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WHIRLING EFFECT ON BEARINGS OF MARINE LINE-SHAFTS UNDER EXTERNAL EXCITATIONS

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ABSTRACT

Due to the ecentricity of thrust force acting on propeller blades, marine shafts experience whirl while rotating. This directly affects the reactions of the different supports, and consequently causes wear of shaft bearings. This paper is concerned with simplifying the concept for the shafting system in order to estimate the frequencies of whirl and propeller tip deflections. The work is dealing with both; three and four span systems with overhaged propeller. The problem is also studied for both mass and massless shafts. For massless shafts systems of any number of spans, general formulae for calculating the frequency of whirl and the deflection of propeller tip are proposed. For shaft systems of distributed mass, a procedure is suggested to calculate frequencies as well as deflections at different supports.

INTRODUCTION

A shafting system is a line-shaft joining the propelsive plant at one end, with the propeller at the other end. This shaft is supported by a number of bearings of different flexibilities. Such a system is subjected to the different forces coming from various sources of excitation directly or through the structure. The external moment variations applied on the actuating propeller and the shafting system will induce a whirling phenomenon. These would also cause stress variations on the system, whichwere illustrated mathematically by Jasper and Rupp [1] . If the wirling frequencies are somewhat close in value to the frequencies of any of the excitation sources, one should expect resonance. Hence extra vibrational stresses not only on the propeller and tail shaft but also on the whole after part of the ship's hull. The latter type of vibrations always been the source of trouble to the senstive equipment installed on board ship and inconvenience to the crew.

Problems Arise As a Result of Whirling Phenomenon:

The dynamic nature of this phenomenon would cause additional fatigue stresses in the propeller shaft and the tail shaft, especially at the front end of the propeller key way. The bearing reactions would be also magnified causing the vibration of the after body of the ship. The first bearing just forward of the stern tube would be

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damaged in a wiped up and heated upper half, while the lower half is seemed undamaged. In general the stern tube would suffer the most, leading to any or many of undesirable behaveiours, such as: prematurely worn lignum vitae staves in the after bearings, fracture of some of the bushes, a rapid wear and/or the deterioration of tail-shaft, seizing of the metallic bushes, heavy cavitation and galvanic corrosion in the central annual space of the stern-tube, cavitation erosion marks could be traced on the tail-shaft in way of the after vitae bearing bush, last but not least, damage of the stern-tube as a whole due to exceessive bearing reactions. These problems and many others could be experienced by marine engineers working on board ships or could be seen as illustrated in Solumsmoen [2] and Svenson[3]

PROPELLER EXCITATIONS

When a propeller is situated behind a ship's hull, it tends to be irregularly fed due to the non-uniform wake pattern, and the turbulent layer of the water streams affected by the after hull. In such case the torque caused by the propulsive plant when transmitted into thrust at the propeller blades will induce variations of forces and moments which consequently influence the behaviour of the shaft.

A study by Frivold [4], on four and five bladed propellers showed that not only there are variations of torque and moments, with respect to the angles of rotation, but that there are also variations in the values of thrust creating horizontal and vertical forces together with eccentricties acting on the propeller shaft. These variations in thrust with eccentricities are the cause of the whirling phenomenon or the so called "lateral vibrations".

The frequencies of these forces are equal to the propeller blade rate -i.e. the propeller blade frequency-and its multiples- e.g. for a four bladed propeller running at 180 rpm., the frequencies for the first harmonics will be: 120 Hz for the fundamental harmonic and 240 Hz for the second harmonic.

FREQUENCIES OF LATERAL VIBRATIONS

The method used here is based on the known simple beam theory given by Timoshinko [5]. The calculations are considering the following assumptions throughout the calculations:

The eccentric force acting on the blade is assumed of a max of 10 % of the mean thrust, and is applied at a mean distance of eccentricity.

The rotios between deflections and slopes at any point remain constant.

The first support which is meant to be the long stern tube bearing is considered fixed relative to the overhanged propeller shaft.

The shaft is considered elastic and massless relative to the mass of propeller. Hence the static bending moment of each span is neglected.

The effect of axial thrust force is neglected.

A SYSTEM OF THREE MASSLESS SPANS

Consider the case of supported shaft as shown in Fig. (1-a) A thrust moment M is applied statically at the free end A, causing a deflection as shown in Fig. (1-b).

To study the system, one can separate the three spans and examins each span individually and then relate the effect of each span, on its adjacent.

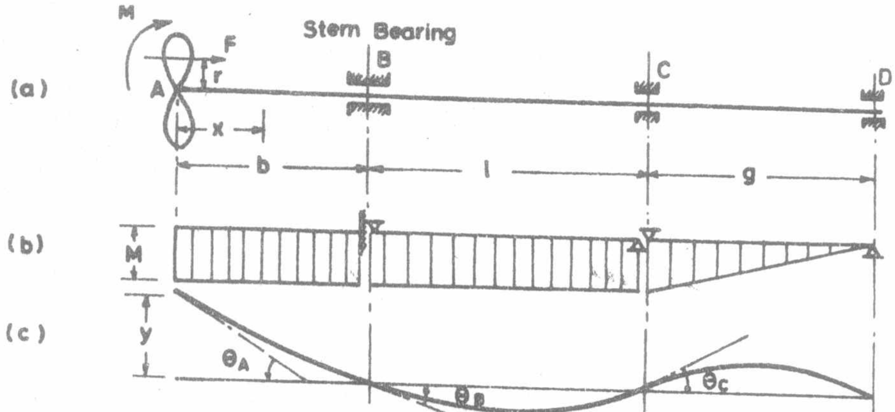

Fig. 1 Shaft system of three spans.

Span CD

The bending moment, the slope and the deflection at any point x measured from support C are given by:

$$M_x = M \frac{g - x}{g} \tag{1}$$

$$\theta_x = \frac{M}{EI} \left(x - \frac{x^2}{2g} \right) + C_1 \tag{2}$$

$$y_x = \frac{M}{EI} \left(\frac{x^2}{2} - \frac{x^3}{6g} \right) + C_1 x + C_2 \tag{3}$$

where ;

E = Young's modulus,
I = moment of inertia of shaft section.
C_1 & C_2 are constants of integration.

Applying the boundary conditions for span CD, we get :

$$C_2 = 0 \quad \text{and} \quad C_1 = -\frac{M}{EI} \cdot \frac{g}{3}$$

Thus, the slope equation of span CD becomes;

$$\theta = \frac{M}{EI} \left(x - \frac{x^2}{2g} - \frac{g}{3} \right) \tag{4}$$

The deflection equation for span CD becomes;

$$y = \frac{M}{EI} \left(\frac{x^2}{2} - \frac{x^3}{6g} - \frac{g.x}{3} \right) \tag{5}$$

At support C i.e. at x = 0 ,

$$\theta_c = \frac{-M}{EI} \frac{g}{3} , \quad y_c = 0 \tag{6}$$

Span BC

Applying the general equations for bending moment, slope, and deflection at any point x from support B, we get the same relations as (1) , (2) & (3)

Applying the boundary conditions at point C, we get the general slope and deflection equations, respectively;

$$\theta_{BC} = \frac{M}{EI}\left(x - \frac{1}{2} \right) \tag{7}$$

$$y_{BC} = \frac{M}{EI}\left(\frac{x^2}{2} - \frac{1.x}{2}\right) \tag{8}$$

At point B, we have

$$\theta_B = \frac{M}{EI} \cdot \frac{1}{2} \quad (+ \text{Ve for convention}) \tag{9}$$

Positive in + Ve clock wise rotation

From the above discussion, one can say that the total slope at point B is the sum of the slopes to the two spans to the right BC & CD.

Thus from Eq. (6) & (9), the total slope

$$\theta_{BB} = \theta_B + \theta_C \quad = \frac{M}{EI}\left(\frac{1}{2} - \frac{g}{3} \right) \tag{10}$$

Span AB

If this span is assumed fixed at point B-Stern Tube Bearing the bending moment, slope, and deflection of any point x measure from support A would be :

$$M_x = M$$

$$\theta_x = \frac{M.x}{EI} + C_1''$$

$$y_x = \frac{Mx^2}{2EI} + C_1''x + C_2''$$

Applying the boundary conditions, the general equations for slope and deflection at any point along span AB, become;

$$\theta_x = \frac{M}{EI}(x-b) \tag{11}$$

$$y_x = \frac{M}{EI}\left(\frac{x^2}{2} - b.x + \frac{b^2}{2} \right) \tag{12}$$

At point A

$$\theta_A = \frac{bM}{EI} \quad (+ \text{Ve for convention}) \tag{13}$$

The Equation of Motion

To lay down the equation of motion we assume that at all times the slope and moments are equal in value at both sides of each support.

Now for the separated span AB, the moments acting to the left of point B are :

Excitation periodic moment = $M_e \sin wt$

where :

M_e = F.r
w = angular frequency.

Inertia moment of the propller mass caused by the angular acceleration due to the slope θ_{BB} alone,

$$= \ddot{\theta}_{BB} \cdot (I_d + m_p b^2) \tag{14}$$

where:

I_d = mass moment of inertia of propeller about a diameter corrected for the effect of entrained water, calculated as given by Obrien [6].

m_p = the propeller mass corrected for the effect of the entrained water, Obrien [6] ;

b = length of the cantilever span ,

θ_{BB} = the part of the slope of the elastic line at point A contributed by the rest of the shaft.

Inertia moment of the propeller mass caused by the acceleration due to the slope θ_A alone,

$$= (I_d \cdot \ddot{\theta}_A + m_p \cdot \ddot{y}_A \cdot b) \tag{15}$$

From Eq. (12), the deflection at point A is given by :

$$y_A = \frac{M}{EI} \cdot \frac{b^2}{2} \tag{16}$$

For small deflections we can assume that;

$$\theta_b' = \frac{y_A}{b} = \frac{M}{EI} \cdot \frac{b}{2} = \frac{\theta_A}{2}$$

or;

$$y_A = \frac{b}{2} \cdot \theta_A \tag{17}$$

where:

θ_b' = the part of slope at point A contributed by the length of span AB

Thus:

$$I_d \cdot \ddot{\theta}_A + m_p \cdot \ddot{y}_A \cdot b = I_d \cdot \ddot{\theta}_A + m_p \cdot \ddot{\theta}_A \cdot \frac{b^2}{2}$$

$$= (I_d + m_p \cdot \frac{b^2}{2})\, \ddot{\theta}_A \tag{18}$$

Elastic moment transmitted by the shaft at B

$$M = \theta_{BB} \cdot \frac{E.I}{(\frac{l}{2} - \frac{g}{3})}$$

From equations (10) & (13) the ratios of slopes are:

$$\frac{\theta_A}{\theta_{BB}} = \frac{b}{(\frac{l}{2} - \frac{g}{3})} \tag{19}$$

The equation of motion is then rewritten as:

$$M_e \sin wt = \left(I_d + m_p \frac{b^2}{2}\right).\ \ddot{\theta}_A + \left(I_d + m_p b^2\right).\ \ddot{\theta}_{BB} +$$

$$\theta_{BB}.\left(\frac{E.I}{\frac{l}{2} - \frac{g}{3}}\right) \qquad (20)$$

Substituting for θ_{BB}, we get :

$$M_e \sin wt = \frac{\ddot{\theta}_{BB}.\ b}{\frac{l}{2} - \frac{g}{3}}\left(I_d + m_p \frac{b^2}{2}\right) + \left(I_d + m_p b^2\right).\ \ddot{\theta}_{BB} +$$

$$\theta_{BB}\left(\frac{E.I}{\frac{l}{2} - \frac{g}{3}}\right) \qquad (21)$$

To solve for θ_{BB}, we assume that ;

$$\begin{aligned} \theta_{BB} &= A.\ \sin w\,t + B.\ \cos wt \\ \dot{\theta}_{BB} &= A.w.\ \cos wt - B.\ w.\ \sin wt \\ \ddot{\theta}_{BB} &= -\ A.w^2.\ \sin wt - B.\ w^2.\ \cos wt \end{aligned} \qquad (22)$$

Substitute equations (22) into equation (21), and equate alike terms, we get :

$$M_e = -\ A.w^2\left[\left(\frac{b}{\frac{l}{2} - \frac{g}{3}}\right).\left(I_d + m_p \frac{b^2}{2}\right) + \left(I_d + m_p b^2\right)\right] + A\ \frac{E.I}{\frac{l}{2} - \frac{g}{3}}\ .$$

$$0 = -B.w^2.\left[\left(\frac{b}{\frac{l}{2} - \frac{g}{3}}.\ \left(I_d + m_p \frac{b^2}{2}\right) + \left(I_d + m_p b^2\right)\right] + B\ \frac{E.I}{\frac{l}{2} - \frac{g}{3}}$$

At resonance, the amplitude of θ_{BB} is infinite, hence ;

$$-\ w^2.\left[\left(\frac{b}{\frac{l}{2} - \frac{g}{3}}\ .\ \left(I_d + m_p \frac{b^2}{2}\right) + \left(I_d + m_p b^2\right)\right] + \frac{E.I}{\frac{l}{2} - \frac{g}{3}} = 0 \qquad (23)$$

Finally the natural frequency in cycles per min. is found to be :

$$f = \frac{30}{\pi}\sqrt{\frac{E.I}{I_d.\left(b + \frac{l}{2} - \frac{g}{3}\right) + m_p\, b^2.\left(\frac{b}{2} + \frac{l}{2} - \frac{g}{3}\right)}} \qquad (24)$$

The total deflection of the propeller tip is:

$$Y_{pr} = y_A + \theta_{BB} \cdot b$$

$$= \frac{Mb}{E.I} \cdot \left(\frac{b}{2} + \frac{l}{2} - \frac{g}{3}\right) \qquad (25)$$

A SYSTEM OF FOUR MASSLESS SPANS

If a further span DE of length -g- is added to the previously studied system, we get a four span shafting system. Fig. (2).

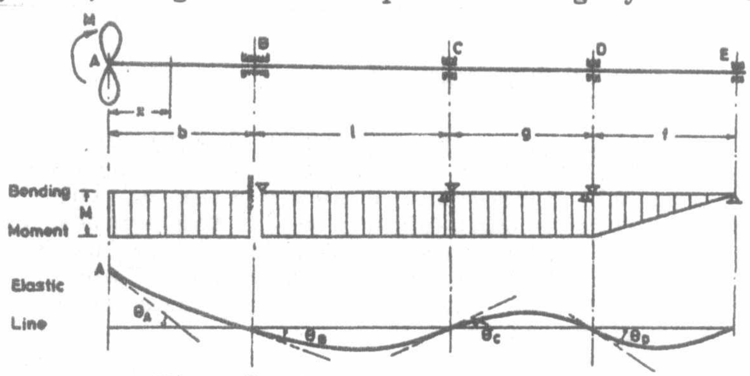

Fig. 2. A System of Four Spans

Starting from the first span to the right -DE- and proceding backwards till we reach the overhanged span -AB-, we would end up with two similar formulae as (24) & (25).

Thus the natural frequency in sycles per min and a tip deflection for a four massless span system would be:

$$f = \frac{30}{\pi}\sqrt{\frac{E.I}{I_d \cdot \left(b + \frac{l}{2} - \frac{g}{2} + \frac{f}{3}\right) + m_p b^2 \cdot \left(\frac{b}{2} + \frac{l}{2} - \frac{g}{2} + \frac{f}{3}\right)}} \qquad (26)$$

$$Y_{pr} = \frac{M}{E.I} \cdot b \cdot \left(\frac{b}{2} + \frac{l}{2} - \frac{g}{2} + \frac{f}{3}\right) \qquad (27)$$

GENERAL FORMULA FOR ANY NUMBER OF MASSLESS SPANS

From the similarity of the Eq. (24) & (26), a general formula could be introduced to calculate the natural frequency of a shafting system of any number of spans.

$$f = \frac{30}{\pi}\sqrt{\frac{E.I}{I_d\left(L_o + \sum_{i=1}^{k}(-1)^i \cdot \frac{L_i}{2}(-1)^n \frac{L_1}{3}\right) + m_p b^2\left(\frac{L_o}{2} + \sum_{i=1}^{k}(-1)^i \frac{L_i}{2} + (-1)^n \frac{L_1}{3}\right)}} \qquad (28)$$

The general equation for propeller deflection would:

$$Y_{pr} = \frac{M}{E.I} . L_o . \left[\frac{L_o}{2} + \sum_{i=1}^{k} (-1)^i . \frac{L_i}{2} + (-1)^n . \frac{L_1}{3} \right] \qquad (29)$$

where:

- n= number of total spans studied,
- k= $n-1$,
- L_o= length of overhang,
- L_1= length of the last span, (forward end)
- L_i = lengths of spans in between, keeping in mind that L_1 should be always taken equal to zero.

E, I, and I_d are as given before.

SHAFT SYSTEMS WITH CONSIDERED MASSES

The following is a method for calculating the natural frequency taking into consideration the effect of the shaft mass. This method based on the assumption that at all times the elastic line of the shaft retains the same configuration, that obtained if the system is considered stationary with a pure static moment applied at point A. This means that the ratios between deflections and slopes at any point of the shaft remain constant. The form of elastic line will define by knowning the slope θ at point B.

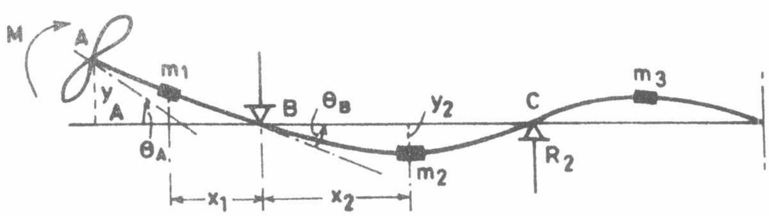

Fig. (3) Equivalent support beam with effect of shaft mass taken into consideration

From Fig. (3), let m_1 represent in general the masses between points A and B including the distributed mass of the shaft, x_1 the distance of these masses from point B , m_2 and x_2 the corresponding quantities to the right of point B.

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Supposing that the application of a static moment M at point A is causing a deformation of the shaft having slope θ_B, and the corresponding slope θ_A and the deflection at the masses m_1 and m_2 are in general y_1 and y_2. Also let R_c represent the reaction force at the support C. According to the original assumption the ratios

$\frac{\theta_A}{\theta_B}$, $\frac{y}{\theta_B}$ and $\frac{R_c}{\theta_B}$ remain constant at all times during a

vibration cycle.

Taking moments for the entire shaft about point B, the differential equation of motion with variable the angle θ which is the instaneous value of the slope at point B.

$$M_e \sin wt = I_d \frac{\theta_A}{\theta_B}\ddot{\theta} + \sum m_1 \frac{y_1}{\theta_B} x_1 \ddot{\theta} \quad \sum m_2 \frac{y_2}{\theta_B} x_2 \theta + + \frac{R_c l}{\theta_B}\theta \qquad (30)$$

where:

I_d = the moment of inertia of the propeller about a diameter.

M_e sin wt= the excitation moment

l = distance of bearing C from support B.

Eq. (30) can be solved in a manner similar to that of massless shaft, and the solution for the natural frequency in cycle per minute will be :

$$f = \frac{30}{\pi}\sqrt{\frac{R_c l}{I_d \theta_A + \sum m_1 y_1 x_1 + \sum m_2 y_2 x_2}} \qquad (31)$$

If more than one span between bearings is to be taken into consideration, the same principle of assuming the configuration of the elastic line remaining proportionalety the same can be accepted, and after the elastic line is drawn for some value of M the quantities involved in the equation of the elastic line can be obtained the equation then takes the form:

$$f = \frac{30}{\pi}\sqrt{\frac{\sum Rl}{I_d \theta_A + \sum myx}} \qquad (32)$$

where :

l = distance of any bearing from support B, (Stern-tube bearing)

R = the reactions at each support,

m = mass of shaft element,

y = corresponding deflection of shaft element,

x = distance from each element to support B, For the following cases, the numerator and the denominator are analysed separately and then subtituted in Eq. (32) to get the fundamental frequency of whirl.

Case of Three Spane

The propeller mass only causes deflection y_{AA}, Eq. (25) Hence,

$$(myx)_p = m_p \cdot \frac{Mb^2}{E.I} \cdot b^2 \left(\frac{b}{2} + \frac{1}{2} - \frac{g}{3} \right) \quad (33)$$

For mass of span AB

The equation of deflection:

$$y_1 = y_x + \theta_{BB} \cdot x_1$$

Using Eq. (12) & Eq. (10) and substituting for $x = b - x_1$, we get;

$$\sum m_1 y_1 x_1 = \mu_1 \frac{M}{E.I} \cdot b^3 \left(\frac{b}{8} + \frac{1}{6} - \frac{g}{9} \right) \quad (34)$$

For mass span BC

$$y_2 = \frac{M}{EI} \left(\frac{x_2^2}{2} + \frac{1.x_2}{2} - \frac{g.x_2}{3} \right)$$

$$\sum m_2 y_2 x_2 = \mu_2 \cdot \frac{M}{E.I} 1^3 \left(\frac{7}{24} 1 - \frac{g}{9} \right) \quad (35)$$

For mass of span CD

Using Eq. (5) & having $x = x_3 - 1$, we get ;

$$\sum m_3 y_3 x_3 = \mu_3 \frac{M}{E.I} \left(-\frac{7}{360} \cdot g^4 + \frac{1}{45} \cdot 1.g^3 - \frac{1}{20} 1^2 \cdot g^2 + \frac{3}{4} 1^3 g + \frac{1}{6} 1^4 \right) \quad (36)$$

Having defined the terms I_d & θ_A we may conclude the denominator of Eq. (32) to be:

$$= \frac{M}{EI} \left(\left(b + \frac{1}{2} - \frac{g}{3}\right) . I_d + m_p b^2 \left(\frac{b}{2} - \frac{1}{2} - \frac{g}{3} \right) + \mu b^3 \left(\frac{b}{8} + \frac{1}{6} - \frac{g}{9} \right) + 1^3 \left(\frac{71}{24} - \frac{g}{9} \right) \mu + \right.$$

$$\left. + \mu \left(\frac{7 g^4}{360} + \frac{1g^3}{45} - \frac{1^2 g^2}{20} + \frac{31^3 g}{4} + \frac{1^4}{6} \right) \right) \quad (37)$$

where :

μ_1, μ_2, μ_3 = mass per unit length of diffcrent spans

μ = mass per unit length of all spans where b,l,g, are the equivalent lengths.

The structure is then solved to calculate the reactions at the different supports.
Hence:

$$\sum R.L = R_C . 1 + R_D . (1+g) \quad (38)$$

Substituting back in Eq. (38) we can calculate the first frequency of vibration.

Case of Four Spans

The same procedure is applied and the different corresponding elements of Eq. (32) would be :

$$(myx)_p = m_p \cdot \frac{Mb^2}{EI} \left(\frac{b}{2} + \frac{l}{2} - \frac{g}{2} + \frac{f}{3}\right) \tag{39}$$

$$\sum m_1 y_1 x_1 = \mu_1 \frac{M}{EI} \cdot b^3 \left(\frac{b}{8} + \frac{l}{6} - \frac{g}{6} + \frac{f}{9}\right) \tag{40}$$

$$\sum m_2 y_2 x_2 = \mu_2 \frac{M}{E.I} \cdot l^3 \left(-\frac{l}{24} - \frac{g}{6} + \frac{f}{9}\right) \tag{41}$$

$$\sum m_3 y_3 x_3 = \mu_3 \cdot \frac{M}{E.I} g.\left(-\frac{g^3}{24} - \frac{25}{12} l^3 - 2lg^2 + \frac{flg}{6} + \frac{fg^2}{9}\right) \tag{42}$$

$$\sum m_4 y_4 x_4 = \mu_4 \frac{M}{E.I} \left(\overline{Gf}^2 - \overline{Gl}^2\right)\left\{-\frac{1}{6f}\left[\frac{\overline{Gf}^3 - \overline{Gl}^3}{5} - \frac{\overline{Gl}^3}{2} - \frac{3}{4}\overline{Gl}\left(\overline{Gf}^2 - \overline{Gl}^2\right) + f.\overline{Gl}^2\right] + \right.$$

$$\left. \frac{\overline{Gf}^2 - \overline{Gl}^2}{8} + \frac{\overline{Gl}^2}{4} - \frac{f.\overline{Gl}}{3} - \frac{f^2}{9} - \frac{f.\overline{Gl}}{6}\right\}$$

where ; $\overline{Gl} = g + l$

$\overline{Gf} = g + l + f$

The equation for $\theta_A = \frac{M}{E.I}\left(b + \frac{l}{2} - \frac{g}{2} + \frac{f}{3}\right)$

Then the whole structure is solved to get the values of the nominator;

$$\sum R.L = R_C \cdot l + R_D\,(l+g) + R_E\,(l + g + f)$$

Finally the frequency equation is found.

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