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DISCONTINUOUS BOUNDARY ELEMENTS FOR TWO DIMENSIONAL INVISCID LAMINAR FLUID FLOW

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ABSTRACT

In the Boundary Element Method the familiar infinite system of integral equations is discretized by the introduction of boundary elements here ranging from constant through linear, quadratic ... etc.

Hitherto, in the case of linear and higher order elements, interelement continuity of the unknown functions has been imposed in applications, undoubtedly in analogy with finite element practice. In the Boundary Element Method there is no such requirement; so that interelement continuity is simply not necessary.

INTRODUCTION

The Boundary Element Method is established as a valid numerical technique for the solution of field problems equal to the finite Element Method in generality and surpassing it in computational effeciency in some cases[1].

In this method, the field problem is reduced to a problem on the boundary and is expressed in the form of an infinite system of boundary integral equations. In the numerical solution of a problem it is discretized by the introduction of boundary elements similar in context to finite elements; the integrals are evaluated numerically and the resulting system of algebraic equation is solved by familiar methods. As with the finite element method, a heirarchy of elements can be defined starting with a constant element and ranging through linear, quadratic and cubic element, etc. interelement continuity is unnecessary and gives rise to at least three problems [2] :

First, at a point where the Boundary is not smooth, the normal is not defined, thus the freedom there demands a valid normal (figure 1.a).

Second, At the boundary (or interface) where there is a change in the nature of boundary conditions (say between potential (stream function) and its normal derivative (velocity along boundaries)) it is apparent that both types of freedoms are constrained (figure 1.b).

Third, when the problem is partitioned into subdomains, there can be excessive constraints where several surfaces meet (figure 1.c). Accommodation

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to the second and third problems has been deviced by appropriate supression of degrees of freedoms and to the first by the introduction of two freedom nodes close to the geometric singularity, on either side. The last approach is objectionable since, if the nodes are not closely spaced the boundary integrals are not well discretized while if they are, the resulting algebraic equations are ill-conditioned. This is becuase proximities of freedom nodes implies linear dependance in the algebraic equations. The root of the problem is interelement continuity. This is necessary with finite elements [3] to maintain positive definiteness but is not a requirement in the boundary element method.

Discontinuous elements obviate these problems. A quadratic element is developed here and successfully applied to a number of test problems.

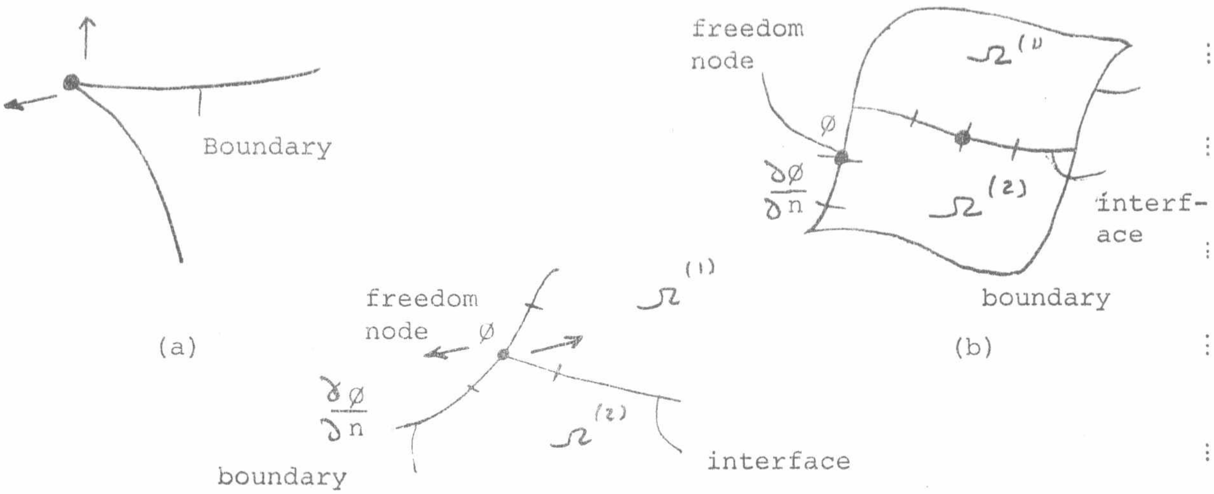

Figure 1. Problems facing continuous elements

THEORY

The Boundary Element Method can be interpreted as a weighted Residual Formulation [4] in which a solution, satisfying the governing equation, but otherwise arbitrary, was chosen as our approximate solution. Let us now adopt that aproach to study the Boundary Element Method with a certain type of influence solution: this solution represents an applied charge, concentrated at a point and called the fundamental solution (or kernel function) [5].

The inviscid laminar fluid flow is governed by laplace equation. Let us consider a scalar field function $\emptyset$ (stream function), defined over a domain Ω and its boundary Γ, which satisfies the following governing equation within the domain (figure 2),

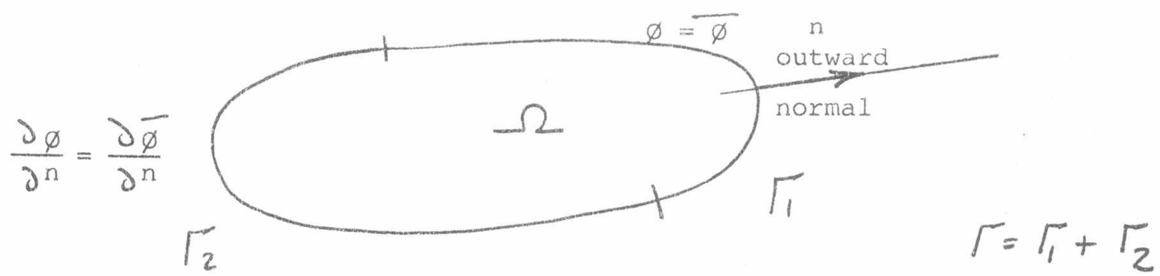

Figure 2. Notation of the flow problem