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OPTIMUM DIMENSIONS FOR AN AXIAL TORBOMACHINE STAGE

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ABSTRACT

The main dimensions of an axial turbomachine stage may be estimated to have a maximum hydraulic effeciency. Design formulae are obtained by writing the momentum equation for the rotor and stator of the machine, and by assuming that the head produced at any section is proportional to the radius. The only variable has to be chosen is the mean vane angle of the stator, such that the hydraulic effeciency is maximum, while the other main dimensions are calculated.

INTRODUCTION

An axial turbomachine stage consists mainly of an axial rotor and a stator. It is of practical importance to estimate the main dimensions of the rotor, as well as the stator to obtain the best hydraulic effeciency and performance. The main dimensions have to estimated are the hub to outer diameter ratio, the relative and absolute velocities angles, at the mean radius, and the form and number of vanes for both the rotor and stator,, [1]. Herein, the diameter ratio and the velocity triangles have been estimated analytically to have the maximum hudraulic effeciency of the stage. The design calculations are assumed for zero whirl velocity at inlet and outlet of the stage.

FUNDAMENTAL RELATIONS

For two dimensional flow, the momentum equation, for the rotor and stator cascades, may be written as, [2], Fig.1,

$$\text{rotor}: \rho Q(\overline{w_2} - \overline{w_1}) = \overline{p}\,S + (\overline{-F_1})$$

$$\text{stator}: \rho Q(\overline{c_3} - \overline{c_2}) = \overline{p}\,S + (\overline{-F_2}) \qquad (1)$$

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where ρ is the fluid mass density, $\overline{w}$ is the relative velocity, $\overline{c}$ is the absolute velocity, p is the pressure, $\overline{S}$ is the flow area, and $\overline{F}$ is the fluid force. The volume flow rate Q is,

$$Q = S\ c_a = \pi \cdot (R_o^2 - R_h^2)\ c_a \quad ,$$

where c_a is the axial flow velocity, and assumed to be constant through the pumping stage. The subscribts 1 and 2 are for the rotor inlet and outlet, while 3 is for the stator outlet. The hub and outer diameters are R_h and R_o.

Assuming that the whirl component $c_{u3} = 0$, and by using the geometrical relations given in Fig. 2, equation (1) may be written as, [3],

$$p_2 - p_1 = \rho c_a^2 \tan\alpha_2 \cdot \tan(\beta_m - \varepsilon_1) \quad ,$$

$$p_3 - p_2 = \rho c_a^2 \tan\alpha_2 \cdot \tan(\beta_m - \varepsilon_2) \quad , \qquad (2)$$

where ε is the glide angle, [4]. The relations between the different angles of the stage may be given as, Fig. 2,

$$\tan\beta_m = (\tan\beta_1 + \tan\beta_2)/2 \quad ,$$

$$\tan\alpha_m = (\tan\alpha_2 + \tan\alpha_3)/2 \quad ,$$

$$\tan\beta_1 = \tan\beta_1 + \tan\beta_2 \quad . \qquad (3)$$

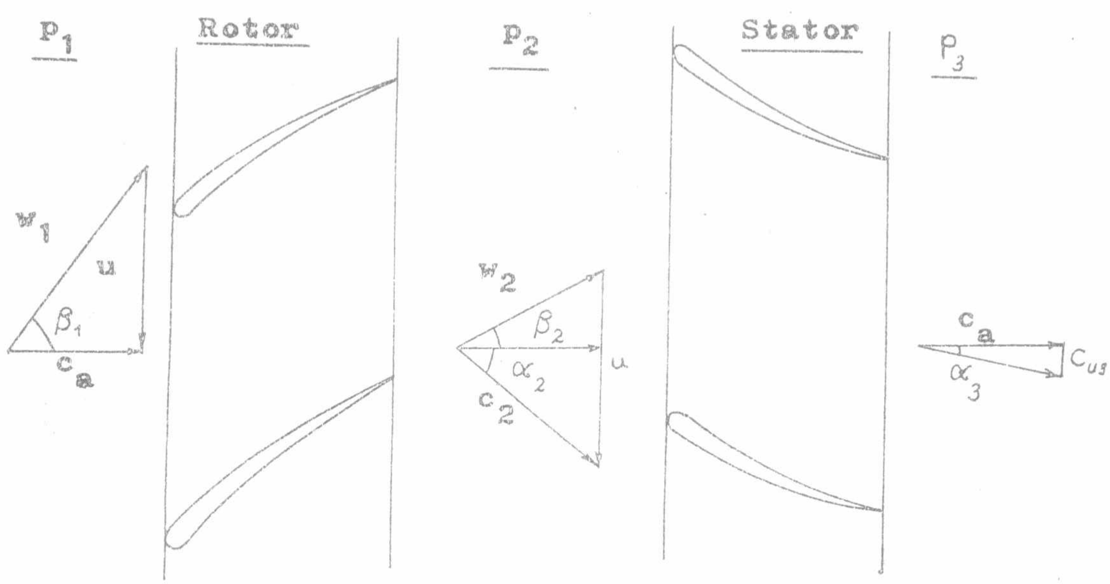

Fig. 1. Axial Pump Stage

6

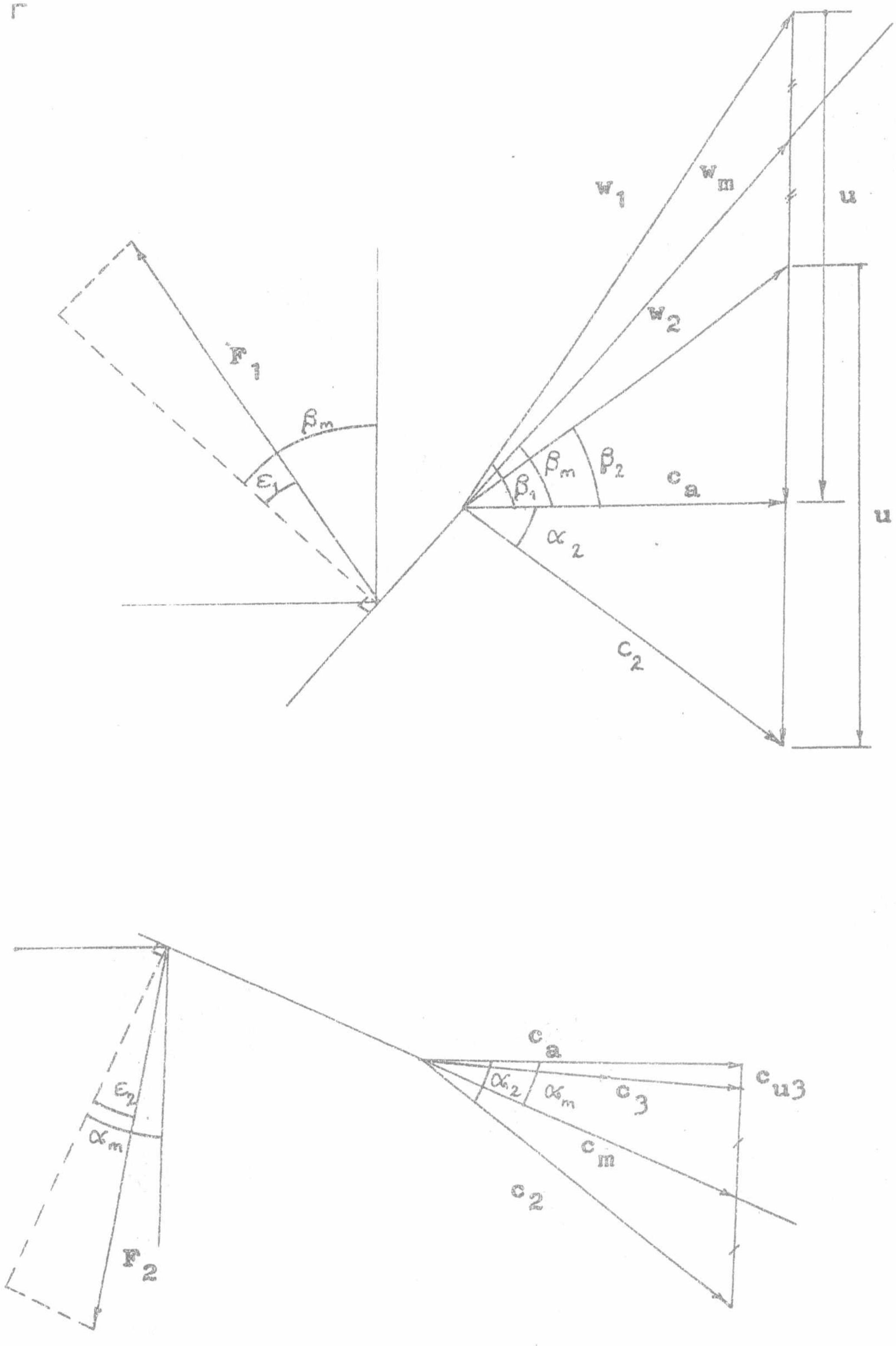

Fig. 2. Velocity Triangles at Inlet and Outlet of Rotor and Stator

The pumping head H for an axial stage is given by,

$$H = (p_3 - p_1) / \rho g \qquad (4)$$

and by using Eq. 2 and 3,

$$H = \frac{c_a^2}{g} \tan\alpha_2 (\tan(\beta_m - \varepsilon_1) + \tan(\alpha_m - \varepsilon_2)) \qquad (5)$$

MAIN DIMENSIONS OF AN AXIAL FLOW PUMP

The average puming head H for an axial pump stage may be calculated in assuming that,

- the flow velocity c_a is constant for all radii,
- the friction losses are negligible as compared to pumping head H, and hence the glide angle ε,
- the head is a linear function of the radius R. That means that the average pumping head is the head generated at the mean radius $\bar{R} = (R_h + R_o)/2$.

Then, the average head H is,

$$H = H(\bar{R}) = \frac{1}{R} \int_{R_h}^{R_o} H.R.dR \qquad (6)$$

writing the variables at mean radius as $\bar{\beta}_m$, $\bar{\alpha}_m$, $\bar{\beta}_2$, $\bar{\beta}_1$, and $\bar{\alpha}_2$, and by using Eq.(5), the average head is,

$$H = \frac{c_a^2}{g} \tan\bar{\alpha}_2 (\tan(\bar{\beta}_m - \varepsilon_1) + \tan(\bar{\alpha}_m - \varepsilon_2)) \qquad (7)$$

which can be written as,

$$H = K c_a^2 / g \qquad (8)$$

where K is constant, that depends only on α_m, β_m.

The pump shape number Ω, and the specific radius Λ are defined as, [3],

$$\Omega = \frac{\omega \sqrt{Q}}{(gH)^{3/4}} \qquad (9)$$

$$\Lambda = \frac{R_o (gH)^{1/4}}{\sqrt{Q}} \qquad (10)$$

Substituting for the flow rate Q, and for the axial velocity

$$c_a = \omega \bar{R} / \tan\bar{\beta}_1 \qquad (11)$$

Eq.(9) gives,

$$T = (K1 - \Omega^2) / (K1 + \Omega^2) \qquad (12)$$

where T is the diameter ratio ($T = R_h/R_o$), and K1 is,

$$K1 = 4\pi \tan^2 \beta_1 \,/\, K^{1.5} \tag{13}$$

Similarly the specific radius may be given as,

$$\sigma = K2\ (\ K1 + \Omega^2\ )\ /\ 2 \tag{14}$$

where $$K2 = K^{1/4}\ /\ K1\sqrt{\pi} \tag{15}$$

Equation (12) gives the relation between the diameter ratio T and the shape number Ω. The relative velocity angle β_1 can be given as, (cf. Fig. 2),

$$\tan \beta_1 = \tan \beta_m + \tan \alpha_m \tag{16}$$

CHOICE OF THE ANGLES α_m, β_m

From the previous equations, it is obvious that the diameter ratio T as well as the euter radious R_o may be estimated if the angles α_m, and β_m are known. The optimum value of β_m may be found by assuming a reasonable value of α_m, (to have maximum hydraulic effeciency η_h), and by relating the propeller hydraulic effeciency to the vane angles.

The hydraulic effeciency η_h is defined as the ratio between the actual pumping head, and the theoretical head of an ideal fluid,

$$\eta_h = \frac{\tan\ (\ \beta_m - \varepsilon_1) + \tan\ (\alpha_m - \varepsilon_2)}{\tan \beta_m + \tan \alpha_m}$$

By neglecting the effect of the drag, and hence the angles ε_1 and ε_2, the condition for maximum effeciency is,

$$\begin{aligned} \partial\eta/\partial\beta_m &= 0 \\ &= \tan^2 \beta_m - \tan^2 \alpha_m + 2 \tan \alpha_m . \tan \beta_m - 2 \end{aligned} \tag{17}$$

and from which the choice of angles α_m and β_m is reduced to the choice of α_m. The angle α_m is usually chosen in the order of 45^o to have the optimum hydraulic effeciency.

DESIGN PROCEDURE

For an axial pump, of given flow rate Q, head H and speed n, the main dimensions may be estimated to have a maximum hydraulic effeciency η_h. The following procedure as used,

1) Calculate the shape number Ω, Eq.(9).
2) Choose the angle α_m to be in the order of 45^o, to obtain the best hydraulic effeciency.

3) For a given α_m, the specific radius $\sim$ as well as the diameter ratio T are calculated by using Eq. (12 and 13). Hence the radii R_h and R_o are calculated.
4) The vane angle β_m is calculated by using Eq.(17).
5) By using Eq.(2 and 3), the angles β_1, β_2 and α_2 can be calculated.
6) Now, the velocity triangles can be drawn and the vanes shape can be chosen, [3].

CONCLUSIONS

The main dimensions of an axial pump stage, (rotor and stator) may be estimated by assuming a linear relation between the pumping head and the radius of the rotor, as a function of the pump flow rate, head and speed. Only, the stator mean vane angle α_m has to be assumed, on base of maximizing the hydraulic effeciency. The larger the tangent of the angle, the best effeciency is obtained. By knowing the main radii and the velocity triangles, the design procedure is completed by the choice of the vanes shapes and numbers.

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