MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

A NEW METHOD FOR CALCULATING COMPRESSIBILITY EFFECT ON AIRFOILS

M.E. ESSAWY*

ABSTRACT

Numerical methods for calculating compressibility effect on airfoils play a significant role in aeronautics because the experimental investigation in wind tunnels is more expensive. In literature, there are three well known methods i.e. method of Prandtl-Glauert, Karman-Tsien and Christianovitch. The last method is the most accurate one but it has some weak points.

This paper presents a new method based on a non linearized approach in the Hodograph plane. The local velocity distribution and the pressure coefficient in the compressible fluid flow can be determined in terms of their corresponding values in the incompressible flow and the required Mach number and vice versa; because the formulae are explicit. This method is programmed on a digital computer consequantly, the compressibility effect is determined in a short time. The present method exhibits an order of magnitude reduction in computing time over other methods with comparable accuracy.

NOMENCLATURE

A, B	Two functions defined by equation (1)
C_p	Coefficient of pressure
K	Constant
M	Mach number
P	Local static pressure
$\bar{P}$	relative pressure P/P_o
γ	ratio of specific heats
λ	dimensionless velocity (V/V_{cr})

Subscripts

C	Compressible
cr	critical
i	incompressible
O	stagnation
∞	free stream

* Lecturer of Aerodynamics, Dpt. of Aeronautics, Military Technical College, Cairo, Egypt. (Member AIAA).

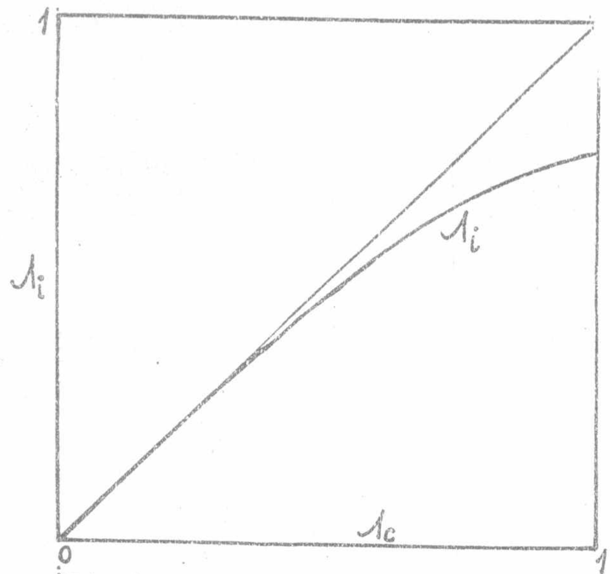

Fig.1, Christianovitch Diagram

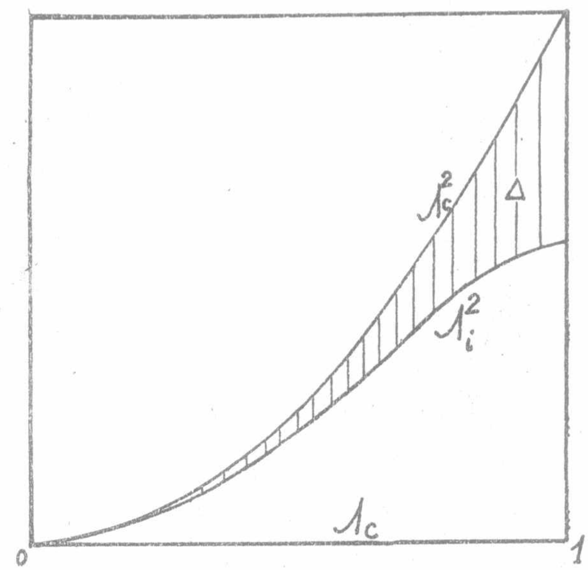

Fig.2. Deduction Diagram

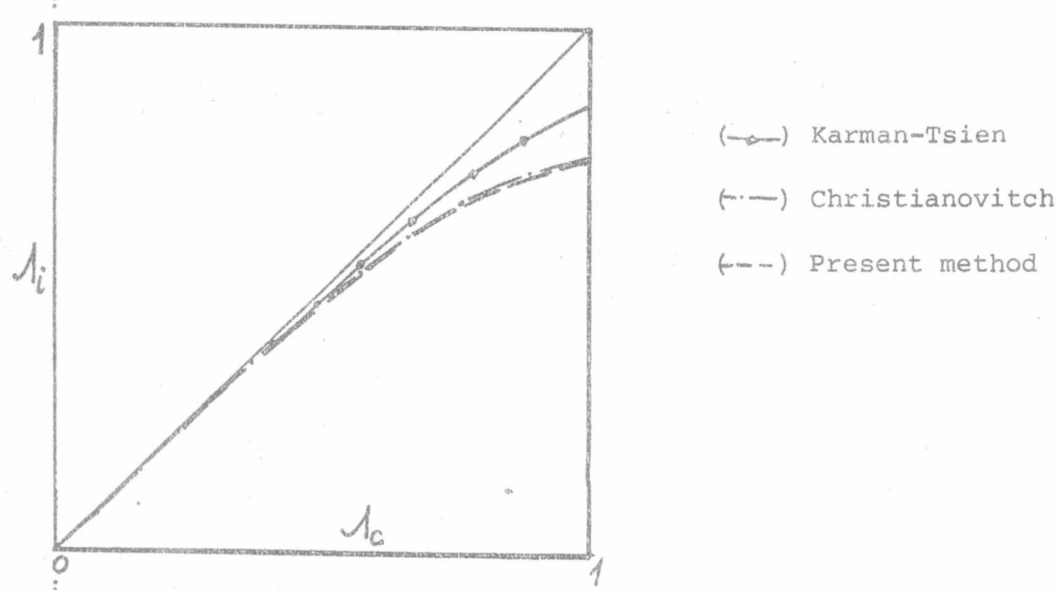

Fig.3. Comparison Diagram.

Following way:

The diagram of Christiancvitch (Fig.1.) is replotted such that λ_c^2 and λ_i^2 are plotted versus λ_c (Fig.2.).
It is evident that:

$$\lambda_i^2 = \lambda_c^2 - \Delta \qquad (2)$$

It is evident that eq (8) is a simple algebriac equation in terms of C_{p_i} and M_∞. A computer program is prepared such that C_{p_c} is tabulated at arbitrary constant values of M_∞ and C_{p_i}

COMPARISON

The present method is compared with other available methods in the following two ways:

a- The value of $\bar{p}$ at stagnation point and at critical state are calculated and following results are obtained:

i - It could be proved easily that at the stagnation point both the exact and present method have the same value :

$$(C_{p_c})_o = \left[(1+0,2\ M_\infty^2)^{3,5} - 1\right] / 0,7\ M_\infty^2$$

ii- Regarding the critical state, a small difference in the value of $(\bar{P})_{cr}$ determined by the present method and its exact value .

$(\bar{p})_{cr} = 0,5283$ exact value

$(\bar{p})_{cr} = 0,5256$ present method.

∴ The relative difference is equal to 0,51 % only that is very acceptable percentage of error. Consequently, the present method is in good agreement with the exact method.

b- The second way of comparison is shown by plotting the relation $\lambda_c = f(\lambda_i)$ by different methods. Fig. 3 shows good agreement between the present method and diagram of Christianovitch that is considered as an accurate approach.

Moreover, the present method gives an order of magnitude time reduction than the method of Christianovitch, because values of C_{p_c} corresponding to given values of C_{p_i} and M_∞ could be, easily, read out of prepared tables . In other words, the present method is more convenient for computational field than that of Christianovitch with a comparable degree of accuracy.

CONCLUSION

Regarding the determination of compressibility effect, the method presented in this paper determines the local velocity distribution of compressible fluid flow two-dimensional airfoils by a simple explicit function. Consequently, the pressure coefficient is determined by means of a simple explicit function, too, in M_∞ and C_{p_i} . As a final result, the values of C_{p_c} corresponding to different arbitrary constant values of C_{p_i} and M_∞ became available in a tabulated form.

The present method is more convenient for computational aerodynamics than other methods with comparable accuracy to the most accurate method of Christianovitch. It exhibits an order of magnitude reduction in computing time than the latter.

ACKNOWLEDGEMENT

The autrer would like to express his appraciation to Prof. V.Brož, Aeronautical Department, Charl's University, Prague, for his helpful discussions and guidance.