ELASTIC CONTRIBUTION OF A NOTCH TO A CRACK AT ITS ROOT

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ABSTRACT

The purpose of the present work is to estimate the extent to which elastic stress-strain field of a notch has some contribution to the behaviour of a crack emanated from the notch root . A current work necessitates the analysis of some non-standard notch geometries with no data available in the literature .

Two dimensional elastic finite element technique is used for such a study . The states of plane stress and plane strain are analysed . In the present approach a model based on the concept of equivalent cracks applied to the relative movement of the two surfaces of Irwin's elastic crack is suggested . Alternatively , and for comparison , stress intensity factors for all cracks analysed are computed by making use of the energy release rate concept .

INTRODUCTION

Examination of the elastic stress field near the tip of a crack [1] proves that the stress intensity factor (SIF) , K_N , invoked in fracture mechanics analysis [2] and expressed as

$$K_N = Y \cdot \sigma \cdot \sqrt{\pi l} \quad (1)$$

is a general distinguishing parameter to measure the severity of different cracks . Here , N indicates the relative movement of the crack surfaces [3] , σ is a stress determined by the loading , l is a crack length and Y is a geometrical factor depending on the type of loading and the geometry both of the component and the crack [4].

During the last four decades considerable effort has been devoted to evaluating Y both theoretically and experimentally contributing over 250 solutions [5,6] . As an example Fig.1 illustrates a crack of length 2a contained within a plane elastic body of indefinite extent . Locations remote from the crack are stress-free and a uniform stress , σ , is applied perpendicular to the crack faces .

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The opening mode SIF, K_I , of such a crack system is given by [7],

$$K_I = \sigma \cdot \sqrt{\pi a} \quad (2)$$

This crack system is denoted in this work as S1.

The usefulness of the SIF in the analysis of fatigue crack growth data is now well established [8]. A current research was designed to assess fatigue behaviour due to the introduction of a notch at the tip of a propagating mode I crack [9]. Such a geometry is not standard and has no data available in the literature [5,6]. An initiated crack at the root of a notch propagates within the notch stress-strain field [10]. Such a field offers the tip of the present crack elastic and plastic contributions [11]. Elastic contribution is a reflection of the SIF of the system.

In the present work, a new model is developed to evaluate the SIF of a crack system. A cracked slotted notch located at the edge of a finite sheet is analysed to test the present approach.

PRESENT MODEL

For S1 the displacement component of the upper crack face in the direction perpendicular to the crack plane, v , is given by [12],

$$\begin{aligned} v &= \frac{2\sigma}{E} \cdot \sqrt{a^2 - x^2} && \text{for plane stress} \\ &= \frac{2\sigma(1-\mu^2)}{E} \cdot \sqrt{a^2 - x^2} && \text{for plane strain} \end{aligned} \quad (3)$$

with the origin corresponding to the mid-point of the crack. The modulus of elasticity and Poisson's ratio are respectively E and μ .

Eq. (3) is suggested to calculate K_I of an arbitrary crack system, S2, based on the following proposal. Two crack systems, S1 and S2, are assumed equivalent should they have the same v at two identical crack surface locations, v_r , relative to the corresponding tips of their cracks, i.e. at the same distance r as schematically depicted in Fig.2. Thus, if the two systems are subjected to the same stress, σ , two cracks of lengths l and a respectively contained within S1 and S2 are equivalent to be having the same K_I . The schematic of Fig.2 illustrates the application of this approach.

COMPUTATIONAL ANALYSIS

A suitable standard two dimensional elastic finite element programme was used. The programme is being stored in the library of the scientific computing center of Cairo University [13]. Both plane stress and plane strain were analysed. Elastic properties typical of steels were given as input data to the programme. The flow sequence of the programme is presented elsewhere [9].

Cracks of different depths, l , emanated from the root of the notch in the plate of the geometry shown in Fig.3 were considered for

evaluating the geometrical factor, Y , expressed in the mathematical form of the opening mode SIF, K_I , given by eq.(1). The mostly used linear displacement triangular elements [14] were adopted with a mesh consisting of 320 nodal points generating 350 elements most of them are located around the tip of the crack as higher order degree elements. The boundary conditions fitted to the present problem are listed in Fig.3 [9].

At a specific stress, σ , values of crack opening displacements, v_r , at a distance r ($= 0.1$ mm) measured from the crack tip were computed. For a crack in this system, S2, of length l , the corresponding equivalent crack length in S1, a , is given by eq.(3) as

$$\begin{aligned} a &= \frac{1}{2r} \left[\left(\frac{E v_r}{2\sigma} \right)^2 + r^2 \right] && \text{for plane stress} \\ &= \frac{1}{2r} \left[\left(\frac{E v_r}{2\sigma(1-\mu^2)} \right)^2 + r^2 \right] && \text{for plane strain} \end{aligned} \quad (4)$$

By making use of equations (1) and (2), Y follows

$$Y = \sqrt{a/l} \quad (5)$$

For the sake of comparison, values of mode I strain energy release rates, G_I , [15] were also computed for different crack lengths. Linear elastic fracture mechanics dictates a relationship between G_I and K_I in the form

$$\begin{aligned} K_I &= \sqrt{E G_I} && \text{for plane stress} \\ &= \sqrt{E G_I / (1-\mu^2)} && \text{for plane strain} \end{aligned} \quad (6)$$

Fig.4 presents a comparison of K_I values obtained by the present model and the concept of energy release rate. The results of this work indicates that in both cases, plane stress and plane strain data are closely in agreement.

DISCUSSION

The notched plate shown in Fig.3 was analysed numerically to test the present model. Fig.4 compares the computational results produced by the application of the proposed model of equivalent cracks and the well known energy release rate concept. The data exhibit a close agreement.

To check the accuracy of the adopted finite element analysis, an edge sharp crack in a plate of the same width was considered. Generally, this geometry is in agreement with those of single edge sharp notched plates calibrated for the calculation of K_I [16]. The value of K_I can be calculated by a compliance function claimed to be accurate within 0.4% if l/w is less than 0.6, i.e.

$$K_I = \sigma \sqrt{l} \left[1.99 - 0.41(l/w) + 18.7(l/w)^2 - 38.48(l/w)^3 + 58.85(l/w)^4 \right] \quad (7)$$

In analysis, such a plate was subject to the same sources of simulation and computational errors. Fig.5 illustrates a comparison of the present data and the compliance function of eq.(7). Within the range of the present data, a negligible difference is exhibited irrespective of l/w . Should this be taken into consideration the application of the present model is valid.

Solutions exist but for some standard notch profiles [5,6]. The notch under consideration in this work is of a non-standard shape. However, experimental and numerical methods provide a solution for any one particular problem with a relatively expensive cost. Thus, the present work was carried out in the hope that it would lead to an applicable approximate solution for notches. Fig.6 illustrates the application of the concept of equivalent cracks on the present data of the sharp edge and slotted notched plates. Fig.5 in conjunction with eq.(5) concluded the data of the former notch. The latter notch data were direct output of the finite element programme. In Fig.6, a is the length of the crack in S1 equivalent to a crack of length l in S2. Obviously, a increases with an increase in l . At $l=0$, the extent of a is zero. Generally, the present data of the slotted notch can be divided into two regions. The short crack regime extends up to $l=\lambda$. A crack shorter than λ behaves as a crack longer than its actual length. Cracks deeper than λ represent the second region. As shown in Fig.6, a straight line is a reasonable representation of this regime in which the data of both notches match each other.

Two important points can be concluded. First, the notch offers no effect to cracks longer than λ . This means that a crack deeper than λ behaves as a crack having a length of its total depth. The second conclusion is that the notch contributes elastically to a crack of a tip lying within λ . Thus, λ may be called the extent of the notch elastic field. It is a geometrical factor depending on the notch geometry and type of loading [17]. For the present type of bulk stress, Smith and Miller [18] concluded both numerically and experimentally an approximate value of $\lambda=0.13\sqrt{DP}$, where D and P are respectively the notch depth and root radius. Fig.6 gives $\lambda=0.7$ mm for the notch under consideration indicating a value of $\lambda=0.22\sqrt{DP}$. Recently, Smith and Cameron [19] modified the previous finding of λ to be $0.25\sqrt{DP}$ justifying the present result.

CONCLUSIONS

The applicability of the proposed concept of equivalent elastic cracks on the present notch problem has been justified. The concept is based on crack opening displacement. The present data conclude that a notch offers no contribution to a crack longer than its elastic field, λ . A crack shorter than λ behaves due to the presence of the notch as a crack longer than its actual length.

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Fig.1 Schematic illustration of Sl crack system .

Fig.2 Concept of equivalent cracks suggested by the present model

BOUNDARY CONDITIONS

1. Along JK : $P =$ vertical nodal forces , $0 =$ nodal horizontal disps.
2. Along AM : $P =$ vertical nodal forces , $0 =$ nodal vertical disps.
3. Otherwise : $0 =$ external nodal forces

Fig.3 Finite element idealisation applied
on the present slotted notched plate

Fig.4 Comparison of the computation SIF results produced by the application of the present model and the strain energy release rate concept

Fig.5 Comparison of the present data on single sharp edge-notched plate and the compliance function of eq.7

Fig.6 Application of the concept of equivalent cracks on the present data of the sharp edge and slotted notched plates