affecting the static stiffness have been investigated. A mathematical analysis of the design parameters have been developed. They are ; normal and shear loads, geometry and dimensions of the two mating surfaces. Determination of the static joint stiffness and some basic optimum values would then be within the reach of the designer.

1- INTRODUCTION

Static stiffness of the bolted connections, defined by the deflection of their elements, is of great importance for precision machine tools. The deflection of a specimen consisting of two parts connected by a joint face is composed of the deflection of the solid material, the deflection of the surface asperities of the joint interface and waves on the joint surface.

The relationship between the normal stress "σ_n" and the normal deformation "δ_n" Can be written as : [7]

$$\delta_n = \alpha . \sigma_n^m \qquad (1)$$

Where α and m are constants depending upon the type of materials and the surface finish (Table 1).

The shear compliance was analysed by Kirsanova [6] and for repeated loadings; the relationship between shear stress "σ_s" and shear deformation "δ_s" was given by :

$$\delta_s = K_s . \sigma_s \qquad (2)$$

Where K_s is the shear factor in $\mu m\ mm^2/N$. Also, he reported that the shear compliance of the machined surfaces is dependent upon the surface finish

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and that it decreases with the increase of the normal stress. From the results reported in references [1 , 6] , it can be seen that the relationship between the shear compliance and the normal stress can be written as

$$K_s = R / \sigma_n^s \qquad (3)$$

Where R and s are constants dependent on the joint materials and surface finish (Table 1). The effect of surface finish on the measured shear compliance of C.I and steel surfaces is as shown in Fig. 1.

To determine the static stiffness properties, it is necessary to know all design parameters influencing the stiffness magnitude. These parameters may be :

1- Material properties of the parts to be connected.
2- Dimensions and geometry of the joint contact surface.
3- Roughness and flatness of the machined mating surfaces.

The parameters affecting the stiffness of the machined surfaces have been amalysed before in references [1 , 4 , 8 , 10]. More recently details can be found in references [5 , 9] . It was found that, there is no previous work to compute the static joint stiffness due to the mromal and shear loads. Therefore, the objective of this paper is mainly to approach as closely as possible to some appropriate mathematical model to calculate the stiffness using design parameters of the joint.

2- NORMAL STIFFNESS OF THE JOINT

2.1 : Factors Affecting the Normal Stiffness

From equation (1) it can be seen that the magnitudes of the constants α and m influence the normal static stiffness. These constants depend on ;

a- The mating surface material : In general, when different materials are used, the constant "α" of equation (1) varies inversely with the modulus of elasticity of the material, while the constant "m" remains at approximatelly 0.5 [1] .

b- Surface finish and lay orientation : The roughness of the surface of a given material depends on the machining process and cutting conditions. Theoretical models of contact conditions have been considered and the results showed that the surface stiffness is increased with the reduction in surface roughness. Experimental investigations [2] using mild steel specimens showed that, the stiffness is inversely proportional to the surface roughness.

c- Hardness : The effect of hardness on stiffness has been investigated using cast-iron specimens at relatively low interface pressures. The results indicated that the hardness had no effect on surface stiffness. A similar investigation was carried out [3] which included different surfaces finish and contact stresses up to 160 N/mm^2. It appears that elastic stiffness increases with decreased hardness.

d- Flatness deviation and surface size : In practice the flatness deviation of a surface will increase with the size of the surface. An investigation [4] on mild steel specimens of constant area in the form of an annulus, with different outside dimeters showed that the flatness

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deviation is increased with the size of the specimen. Recent work by Schofield [8] suggested that the stiffness of large surfaces which were subjected to flatness deviation, could be increased by the use of rougher surfaces becouse it will be less sensitive to flatness deviation.

2.2 : Mathematical Model :

It is apparent from the factors discussed previously that the stiffness would be proportional to the apparent area of the contact surfaces provided that the surfaces are flat. Therefore, the stiffness of a flat joint can be determined directly from the dimensions and the geometry of the joint contact surfaces. The exponential relationship (1) between the normal stress "σ_n" and the normal deflection "δ_n" may be used to compute the normal stiffness, where ;

$$C_n = \partial F / \partial \delta = \sigma_n^{1-m} \cdot A_F / \alpha \cdot m \qquad (4)$$

For hand-scraped contact joint surfaces (cast-iron) with h = 6-8 um and z = 2-3 Spots/cm^2 as usually used in machine tools, the constant "m" may be taken about 0.5 ; and when one of the joint surfaces recessed (see Fig. 2), the normal stiffness may be calculated by the following equation.

$$C_n = \frac{2\sqrt{\sigma_n}}{\alpha} a_f \cdot b_f (1 - R_a \cdot R_b) \qquad (5)$$

3- JOINT STIFFNESS UNDER SHEAR LOAD

Fig. 2 shows the geometrical model of the joint with the shear loads (F_x , F_y). To derive the proposed mathematical model, it may be assumed that, the deflection of the elements to be connected is smaller than the joint shear deformation. It may, thereforem be neglected. Equation (2) presents the linear relationship between the shear deformation and the applied shear stress, which may be calculated as ;

$$\sigma_s = F_s / A_F \qquad (6)$$

Therefore, the shear stiffness of the joint may be calculated with the following linear equation;

$$C_s = F_s / \delta_s = A_F / K_s \qquad (7)$$

The factor K_s is dependent on the normal stress (equation 3) therefore, the shear stiffness may be calculated with the following equation.

$$C_s = \frac{a_f \cdot b_f}{R} \sqrt{\sigma_n} (1 - R_a \cdot R_b) \qquad (8\text{-a})$$

or,

$$C_s = \frac{a_f \cdot b_f}{R} \sqrt{F_i (1 - R_a \cdot R_b)} \qquad (8\text{-b})$$

4- COMPARISON AND DISCUSSION

To compare the calculated stiffness with the available experimental results [5].

The experiment has been carried out to study the static deformation behaviour of four types of models. These models were fabricated by welding process. Fig. 3 shows the structure model I-C , which is fastened with the base by 12 bolts M8. The base was manufactured from steel 45 with dimensions 500 x 500 x 100 mm. The quality of the two contact surfaces was hand-scraped (3 spots/cm^2) for the column model and ground (C.L.A = 1.0 um) for the base. Fig. 4 shows the comparison between the theoritical and the experimental results. It can be seen that, the closed joint has stiffness magnitude higher than the recessed one, but it has also 30% excess than the calculated results. This difference may be attributed to the different flatness effects. The recessed joint represents a good agreeable comparison. Fig. 5 shows the experimental relation between the shear factor "K_s" and the normal stress "σ_n". The fitting of this experimental data gives the following equation .

$$K_s = R / \sqrt{\sigma_n} \qquad (9)$$

Where $R = 63.5 \times 10^{-3}\ mm^2 / \sqrt{N}$. The experimental results after Kirsanowa [6] gives the R magnitude with $25.3\ 10^{-3}\ mm^2 / \sqrt{N}$. The reasons of this difference may be due to undistributed normal stress on the joint surface. Also, by reducing the joint surface A_F with a constant initial preload F_i, the stiffness decreases (Equ. 8-b). Thereforem the designers may better use $R = 63.5\ 10^{-3}\ mm^2 / \sqrt{N}$ as a good design factor to calculate the shear stiffness . It is valid only for the mentioned quality of joint surface after Iżykows Ki[5]. Figs. 6 and 7 represent the relationship between the shear stiffness and the joint recess ratios (R_a , R_b). It can be seen that, the shear stiffness of the connected elements increases with the reduction of the joint recess ratio. If the normal stress is constant, the relation is linear. It is clear also that, the recess ratios R_a and R_b have no position effect on the stiffness value. When the normal stress in the joint increases through the decreasing of the joint area, the stiffness decreases gradually. At the constant F_i and $R_a = R_b = 0.6$ the decrease in the stiffness value may be about 20% with respect to the stiffness of a closed joint.

5- CONCLUSIONS

The static stiffness of bolted connections is greatly affected by the joint design parameters. Thus, the stiffness of a flat joint can be determined from dimensions, geometry and quality of the joint surfaces Based on the discussion of this workm the following are concluded :

- The designers may better use $R = 63.5\ 10^{-3}\ mm^2 / \sqrt{N}$ as a good design parameter to calculate the shear stiffness for Hand-Scraped steel surfaces (3 Spots/cm^2) / Ground (C.L.A = 1 μm)

- The recess ratios R_a and R_b have no position effect on the stiffness magnitude. A closed joint has the maximum stiffness value, but the designers may better use $R_b = R_a = 0.6$ to reduce the difficulties in the joint production. In this case, the stiffness is decreased about 20%.

6- ACKNOWLEDGEMENT

The author wishes to express his sincere gratitude to Prof. Dr. Eng. A.A. NASSER, Dean of Faculty of Eng. and Tech. Menoufia Univ. for his continuous helpfull and great interest throughout this work.

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8- NOMENCLATURE

A_F	Joint area (mm^2).
a_f	Length of the mating surface (mm).
b_f	Width of the mating surface (mm).
C_n	Normal Stiffness (N/µm).
C_s	Shear Stiffness (N/µm).
F_i	Initial load (N).
F_s	Shear load (N).
K_s	Shear factor (µm. mm^2/N).
R,S	Constans
R_a	Recess length ratio.
R_b	Recess width ratio.
δ_n	Normal deformation (µm).
δ_s	Shear deformation (µm).
σ_n	Normal stress (N/mm^2).
σ_s	Shear stress (N/mm^2).
α ,m	Constants.

MDB-13-142

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	Surface Quality	Material
1	Ground/Ground	C.I.
2	Hand Scraped/Ground	C.I.
3	Hand Scraped/Ground	St 45

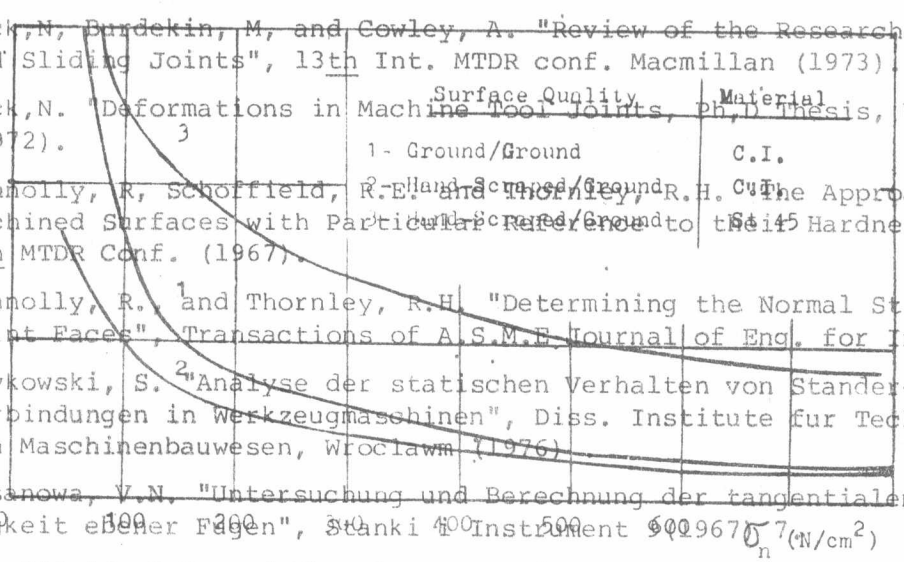
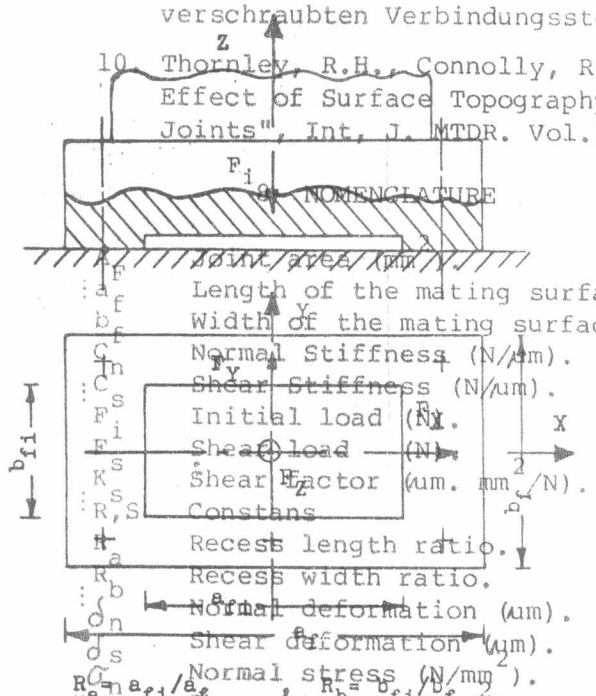


Fig. 1. Comparison of the Shear Compliance.



8- NOMENCLATURE

Joint area (mm²)	
a_f	Length of the mating surface (mm).
b_f	Width of the mating surface (mm).
C_n	Normal Stiffness (N/μm).
C_s	Shear Stiffness (N/μm).
F_i	Initial load (N).
F_s	Shear load (N).
K_s	Shear Factor (μm. mm²/N).
R, S	Constans
R_a	Recess length ratio.
R_b	Recess width ratio.
δ_n	Normal deformation (μm).
δ_s	Shear deformation (μm).
σ_n	Normal stress (N/mm²).
σ_s	Shear stress (N/mm²).

$R_a = a_{fi}/a_f$, $R_b = b_{fi}/b_f$

$F_i = a_f \cdot b_f (1-R_a \cdot R_b)$

Fig.2. Bolted Connection Subject to the Loads.

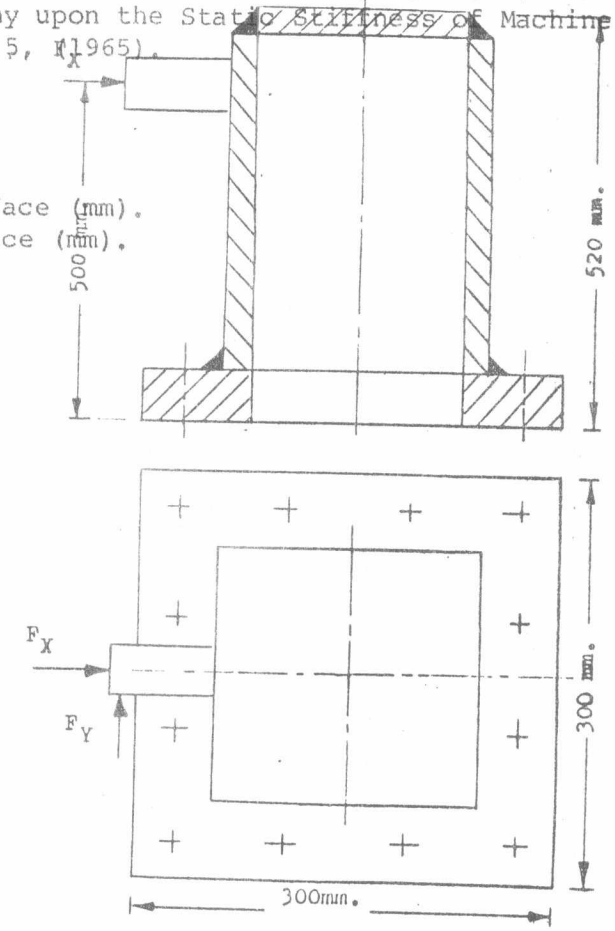


Fig.3. Structure Model.

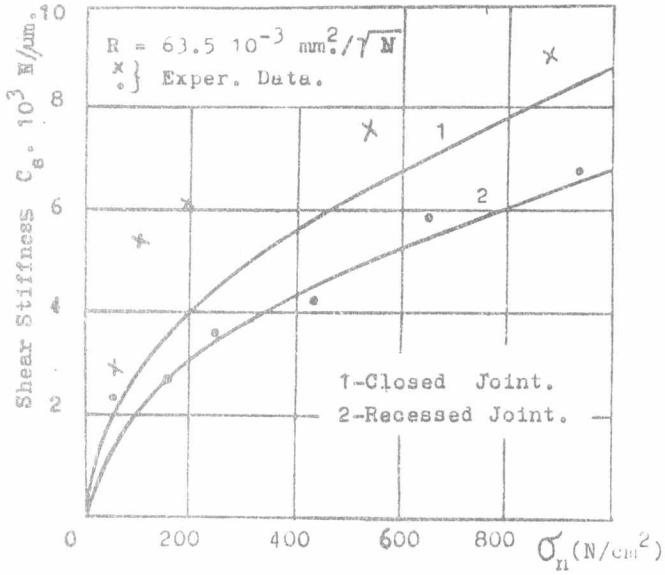


Fig.4. The Comparison of Results.

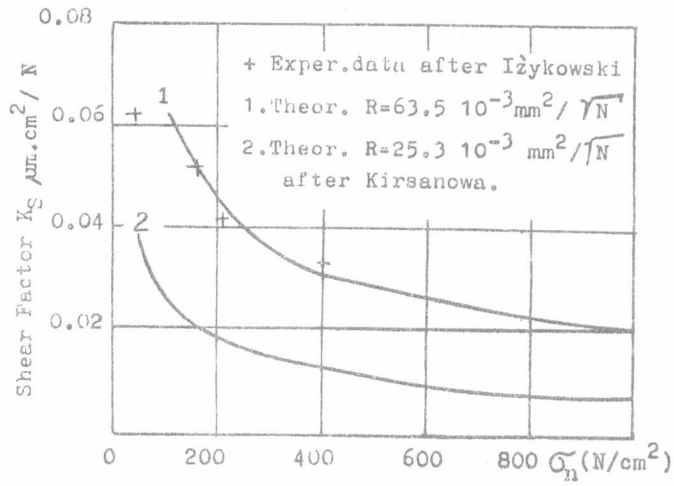


Fig.5.Determination of Shear Factor.

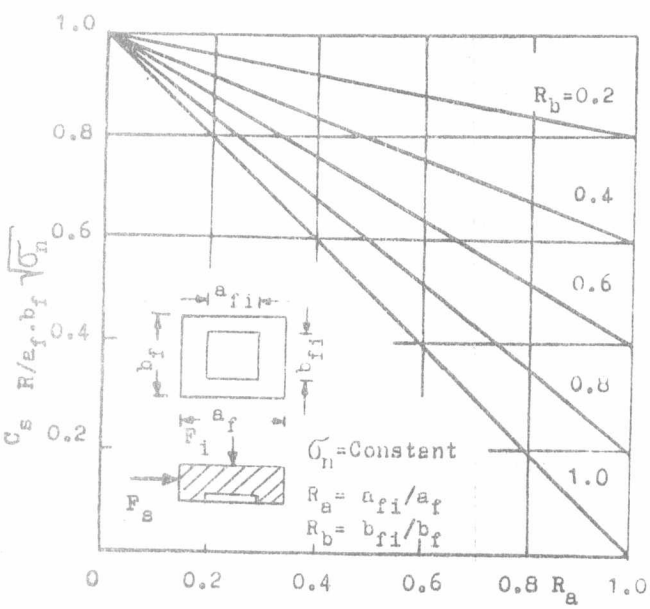


Fig.6. Effect of the Joint Recess.

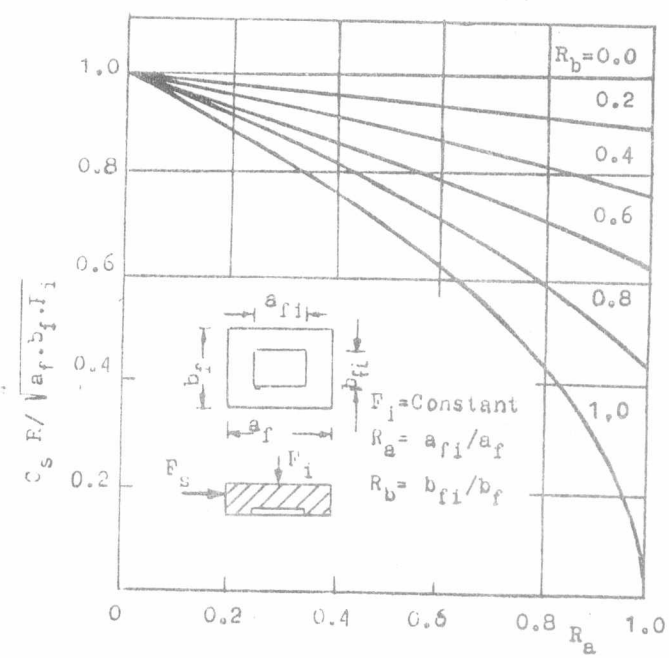


Fig.7. Effect of the Joint Recess.

Table 1. Normal and Shear Compliance Factors of C.I. Surfaces.

Pair of Surfaces in Contact	R	s	α	m
- Hand Scraped/Hand Scraped				
h = 3-5 um, Z = 3-4 Spots/cm^2	0.39	0.5	0.3	0.5
h = 6-8 um, Z = 3-4 Spots/cm^2	0.65	0.5	0.5	0.5
h = 6-8 um, Z = 2-3 Spots/cm^2	1.0-1.3	0.5	0.8-1.0	0.5
h = 6-8 um, Z = 1-2 Spots/cm^2	1.7-2.0	0.5	1.3-1.5	0.5
h = 15-20 um, Z = 1-2 Spots/cm^2	2.0-2.6	0.5	1.5-2.0	0.5
- Hand Scraped, h = 6-8 um, Z = 2-3 Spots/cm^2/ Ground, h = 1.0 μm CLA.	1.0-1.3	0.5	0.8-1.0	0.5
- Peripheral ground / Peripheral ground h = 1.0 μm CLA.	0.8-0.9	0.5	0.6-0.7	0.5
- Finish Planing / Finish Planing	0.78	0.5	0.6	0.5