UTILIZATION OF THE PARAMETRIC TRANSPORTATION PROBLEM IN MECHANICAL ENGINEERING

BY

Lt. Col. Dr. Said A. HASSAN*, Brig. Dr. M.S.A.OSMAN** and

Dr. A.ABOUEL NOUR***

ABSTRACT

In many practical transportation problems, the optimal solution is stated. But the question is always raised by the decision maker: What happens to this optimal solution when changes due to dynamic situations occur and /or due to inaccurate estimation of the system parameters.

This paper presents a computer FORTRAN program solving the practical transportation problem considering possible variations of its parameters both in the objective function and in the constraints. This program is based on theoritical results obtained in another paper written by the same authors that solves this problem directly on the transportation matrix [1].

These results can be used in many practical engineering applications in the distribution of spare parts, raw materials, and final products. As an example, the program was used for solving the distribution of final products from different stores to the individual governorate in Egypt with the objective to minimize the total transportation cost.

* Department of Motor Vehicle, M.T.C, Cairo, Egypt.
** Department of Mathematics, M.T.C., Cairo, Egypt.
*** Professor, Fac. of Engineering, Zagazig University, Egypt.

INTRODUCTION

For certain fixed data of the transmportation problem, an optimal solution can be determined. But any real practical problem is a dynamic one. All the parameters of the problem are continóusly changing and the obtained optimal solution should be discussed once more.

A study of the effect of changes in parameters both in the objective function and in the constraints on the optimal solution and on the optimal value of the objective function is very necessary and valuable. This study enhances the scope of application of the stated results to different variations of the problem and saves the solution of new problems from the beginning.

CHANGES IN UNIT COST OF A BASIC VARIABLE

If the unit cost of a basic variable x_{rs} changes by a value λ_{rs}^{cb}, positive or negative, i.e. $c'_{rs} = c_{rs} + \lambda_{rs}^{cb}$. Accordingly some of the simplex multipliers change which will cause some changes in the marginal costs of non-basic variables. Then there will be a range for the value of λ_{rs}^{cb} in which the optimal solution remains the same. This range is given by the condition that all the new marginal costs δ'_{ij} for all the non-basic variables remain non-negative. Outside this range, the solution becomes nô more optimal and the non-basic variable x_{pq} which has a new marginal cost $\delta'_{pq} = \min \delta'_{ij}$ for all the non-basic variables should be introduced into the basis.

CHANGES IN UNIT COST OF A NON-BASIC VARIABLE

If the unit cost c_{rs} of a non-basic variable is increased by a value λ_{rs} positive, the simplex multipliers remain unchanged. Also the new marginal costs for all the cells are not changed except $\delta'_{rs} = c'_{rs} - u_r - v_s > \delta_{rs}$.
Thus the optimal solution will not change.

On the other hand, if this unit cost decreases, then there will be a range R_{rs} for the reduction of c_{rs} through which the optimal solution remains the same. This range determines the lower limit c_{rs}^L for this unit cost as $c_{rs}^L = c_{rs} - R_{rs}$

The lower limit is limited by the value of the new marginal cost $\delta'_{rs} = c_{rs}^L - u_r - v_s$ reaching zero, i.e.

$$c_{rs}^L = u_r + v_s \qquad (1)$$

Then the range R_{rs} is given by the equation:

$$R_{rs} = c_{rs} - c_{rs}^L = c_{rs} - u_r - v_s \qquad (2)$$

Delow this range of change, the variable x_{rs} should be introduced to the new basis.

CHANGES IN ONE LINE OF THE COST MATRIX

If λ^L, positive or negative, is added to all unit costs of one line L, row or columan, the optimal solution remains unchanged. But the value of the objective function will be changed to z_{new} :

$$z_{new} = z + \lambda^L \sum_L x^L \qquad (3)$$

where

z = Old value of the objective function

$\sum_L$ = Summation over the line L,

x^L = Value of a basic variable on the line L.

CHANGES IN A DESTINATION REQUIREMENTS

If the requirements b_j of the jth warehouse is changed by a value $\lambda_j^b \geqslant -b_j$, there may be the question: Which supply should change his amount in order to balance the problem and in the same time the new value of the objective function will be minimum.

The answer may be obtained by changing the supply a_i of each store in turn and calculate the change value of the objective function from the equation

$$\Delta z = \lambda^b \ (u_i + v_j) \qquad (4)$$

Then that variant with the minimum value $z_{old} + \Delta z$ is chosen

UTILIZATION OF THE PROGRAM

A computer Fortran program [2] is used to solve the classical transportation problem, and a special routine is witten to study the different cases of variations in the transportation problem parameters.

Four cases are studied in this routine:

1. Changes in unit costs of basic variables without affecting the optimal solution, the lower and upper limits for each unit cost is given. The new variable that should be introduced to the new basis when the change is outside the determined range is also given.
2. Changes in unit costs of non-basic variables without affecting the optimal solution, the lower limits for costs of all the non-basic variables are determined.
3. Changes in unit costs of one row, for illustration the transportation rows are affected in turn by the amounts 5,4,3,2,1, -1,-2,-3,-4,-5 unit costs. The new value of the objective function is calculated in each case.
4. Changes in requirements of destinations, for illustration the demand is changed by one unit positive and negative in turn. The change in the objective function corresponding to the assumed change in each supply is calculated.

AN EXAMPLE OF APPLICAATION

The program was used for solving a real problem of destribution of spare parts from twelve different stores in different locations in Egypt to fifty five factories. The available source capacities and the destination demands are shown in Table 1. The cost Coefficients are shown in Table 2. Table 3, gives the optimal solution and the corresponding value of the objective function for the static problem. The effect of variation in problem parameters are shown in Table 4. Fig.1 shows the results of changes in unit costs corresponding to the first four rows and Fig.2 shows the effect of changes in requirements of the first factory.

Table 1. Source capacities and distination demands

AVAILABLE SOURCE CAPACITIES	DESTINATION DEMANDS
S(1) = 12779	D(1) = 1700
S(2) = 6104	D(2) = 196
S(33) = 2322	D(3) = 73
S(4) = 1820	D(4) = 25
S(5) = 2100	D(5) = 73
S(6) = 15	D(6) = 56
S(7) = 14	D(7) = 105
S(8) = 179	D(8) = 85
S(9) = 41	D(9) = 270
S(10) = 177	D(10) = 50
S(11) = 746	D(11) = 196
S(12) = 68	D(12) = 88
	D(13) = 90
	D(14) = 607
	D(15) = 30
	D(16) = 20
	D(17) = 85
	D(18) = 52
	D(19) = 25
	D(20) = 17
	D(21) = 27
	D(22) = 241
	D(23) = 8
	D(24) = 15
	D(25) = 20
	D(26) = 22211

Table 2. The cost coefficients.

THE COST COEFFICIENTS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1	30	240	210	125	150	195	130	85	60	140	100	80	170	30	127	147	262	377	900	900	900	900	597	900	148	0
2	25	230	200	120	140	185	120	75	50	130	90	70	160	20	117	140	252	370	900	900	900	900	590	900	138	0
3	224	30	900	900	900	215	180	205	170	110	130	160	65	230	900	900	900	900	900	900	900	300	900	900	900	0
4	90	130	184	169	224	180	52	75	40	40	20	30	75	95	205	230	900	900	900	900	900	390	900	900	139	0
5	80	210	150	75	145	120	55	20	35	95	75	64	135	85	195	220	900	900	900	900	900	900	900	900	900	0
6	191	900	20	75	174	65	130	150	199	167	184	195	232	200	900	900	900	900	900	900	900	900	900	900	45	0
7	134	900	174	100	20	200	175	179	260	224	200	169	294	140	249	274	900	900	900	900	900	900	900	360	130	0
8	137	900	900	900	900	900	900	207	187	900	237	207	900	135	45	50	120	230	320	460	900	900	450	900	900	0
9	257	900	900	900	900	900	900	900	900	900	900	900	900	240	156	120	50	110	200	340	600	900	330	460	900	0
10	900	900	900	900	900	900	900	900	900	900	900	900	900	900	275	230	110	50	90	230	490	900	220	450	900	0
11	900	900	900	900	900	900	900	900	900	900	900	900	900	900	505	460	340	230	140	50	260	900	450	224	900	0
12	900	900	900	900	900	900	900	900	900	900	900	900	900	900	900	900	600	490	400	260	50	900	900	484	900	0

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Table 3. The optimal solution.

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OPTIMAL SOLUTION
      Z =   214145

NON ZERO VALUES OF X(I,J)
X( 1,26)= 12779
X( 2, 1)=  1700
X( 2, 5)=    59
X( 2,14)=   607
X( 2,25)=    20
X( 2,26)=  3718
X( 3, 2)=   196
X( 3,13)=    90
X( 3,22)=   241
X( 3,26)=  1795
X( 4, 7)=   105
X( 4,10)=    50
X( 4,11)=   196
X( 4,12)=    88
X( 4,26)=  1381
X( 5, 3)=    58
X( 5, 4)=    25
X( 5,,6)=    56
X( 5, 8)=    85
X( 5, 9)=   270
X( 5,26)=  1606
X( 6, 3)=    15
X( 7, 5)=    14
X( 8,15)=    30
X( 8,16)=    20
X( 8,26)=   129
X( 9,17)=    41
X(10,17)=    44
X(10,18)=    52
X(10,19)=    25
X(10,23)=     8
X(10,26)=    48
X(11,20)=    17
X(11,24)=    15
X(11,26)=   714
X(12,21)=    27
X(12,26)=    41
```

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Table 4. Variation in problem parameters.

CHANGES IN UNIT COSTS OF BASIC VARIABLES
====================================

LOWER LIMIT OF UNIT COST C(1,26)= 0
UPPER LIMIT = 5

NEW BASIC VARIABLE IS X(1, 1)

LOWER LIMIT OF UNIT COST C(2, 1)= 0
UPPER LIMIT = 30

NEW BASIC VARIABLE IS X(1, 1)

LOWER LIMIT OF UNIT COST C(2, 5)= 28

NEW BASIC VARIABLE IS X(7,25)

UPPER LIMIT = 145

NEW BASIC VARIABLE IS X(5, 5)

...
...
LOWER LIMIT OF UNIT COST C(10,17)= 50

NEW BASIC VARIABLE IS X(9,26)

UPPER LIMIT = 120

NEW BASIC VARIABLE IS X(8,17)

CHANGES IN UNIT COST OF NON BASIC VARIABLES
==

LOWER LIMIT OF UNIT COST C(1, 1)= 25
RANGE = 5

LOWER LIMIT OF UNIT COST C(1, 2)= 30
RANGE = 210

LOWER LIMIT OF UNIT COST C(1, 3)= 150
RANGE = 60

LOWER LIMIT OF UNIT COST C(1, 4)= 75
RANGE = 50

LOWER LIMIT OF UNIT COST C(1, 5)= 140
RANGE = 10

LOWER LIMIT OF UNIT COST C(1, 6)= 120
RANGE = 75

...
...
...
LOWER LIMIT OF UNIT COST C(12,25)= 138
RANGE = 762

Table 4. Continued.

CHANGES IN UNIT COSTS OF ALL CELLS OF ONE LINE

```
ROW( 1) SUBTRACTING 5 UNITS      Z NEW = 150250
ROW( 1) SUBTRACTING 4 UNITS      Z NEW = 163029
ROW( 1) SUBTRACTING 3 UNITS      Z NEW = 175808
ROW( 1) SUBTRACTING 2 UNITS      Z NEW = 188587
ROW( 1) SUBTRACTING 1 UNITS      Z NEW = 201366
ROW( 1) ADDING      1 UNITS      Z NEW = 226924
ROW( 1) ADDING      2 UNITS      Z NEW = 239703
ROW( 1) ADDING      3 UNITS      Z NEW = 252482
ROW( 1) ADDING      4 UNITS      Z NEW = 265261
ROW( 1) ADDING      5 UNITS      Z NEW = 278040
...
...
...
ROW(12) ADDING      5 UNITS      Z NEW = 214485
```

CHANGES IN DEMANDS OF DESTINATIONS

```
CHANGE IN DEMAND OF DESTINATION NO 1 BY 1

STORE NO 1   DELTA Z =  25
STORE NO 2   DELTA Z =  25
STORE NO 3   DELTA Z =  25
STORE NO 4   DELTA Z =  25
STORE NO 5   DELTA Z =  25
STORE NO 6   DELTA Z =-105
STORE NO 7   DELTA Z = -95
STORE NO 8   DELTA Z =  25
STORE NO 9   DELTA Z = -35
STORE NO10   DELTA Z =  25
STORE NO11   DELTA Z =  25
STORE NO12   DELTA Z =  25

CHANGE IN DEMAND OF DESTINATION NO 1 BY -1

...
...
...
CHANGE IN DEMAND OF DESTINATION NO26 BY -1

...
...
...

.EOF
```

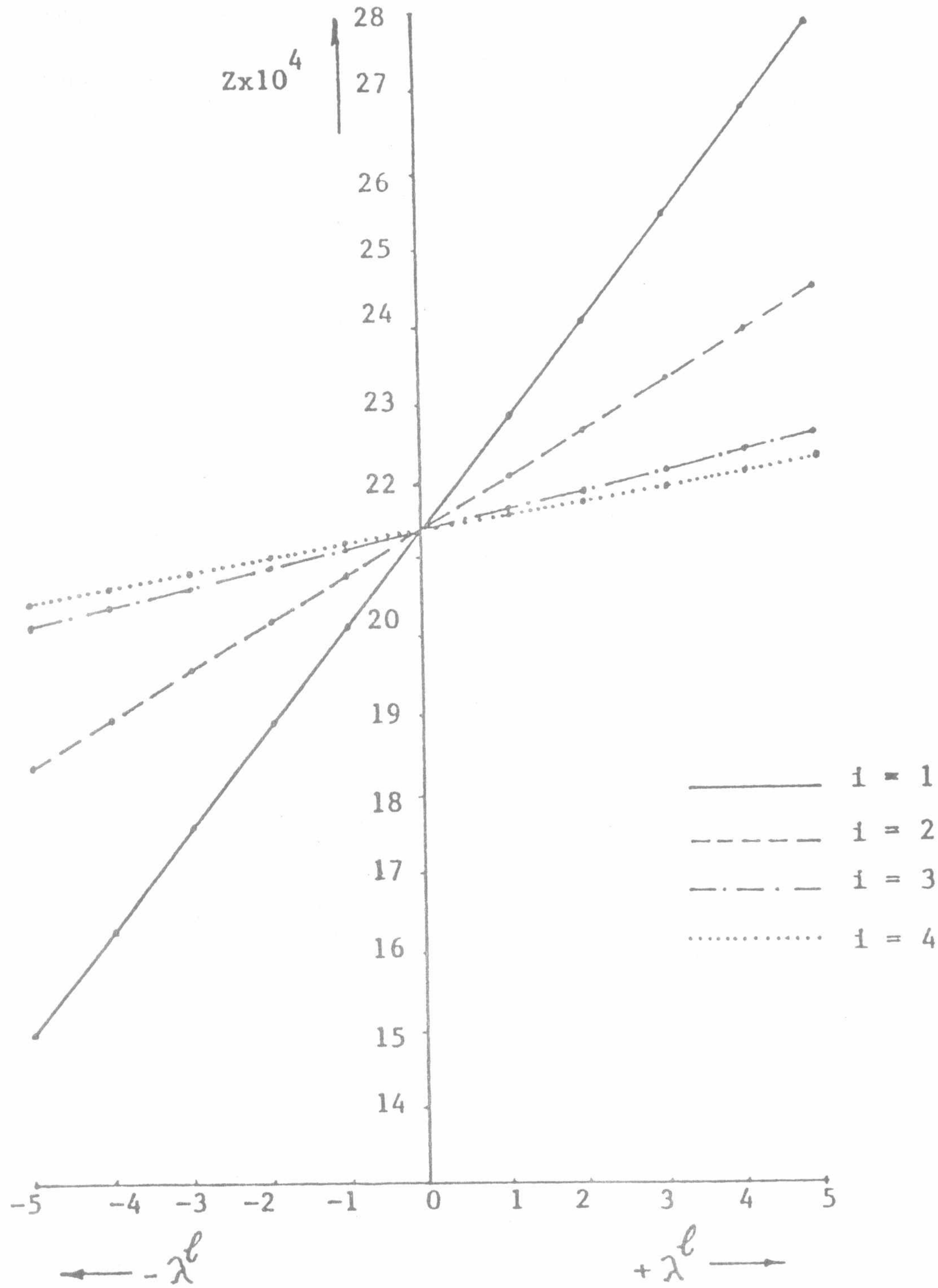

Fig.1. Results of Changes in Cost of the First 4 Rows.

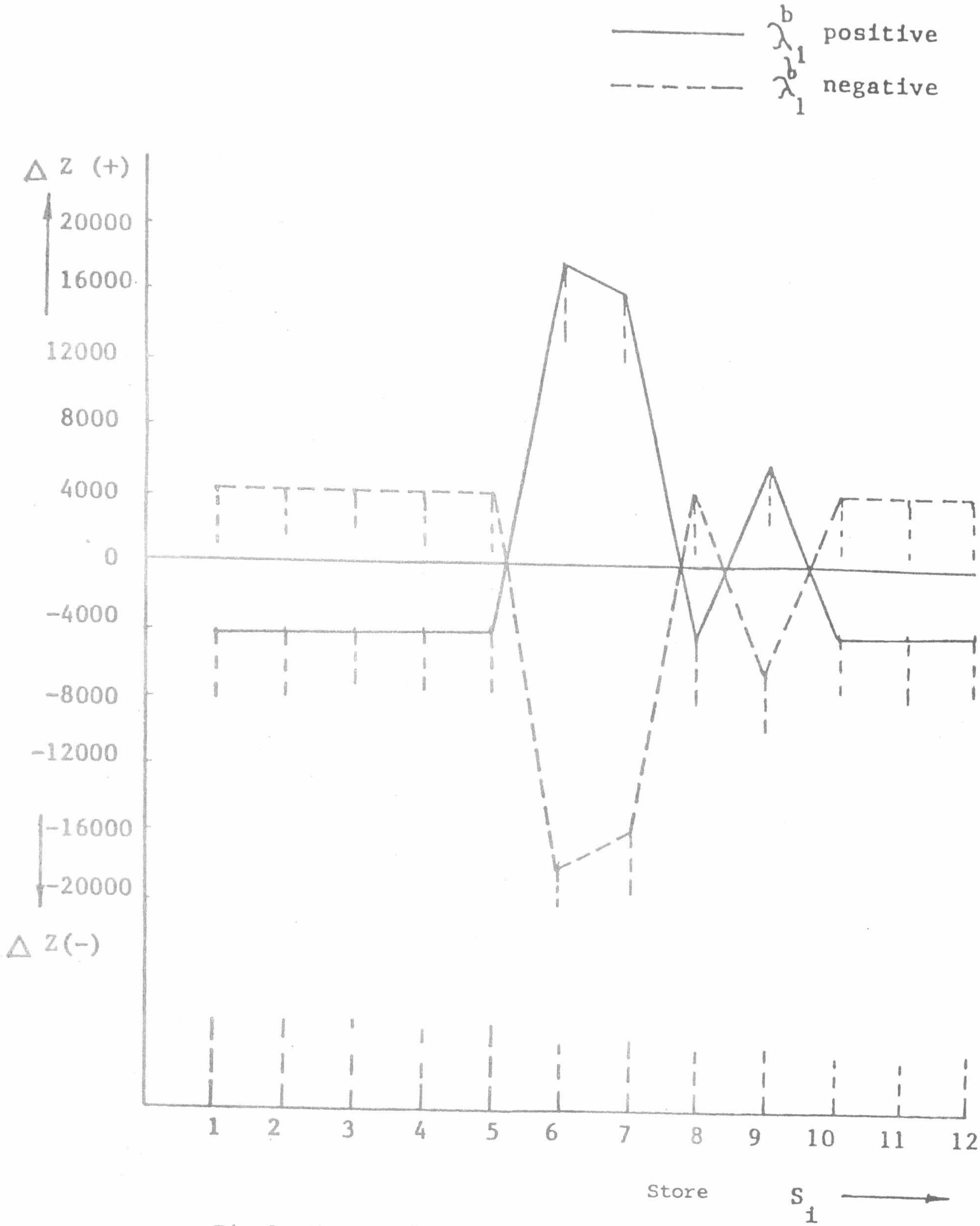

Fig.2. Changes in requirement of the first factory.

CONCLUSION AND FURTHER POINTS FOR RESEARCH

The presented program can be efficiently used to study the effect of changing the transportation problem parameters both in the objective function and in the constraints on the optimal solution and on the value of the objective function.

The obtained results may also be extended to be applied for the transportation problem with transhipment after its reformulation as a direct shipment transportation problem.

The parametric analysis may be used by the decision maker directly in many practical applications to adjust his decision to various changes of the problem parameters without the need to solve a new problem from the beginning.

This work can be extended in future to construct similar computer programs for the parametric study of multistage and multiobjective transportation problems.

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