OPTIMUM DIMENSIONS FOR AN AXIAL TORBOMACHINE STAGE

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ABSTRACT

The main dimensions of an axial turbomachine stage may be estimated to have a maximum hydraulic efficiency. Design formulae are obtained by writing the momentum equation for the rotor and stator of the machine, and by assuming that the head produced at any section is proportional to the radius. The only variable has to be chosen is the mean vane angle of the stator, such that the hydraulic efficiency is maximum, while the other main dimensions are calculated.

INTRODUCTION

An axial turbomachine stage consists mainly of an axial rotor and a stator. It is of practical importance to estimate the main dimensions of the rotor, as well as the stator, to obtain the best hydraulic efficiency and performance. The main dimensions have to be estimated are the hub to outer diameter ratio, the relative and absolute velocities angles, at the mean radius, and the form and number of vane for both the rotor and stator.[1] Herein, the diameter ratio and the velocity triangles have been estimated analytically to have the maximum hydraulic efficiency of the stage. The design calculations are assumed for zero whirl velocity at inlet and outlet of the stage.

FUNDAMENTAL RELATIONS

For two dimensional flow, the momentum equation, for the rotor and stator cascades, may be written as, [2], Fig.1,

rotor : \( \rho Q\left(\bar{v}_2 - \bar{v}_1\right) = \bar{p}S + \left(-\bar{F}_1\right) \)

stator: \( \rho Q\left(\bar{c}_3 - \bar{c}_2\right) = \bar{p}S + \left(-\bar{F}_2\right) \) (1)

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where \( \rho \) is the fluid mass density, \( \bar{V} \) is the relative velocity, \( \bar{C} \) is the absolute velocity, \( p \) is the pressure, \( S \) is the flow area, and \( F \) is the fluid force. The volume flow rate \( Q \) is,

\[
Q = S c_a = \pi \cdot \left( R_o^2 - R_h^2 \right) c_a
\]

where \( c_a \) is the axial flow velocity, and assumed to be constant through the pumping stage. The subscripts 1 and 2 are for the rotor inlet and outlet, while 3 is for the stator outlet. The hub and outer diameters are \( R_h \) and \( R_o \).

Assuming that the whirl component \( c_3 = 0 \), and by using the geometrical relations given in Fig. 2, equation (1) may be written as,

\[
\begin{align*}
P_2 - P_1 &= \rho c_a^2 \tan \alpha_m \cdot \tan \left( \beta_m - \epsilon_1 \right), \\
\epsilon_2 &= \rho c_a^2 \tan \alpha_m \cdot \tan \left( \beta_m - \epsilon_2 \right),
\end{align*}
\]

(2)

where \( \epsilon \) is the glide angle. The relations between the different angles of the stage may be given as, Fig. 2,

\[
\begin{align*}
\tan \beta_m &= \left( \tan \beta_1 + \tan \beta_2 \right) / 2, \\
\tan \alpha_m &= \left( \tan \alpha_2 + \tan \alpha_3 \right) / 2, \\
\tan \beta_1 &= \tan \beta_1 + \tan \beta_2.
\end{align*}
\]

(3)

Fig. 1. Axial Pump Stage
Fig. 2. Velocity Triangles at Inlet and Outlet of Rotor and Stator
The pumping head \( H \) for an axial stage is given by,

\[
H = \frac{p_3 - p_1}{\rho g}
\]  

and by using Eqs. 2 and 3,

\[
H = \frac{a^2}{g} \tan \alpha_2 \left( \tan \left( \beta_m - \epsilon_1 \right) + \tan \left( \alpha_m - \epsilon_2 \right) \right)
\]  

MAIN DIMENSIONS OF AN AXIAL FLOW PUMP

The average pumping head \( \bar{H} \) for an axial pump stage may be calculated in assuming that,

- the flow velocity \( c_a \) is constant for all radii,
- the friction losses are negligible as compared to pumping head \( H \), and hence the glide angle \( \epsilon \),
- the head is a linear function of the radius \( R \). That means that the average pumping head is the head generated at the mean radius \( R = (R_h + R_o)/2 \).

Then, the average head \( \bar{H} \) is,

\[
\bar{H} = H \left( \frac{R}{R_0} \right) = \frac{1}{R} \int_{R_h}^{R_o} H R \, dR
\]  

writing the variables at mean radius as \( \bar{\beta}, \bar{\alpha}, \bar{\alpha_2} \), and \( \bar{\alpha_2} \), and by using Eq. (5), the average head is,

\[
\bar{H} = \frac{a^2}{g} \tan \bar{\alpha_2} \left( \tan \left( \bar{\beta_m} - \epsilon_1 \right) + \tan \left( \bar{\alpha_m} - \epsilon_2 \right) \right)
\]

which can be written as,

\[
\bar{H} = K \frac{a^2}{g}
\]

where \( K \) is constant, that depends only on \( \alpha_m, \beta_m \).

The pump shape number \( n \), and the specific radius \( \lambda \) are defined as,

\[
\Omega = \frac{Q}{(gH)^{3/4}}
\]  

\[
\lambda = \frac{R_o (gH)^{3/2}}{\sqrt{Q}}
\]

Substituting for the flow rate \( Q \), and for the axial velocity \( c_a = \omega R / \tan \beta_1 \)

Eq. (9) gives,

\[
T = \left( K1 - \Omega^2 \right) / \left( K1 + \Omega^2 \right)
\]

where \( T \) is the diameter ratio \( (T = R_h/R_o) \), and \( K1 \) is,
\[ K_1 = 4\pi \tan^2 \beta / K^{1.5} \]  

Similarly, the specific radius may be given as,

\[ \rho = K_2 \left( K_1 + \frac{\rho^2}{K} \right) / 2 \]  
where \[ K_2 = K^{1/4} / K^{1/2} \]  

Equation (12) gives the relation between the diameter ratio \( T \) and the shape number \( \rho \). The relative velocity angle \( \beta \) can be given as, (cf. Fig. 2),

\[ \tan \beta = \tan \beta_m + \tan \alpha_m \]  

**CHOICE OF THE ANGLES \( \alpha_m, \beta_m \)**

From the previous equations, it is obvious that the diameter ratio \( T \) as well as the outer radius \( R \) may be estimated if the angles \( \alpha_m \) and \( \beta_m \) are known. The optimum value of \( \beta_m \) may be found by assuming a reasonable value of \( \alpha_m \), (to have maximum hydraulic efficiency \( \eta_h \)), and by relating the propeller hydraulic efficiency to the vane angles.

The hydraulic efficiency \( \eta_h \) is defined as the ratio between the actual pumping head, and the theoretical head of an ideal fluid,

\[ \eta_h = \frac{\tan (\beta_m - \varepsilon_m) + \tan (\alpha_m - \varepsilon_2)}{\tan \beta_m + \tan \alpha_m} \]

By neglecting the effect of the drag, and hence the angles \( \varepsilon_1 \) and \( \varepsilon_2 \), the condition for maximum efficiency is,

\[ \frac{\partial \eta}{\partial \beta_m} = 0 \]

\[ = \tan^2 \beta_m - \tan^2 \alpha_m + 2 \tan \alpha_m \tan \beta_m - 2 \]

and from which the choice of angles \( \alpha_m \) and \( \beta_m \) is reduced to the choice of \( \alpha_m \). The angle \( \alpha_m \) is usually chosen in the order of \( 45^\circ \) to have the optimum hydraulic efficiency.

**DESIGN PROCEDURE**

For an axial pump, of given flow rate \( Q \), head \( H \) and speed \( n \), the main dimensions may be estimated to have a maximum hydraulic efficiency \( \eta_h \). The following procedure is used,

1) Calculate the shape number \( \rho \), Eq. (9).
2) Choose the angle \( \alpha_m \) to be in the order of \( 45^\circ \), to obtain the best hydraulic efficiency.
3) For a given $\alpha$, the specific radius $R$ as well as the diameter ratio $T$ are calculated by using Eq. (12 and 13). Hence the radii $R_n$ and $R_o$ are calculated.

4) The vane angle $\beta$ is calculated by using Eq. (17).

5) By using Eq. (2 and 3), the angles $\beta_1$, $\beta_2$ and $\alpha_2$ can be calculated.

6) Now, the velocity triangles can be drawn and the vanes shape can be chosen, [3].

CONCLUSIONS

The main dimensions of an axial pump stage, (rotor and stator) may be estimated by assuming a linear relation between the pumping head and the radius of the rotor, as a function of the pump flow rate, head and speed. Only, the stator mean vane angle $\alpha_m$ has to be assumed, on base of maximizing the hydraulic efficiency. The larger the tangent of the angle, the best efficiency is obtained. By knowing the main radii and the velocity triangles, the design procedure is completed by the choice of the vanes shapes and numbers.

REFERENCES


