VIBRATION CONTROL BY DRY FRICTION DAMPER

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ABSTRACT

This work is concerned with the study and use of dry friction dampers for vibration control. First, the characteristics of the friction damper are determined. Then, the friction damper is applied to a hinged-hinged beam. The amplitude-frequency response for a harmonic excitation is determined both theoretically and experimentally. For linearization of the equations of motion, the friction force is replaced by an equivalent linear viscous force dissipating the same energy per cycle. The results obtained from experiments and theory are in good agreement, and it is shown that, by a suitable choice of the ratio of the friction force to applied force the friction damper can be an effective damping device.

INTRODUCTION

Friction damping is always present in assembled structures, but it is very difficult to quantity its effect. The main reasons for that are the variation in material properties, normal force and the non linearity of equations. Amongst the few references on the friction damping the works dealing with slip in joints [6,7,8,9,12,14,15], and a paper on clamped beam damped at the free end with a friction damper [10], may be mentioned.

Here an inexpensive commerical friction damper has been studied. Firstly the behaviour of the friction force under dynamical excitation is investigated experimentally, and a method is described for the determination of the characteristics of the friction damper. Then as beam structures are used for supporting machines [1,11,13], the friction damper has been used to improve the dynamic behaviour of a hinged-hinged beam. Experimental and theoretical results are presented and are shown to be in satisfactory agreement.

DETERMINATION OF THE CHARACTERISTICS OF THE DRY FRCTION DAMPER

Friction Damper:

The damper is included in a system consisting of a mass m, and of a spring of equivalent stiffness k, as shown in Fig. 2. To obtain a continuous

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harmonic response without pauses, the amplitude of the applied force must be higher than the friction force. Thus, the nonlineaity of the friction force does not disturb significantly the harmonic form of the applied force and the response of the system, therefore a heavy mass is used. The mass is supported by two smooth teflon (P.T.F.E.) rollers, rolling on a horizontal smooth plane surface. The mass is excited by an electrodynamic force generator which can give a harmonic force with amplitudes up to 500 Newtons. The force generator is connected to the mass through a piezoelectric force pickup which measures the applied force amplitude. A piezoelectric accelerometer is fixed on the mass to measure the acceleration of the system. The system response which is the ratio of the acceleration amplitude to the applied force in dB is plotted against frequency in Hz. The system response is determined in three cases which will permit the determination of the system characteristics :

a) without the damper,
b) with the damper fixed to the system and blocked by using small exciting force,
c) with the damper fixed to the system, and for different magnitudes of the exciting force which are sufficient to produce slip in the damper.

In both cases a) and b), the system is considered as a linear system with equivalent linear viscous damping ; therefore the acceleration to force ratio is independent of the magnitude of the applied force, and there is no need to keep the amplitude of the force constant during the test. While in case c) the system response is dependent on the amplitude of the applied force, it is therefore necessary to keep the magnitude constant during the frequency sweep test.

Mathematical model of the system [2] , [3] , [4]

The two degrees of freedom mathematical model which incorporate mass m, spring of equivalent stiffness k, equivalent linear viscous damper c, and the friction damper represented by F, Nk and shown in Fig. 3, is used. It is assumed here that the motion of the system is continuous, harmonic and without pauses. The equations of motion of the system with the friction force are non-linear. For linearization of the equations of motion the friction force is replaced by an equivalent linear viscous force [2] , [5], the coefficient of which is determined by equating the energy dissipated per cycle. The equivalent viscous damping coefficient is given by :

$$c_1 = \frac{4\ F}{\pi \Omega |\delta|} \qquad (1)$$

where $\delta = x - x_1$ is the relative displacement between the ends of the the damper. The linearized equations of motion of the system are :

$$m\ \ddot{x} + c\ \dot{x} + c_1\ (\dot{x} - \dot{x}_1) + k\ x = P\ e^{j\Omega t} \qquad (2)$$

$$N\ k\ x_1 - c_1\ (\dot{x} - \dot{x}_1) = 0 \qquad (3)$$

The assumption of continuous harmonic motion, leads to a solution in the form :

$$x = X\ e^{j(\Omega t + \emptyset)} \quad \text{and} \quad x_1 = X_1\ e^{j(\Omega t + \emptyset_1)} \qquad (4)$$

Substituting equations (4) into equations (2) and (3), we get the following

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relations :

$$\delta^2 = X^2 - X_1^{\,2} \tag{5}$$

$$X_1 = \frac{4}{\pi}\frac{F}{Nk} \tag{6}$$

and

$$\left(\frac{X\,k}{P}\right)^2 = \left[\frac{b}{a} - \sqrt{\left(\frac{b}{a}\right)^2 + \frac{d}{a} - \left(\frac{\alpha}{N}\right)^2}\,\right]^2 + \left(\frac{\alpha}{N}\right)^2 \tag{7}$$

where :

$$a = (1 - z^2)^2 + (2\zeta z)^2$$

$$b = 2\zeta\alpha z \quad , \quad z = \frac{\Omega}{\omega} \quad , \quad \zeta = \frac{c}{2m\omega} = \frac{c}{c_{cr}}$$

$$\alpha = \frac{4}{\pi}\frac{F}{P}$$

$$d = 1 - \alpha^2 \left(1 \pm \frac{2}{N}(1 - z^2)\right)$$

For $z = 1$, the system response takes the form :

$$\left(\frac{X\,k}{P}\right)^2 = \left(\frac{1}{2\zeta}\right)^2 \left[\alpha^2 + 1 - 2\alpha\sqrt{1 - \left(\frac{2\alpha\zeta}{N}\right)^2}\,\right] \tag{8}$$

Since the term $\left(\frac{2\alpha\zeta}{N}\right)^2$ is << 1, equation (8) reduces to :

$$\frac{X\,k}{P} = \frac{1 - \alpha}{2\,\zeta} \tag{9}$$

The damper will be blocked when $x = x_1$, from which we can determine the limiting frequency conditions for break-loose and break-in of the damper as :

$$z^2_{1,2} = \left[(1 + N) - 2\zeta^2\right] \pm \sqrt{4\zeta^4 + 4\zeta^2\,(1 + N) + \left(\frac{N}{\alpha}\right)^2} \tag{10}$$

Since ζ^4 and $\zeta^2 << \left(\frac{N}{\alpha}\right)^2$ equation (10) is reduced to :

$$z_{1,2} = \sqrt{1 + N\left(1 \pm \frac{1}{\alpha}\right) - 2\zeta^2} \tag{11}$$

the -ve sign corresponds to the break-loose condition, while the + ve sign corresponds to the break-in frequency.

Results

The system response with and without damper obtained experimentally for different magnitudes of the applied force are shown in Fig. 4. From the response of the system without damper the following characteristics are determined :

a) mass of the system $m = 30$ kg
b) equivalent stiffness of the spring $k = 8.1 \times 10^4$ N/m
c) natural frequency of the system $f_o = 8.27$ Hz
d) equivalent viscous damping ratio of the system $\zeta_1 = 0.01$

From the system response with the damper blocked-in the following characteristics are determined :

e) equivalent viscous damping ratio of the system $\zeta_2 = 0.04$
f) total stiffness of the system $(N + 1)\, k = 2.04 \times 10^6$ N/m
g) natural frequency of the system $f_1 = 41.5$ Hz

From the response of the system slipping for P = 60 N and for P= 70 N and using equation (9) in the form :

$$\frac{a}{P} = \frac{1 - \alpha}{2\zeta m} \qquad \text{at} \qquad z = 1 \tag{12}$$

we can determine :

h) the friction force of the damper = 42,25 N
i) the damping ratio of the system $\zeta = 0.027$.

Using the previous results the system response for the above mentioned cases are calculated by using equation (7) and are shown in Fig.6.

FRICTION DAMPER APPLIED TO A HINGED-HINGED BEAM

Mathematical model of the system

The beam is modelled by using the finite element method as shown in Fig.6. Each element of the beam has 2 nodes and each node has 2 degrees of freedom : displacement and rotation. If the beam is divided into n elements, the total number of degrees of freedom of the beam will be 2(n+1). An additional displacement degree of freedom is added due to the elasticity of the damper, therefore the total number of degrees of freedom of the system will be 2n + 3.

The friction damper force is replaced by an equivalent viscous one. The equivalent viscous damping coefficient is given by equation (1), where in this case the relative displacement between the ends of the damper is given by :

$$\delta = w_a - w_{2n+3} \tag{13}$$

Since the damper is connected to the node a, and the exciting force acting at the node b, and taking into consideration the boundary conditions $w_1 = w_{2(n+1)} = 0$, the displacement vector of the system can be arranged in the form :

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{14}$$

where

$$q_1 = [\phi_1, w_2, \phi_2, w_3, \ldots, \phi_{2(n+1)}]^t \tag{15}$$

and

$$q_2 = [w_a, w_b, w_{2n+3}]^t \tag{16}$$

If the harmonic exciting force has the form :

$$P = (P_r + j\,P_i)\,e^{j\Omega t} \tag{17}$$

The beam response is :

$$q = (q_r + j\,q_i)\,e^{j\Omega t} \tag{18}$$

where the subscript r denotes the real part, and i the imaginary part.

The equations of motion of motion of the system assuming that the beam itself has an equivalent structural damping η, have the form :

$$\left[-\Omega^2\begin{bmatrix} m_{11} & m_{12}\\ m_{21} & m_{22}\end{bmatrix} + j\Omega\begin{bmatrix} 0 & 0\\ 0 & c_1\end{bmatrix} + j\eta\begin{bmatrix} k_{11} & k_{12}\\ k_{21} & k^*_{22}\end{bmatrix} + \begin{bmatrix} k_{11} & k_{12}\\ k_{21} & k_{22}\end{bmatrix}\right]\begin{bmatrix} q_1\\ q_2\end{bmatrix} = \begin{bmatrix} 0\\ P\end{bmatrix} \tag{19}$$

where k^*_{22} represents the submatrix of the beam stiffness corresponding to the coordinate q_2, while k_{22} represents that of the beam and the damper.

Equation (19) is divided into :

$$\begin{bmatrix} k_{11}-{}^{2}m_{11} & -k_{11}\eta\\ \eta k_{11} & k_{11}-\Omega^2 m_{11}\end{bmatrix}\begin{bmatrix} q_{1r}\\ q_{1i}\end{bmatrix} + \begin{bmatrix} k_{12}-\Omega^2 m_{12} & -\eta k_{12}\\ \eta k_{12} & k_{12}-\Omega^2 m_{12}\end{bmatrix}\begin{bmatrix} q_{2r}\\ q_{2i}\end{bmatrix} = [0]$$

i.e.,

$$[A]\begin{bmatrix} q_{1r}\\ q_{1i}\end{bmatrix} + [B]\begin{bmatrix} q_{2r}\\ q_{2i}\end{bmatrix} = [0] \tag{20}$$

and

$$\begin{vmatrix} k_{21}-\Omega^2 m_{21} & -\eta k_{21}\\ \eta k_{21} & k_{21}-\Omega^2 m_{21}\end{vmatrix}\begin{vmatrix} q_{1r}\\ q_{1i}\end{vmatrix} + \left[\begin{bmatrix} k_{22}-\Omega^2 m_{22} & -\eta k^*_{22}\\ \eta k^*_{22} & k_{22}-\Omega^2 m_{22}\end{bmatrix} + \begin{bmatrix} 0 & -\Omega c_1\\ \Omega c_1 & 0\end{bmatrix}\right]\begin{bmatrix} q_{2r}\\ q_{2i}\end{bmatrix} = \begin{bmatrix} P_r\\ P_i\end{bmatrix}$$

i.e.,

$$[D]\begin{bmatrix} q_{1r}\\ q_{1i}\end{bmatrix} + \left[[E] + [H]\right]\begin{bmatrix} q_{2r}\\ q_{2i}\end{bmatrix} = \begin{bmatrix} P_r\\ P_i\end{bmatrix} \tag{21}$$

where

$$c_1 = c\begin{bmatrix} 1 & 0 & -1\\ 0 & 0 & 0\\ -1 & 0 & 1\end{bmatrix} \tag{22}$$

and c is the equivalent viscous damping coefficient given by equation (1).

Substituting equation (20) into equation (21), we get :

$$\begin{bmatrix} q_{2r} \\ q_{2i} \end{bmatrix} = \Big[[E] + [H] - [D][A]^{-1}[B]\Big]^{-1} \begin{bmatrix} P_r \\ P_i \end{bmatrix} \qquad (23)$$

Since the matrix H is function of the beam response, equation (23) have to be solved by using an iterative method. By using the relation given by equation (20) the order of the iterated matrix is always reduced to (6 x 6) whatever be the initial numbers of degrees of freedom of the system.

Experimental test

A steel beam with $E = 2 \times 10^{11}$ N/m^2, ρ = 7800 kg/m^3 and the dimensions (1 x 0.06 x 0.01 m) is attached to a rigid frame. The beam is excited by an electrodynamic force generator, and an accelerometer is fixed below the point of application of the force .

At first the beam response without the damper is determined, from which the natural frequencies (see Table 1) and an equivalent structural damping factor of the beam η = 0.003 are determined. The mode shapes of the beam for the first three natural frequencies are shown in Fig. 7. Then the same damper used previously is attached to the beam at a distance 3/8 L from the left end. The natural frequencies of the beam with the damper blocked-in (small exciting force) and its mode shapes are obtained, (see Table 1 and Fig. 8). Then the beam is excited at a distance (5/8)L from the left end.

The beam response for different magnitudes of applied force was determined, and are given in Fig.9. During each of the frequency sweep tests the amplitude of the applied force is kept constant.

Results

The natural frequencies of the beam without the damper are calculated using 16 finite elements. Then the natural frequencies of the beam are calculated by adding an additional stiffness at a distance (3/8)L, which represents the stiffness of the blocked-in damper. The resulting natural frequencies in both cases are given in Table 1, along with the experimental values.

A program is written for solving equation (23) by matrix iteration on a micro-computer. The number of elements of the beam used is here limited to 3 due to the limited capacity of the memory of the computer. By using equation (21) the order of the iterated matrix is reduced from (14 x 14) to (6 x 6) only. At first, the beam responses are calculated using equation (23) and puting F = 0 and η = 0.003.

Then the beam responses are calculated using equation (23) and considering the damper fixed at a distance (3/8)L from the left end, the friction force = 42.5 Newtons and η = 0.003, and the force applied at a distance 5/8L equals to 1,40,60 and 63 Newtons. The results are given in Fig.10.

The beam responses obtained theoretically and experimentally show that the friction damper as applied here can be very effective. The comparison of the experimental response Fig. 9 and the theoretical one Fig.10 shows

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that the linearized analysis can give a fair prediction of the beam response. The frictionally damped beam has acceptable finite response over an excitation frequency range which includes a number of undamped natural frequencies, as in Fig.9. It may also be observed that the natural frequencies of the structure get altered considerably, when the friction damper is blocked. At these natural frequencies, the friction damper is quite effective in limiting the response, though it is higher than the original structure's response.

CONCLUSION

An inexpensive friction damper has been used to demonstrate its effective use in controlling vibrations of a simple structure. For the damper under consideration it has been observed from the experimental results, that the friction force can be considered as independent of the frequency and the amplitude of the exciting force, in the range studied. From frequency sweep tests, the procedure of determining the characteristics of the friction damper is given. A finite element model has been used, to determine the response of a hinged-hinged beam structure with the damper. The theoretical results obtained show good agreement with experimental values and the damper is very effective in controlling the vibratory ampllitudes near resonant conditions.

Based on the studies made here for a hinged-hinged beam, it might prove that the friction dampers of the type considered here could find more practical application in control of structural vibration.

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NOTATIONS

a	acceleration amplitude (m/sec^2)
c	viscous damping coefficient (Nsec/m)
E	Young's modulus (N/m^2)
F	friction force (N)
f	frequency (Hz)
I	second moment of area of the cross section of the beam (m^4)
k	equivalent stiffness of the spring (N/m)
$[k]$	Stiffness matrix
L	beam length (m)
m	system mass (kg)
$[m]$	mass matrix
N	damper support stiffness/k
P	applied force amplitude (N)
$q, \dot{q}, \ddot{q}$	displacement, velocity, and acceleration vectors of the beam
S	beam cross section area (m^2)
w	displacement (m)
X	displacement amplitude (m)
$z = \frac{\Omega}{\omega}$	frequency ratio
δ	relative displacement between the ends of the damper (m)
$\zeta = \frac{c}{c_{cr}}$	damping ratio
Ω	circular frequency of the force (1/sec)
ω	circular natural frequency of the system (1/sec)
ρ	mass per unit volume (kg/m^3)
η	structural damping factor
$\emptyset$	phase angle

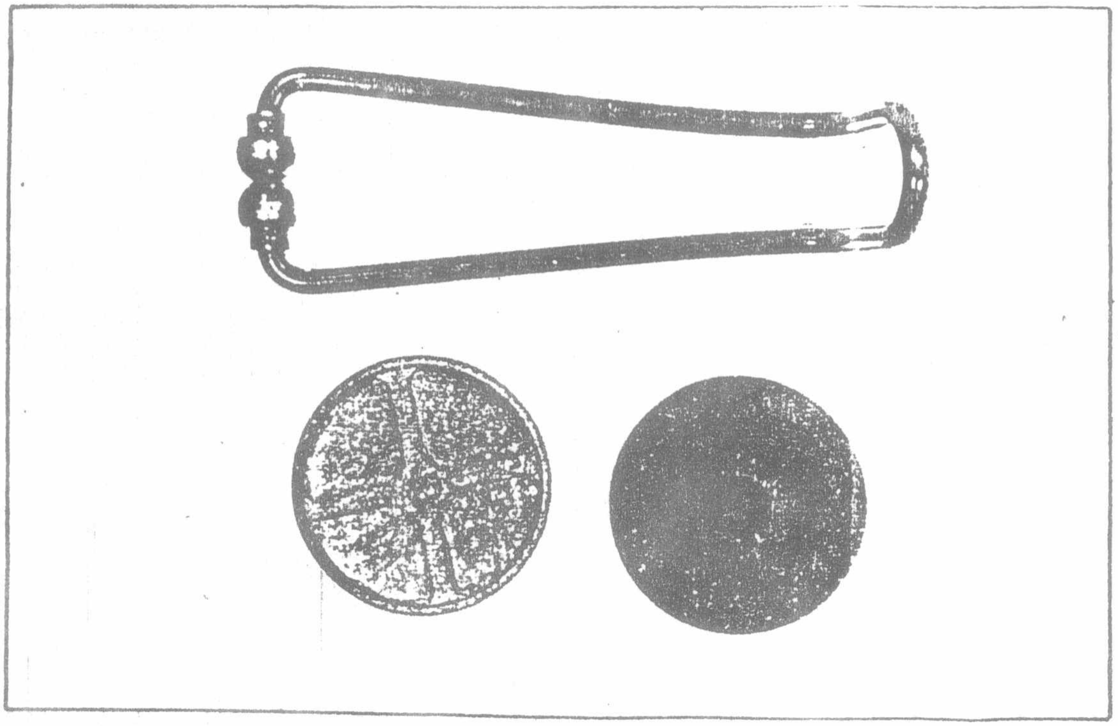

Fig.1. Friction Damper.

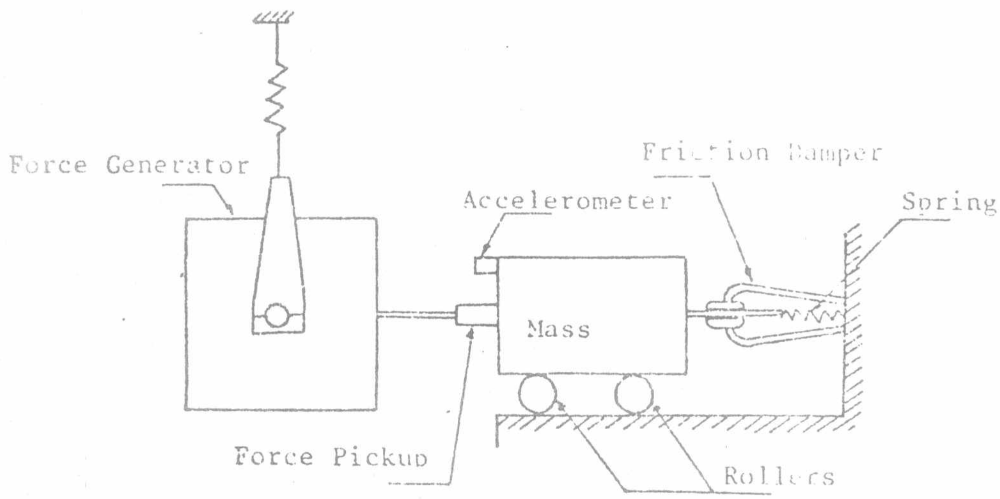

Fig.2 - Experimental arrangement for determination of friction damper characteristics.

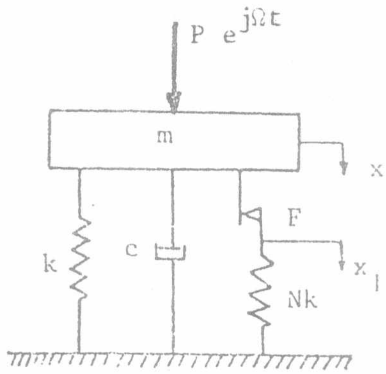

Fig. 3 - Mathematical model for mass and damper.

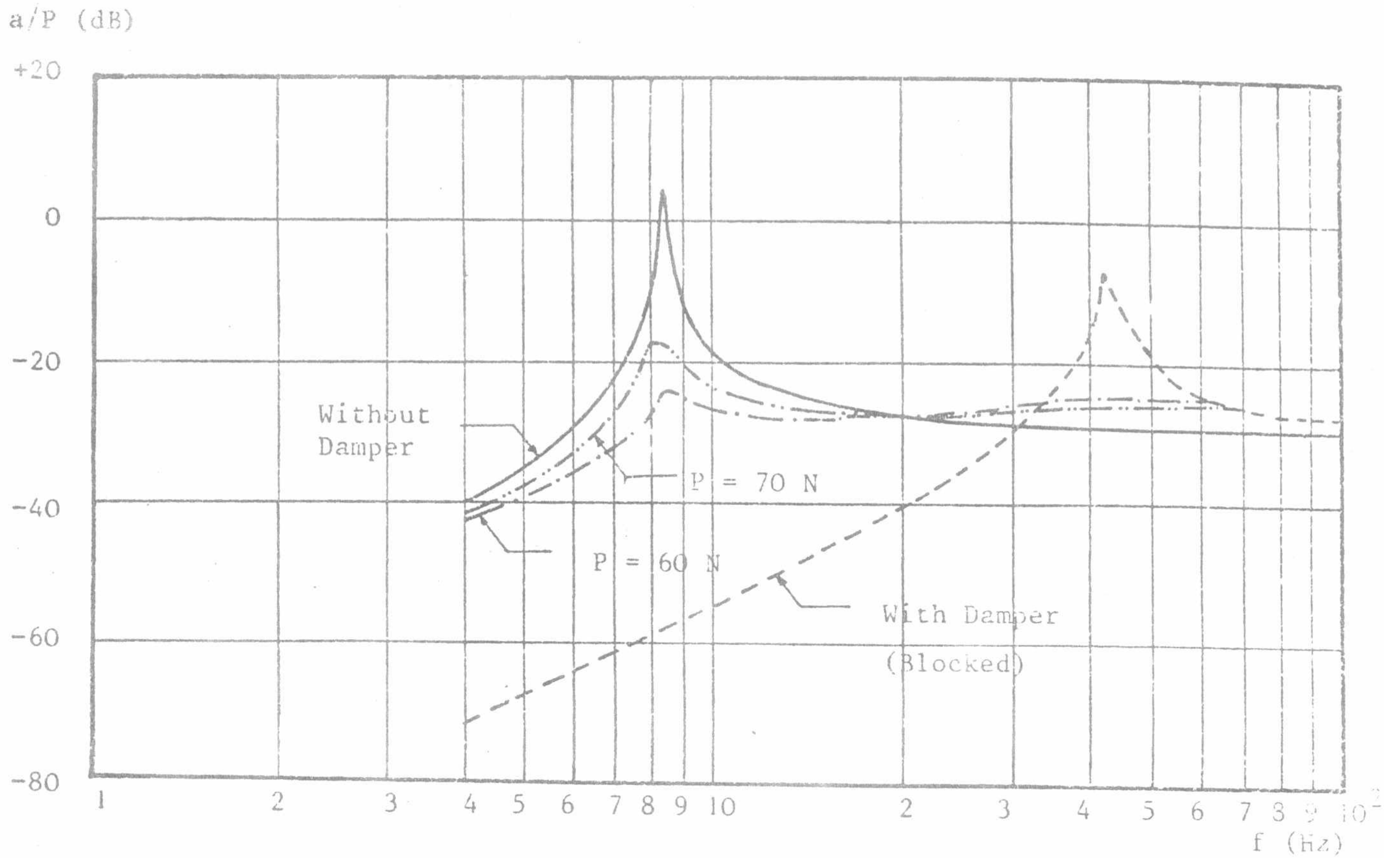

Fig. 4 - Experimental response

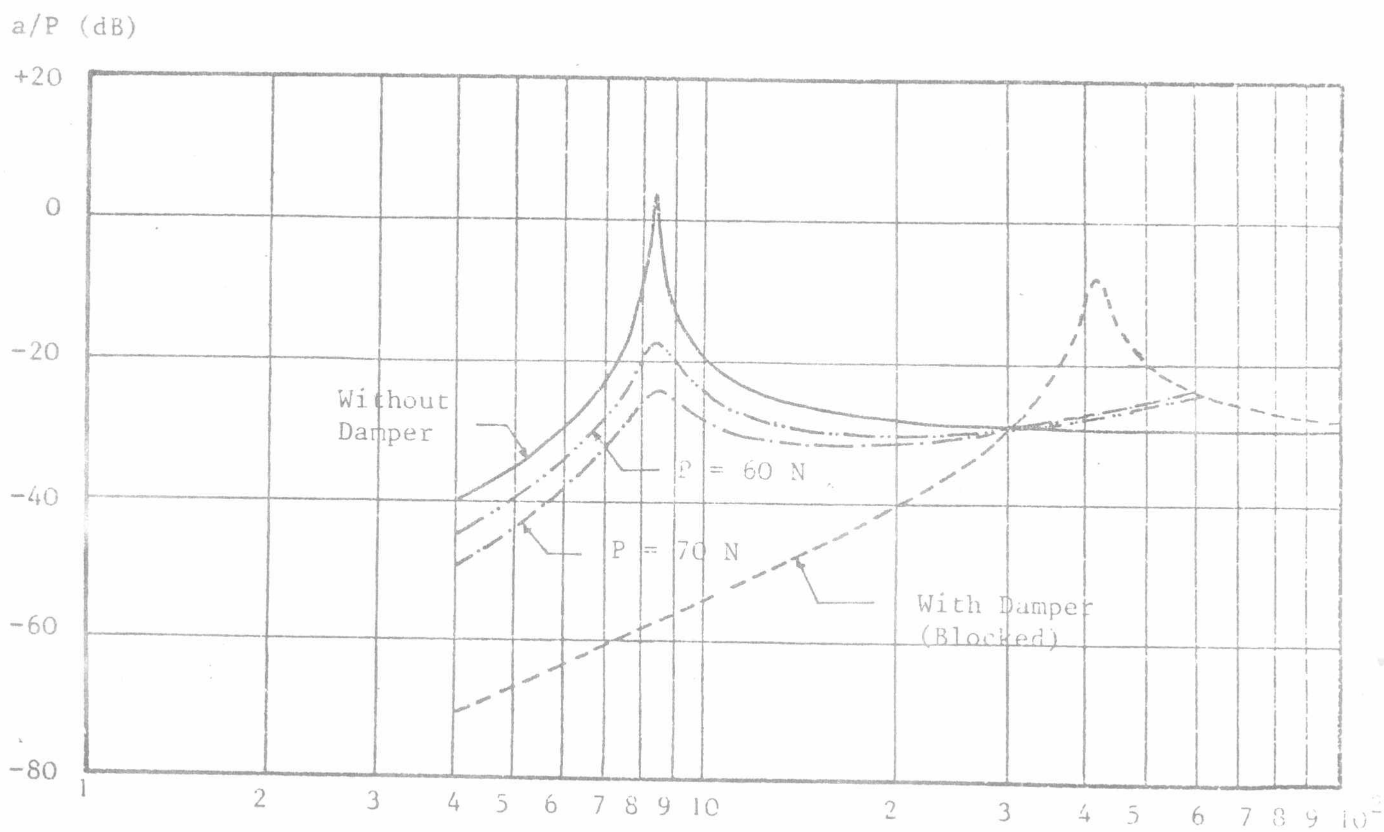

Fig. 5 - Theoretical response

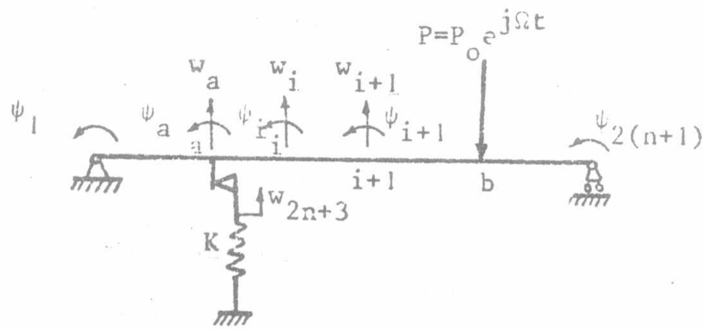

Fig. 6 - Mathematical model of the beam.

Table 1 : Natural frequencies of the system - Hz.

Freq. No.	Beam		Beam + Damper (Blocked)	
	Exp.	Theor.	Exp.	Theor.
1	22.8	22.8	65.2	69.2
2	85.8	91.2	153.5	155.0
3	187.5	204.0	205.0	216.0

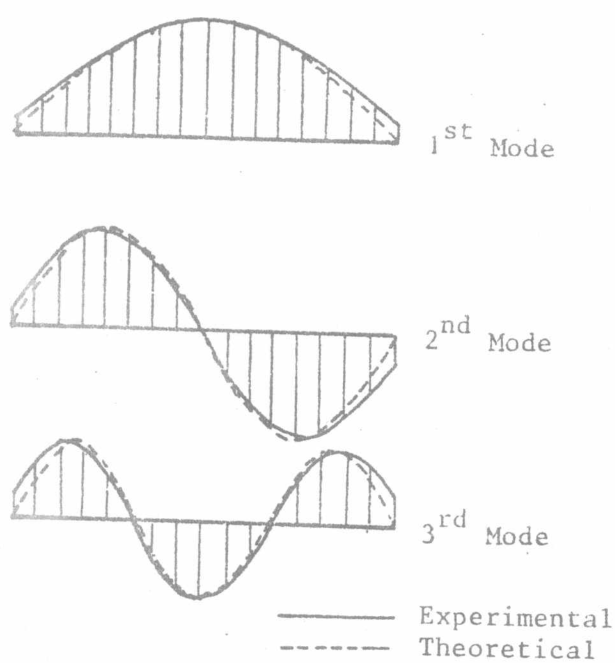

Fig. 7 - Mode shapes of the beam.
(Without damper)

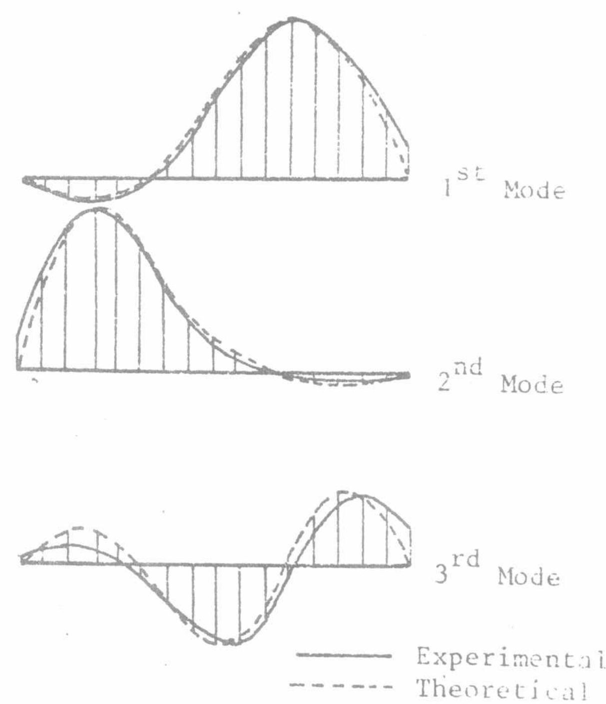

Fig. 8 - Mode shapes of the beam.
(With damper blocked)

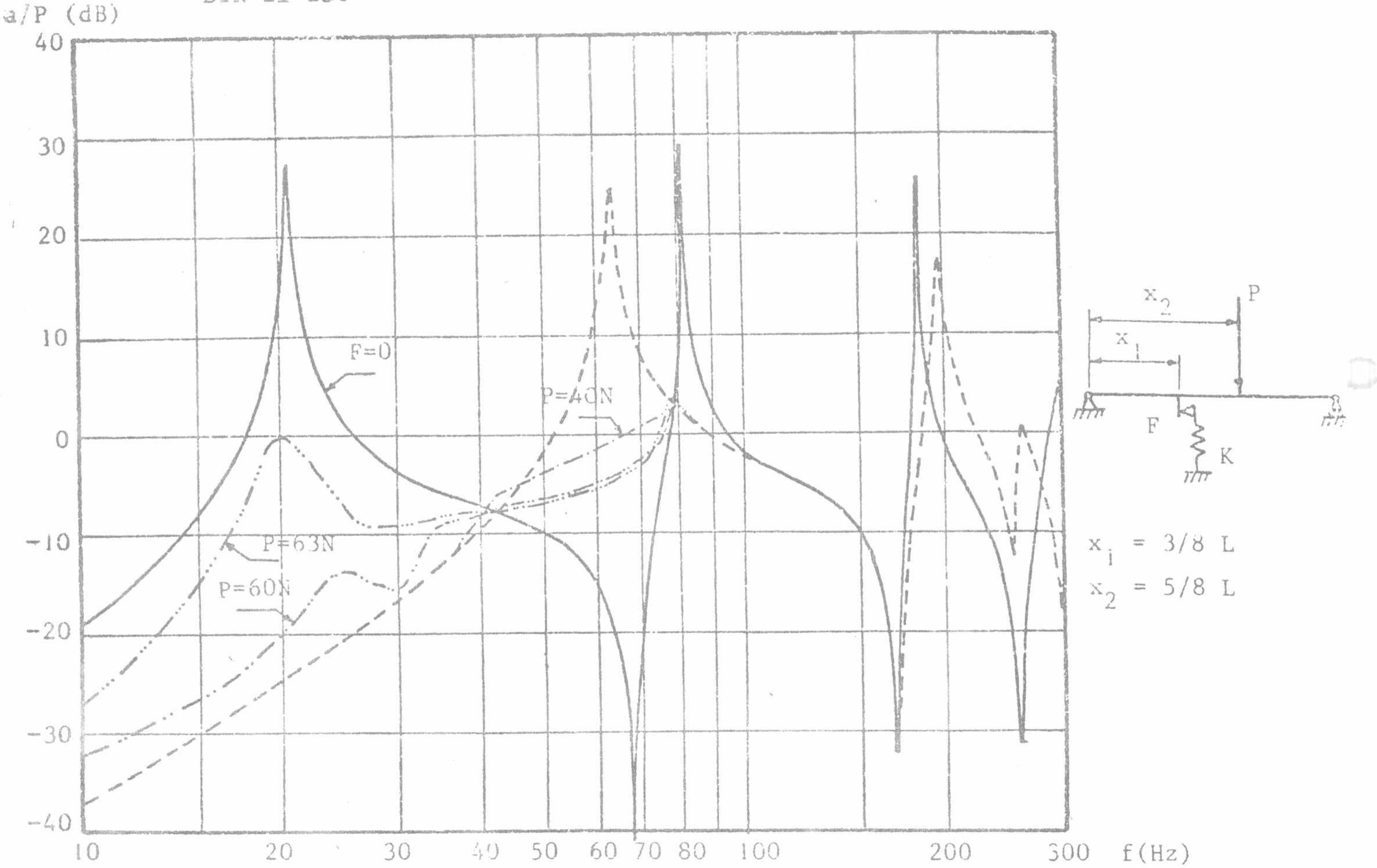

Fig.9 - Experimental response.

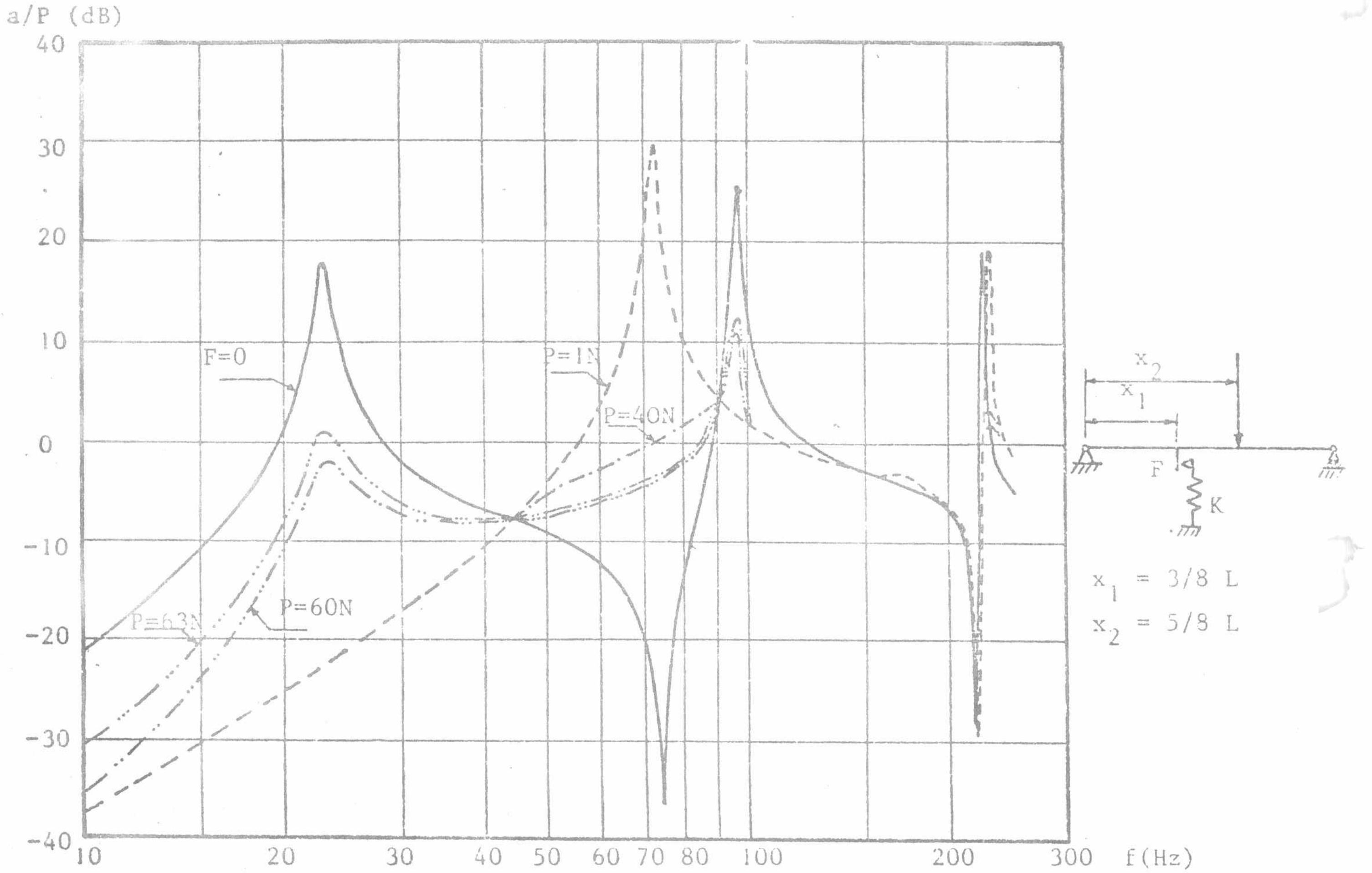

Fig. 10 - Theoretical response.