MILITARY TECHNICAL COLLEGE

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"RELIABILITY EFFECTIVENESS AND LIFE TIMES DETECTION OF COMPLEX "SYSTEMS"

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ABSTRACT

The deterioration of materials and finished products in manufacturing, or in use or in storage is an important problem now in our modern industries to both producers and users. Economical, reliable evaluation, and engineering judgement applied to reliability effectiveness and life times detection provide management with deterioration curves, maintenance requirements and remedy feedback informations to maximize life and product serviceability, thus minimizing costs. To attain these goals some essential elements should be encompassed. Developing and implementing economic new techniques to solve these problems are needed. The ultimate objective of these techniques is to provide reliability assurance for the finished product and to assure optimum reliability costs for that product through the detection of life times.

The formulation of an optimize statistical model of these the basic condition of obtaining objective results. It is important in this case to arrange the method of formulation of these problems in good agreement with the objective of the research taking into account the main and auxilary factors.

By the essence of statistical simulation the process of functioning of the complex system were used to represent a mathematically formulated model which was isomorphic in all essential aspects to the total and required main of research objectives. This model was repeatedly tested to determine the required statistical characteristics, based on the complex stochastic process which was regarded as a sequence of the finite element.

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2. THE PROBLEM

Any complex system might be considered as type of hierarchy in which components are placed into subgroups configurations. Subgroups are assembled into groups of subsystems, and the subsystems are in turn organized into final system whose overall performance is of necessity, dependent on the characteristics of its subgroupings all the way down to the smallest component. Variations in these components are inevitable because of the tolerance of the different components in the mechanism, basic material, manufacturing processes and environmental conditions, etc. Such variations must be considered in reliability effectiveness and life times detection of complex systems.

3. THE APPROACH

The statistical simulation allowed us to standardize the solution of our research problem formulation for the required objectives, with the combination and aids of finite differences and computer systems.

The confergence implies that the finite difference approximation will reduce to the exact solution when the size of the increments employed is made infinitesimally small. Stability implies that errors associated with the use of increments of finite size will not grow as the calculation proceeds. When the finite difference procedure is both convergent and stable, a comparison of calculations made using two different increment intervals is generally a good indication of the reliability that may be assigned to the results. The existence of a solution of the finite difference approximation is assured if the exact initial value problem possesses a solution. The existence of the problem implies that a solution exists, although it is not always easy to determine the relevant boundary conditions. The available evidence indicates that a finite difference method which satisfies the problem under study for stability will prove to be a convergent solution. So sufficient conditions for stability of the finite difference equation:

$$V_s = C_1 V_{ss} + C_2 V_{sg} + C_3 V_n$$

where V_s is the applicable dependent variable of the complex system
V_{ss} is the applicable dependent variable of the complex system
V_{sg} is the applicable dependent variable of the complex system
V_n is the applicable dependent variable of the complex system
C_1, C_2, C_3 are constants related to failure rates λ_{ss}, λ_{sg} & λ_n

with the boundary conditions

V_s is a function of λ_{ss} V_{ss}

V_{ss} is a function of λ_{sg} V_{sg}

V_{sg} is a function of λ_n V_n

require $C_1 + C_2 + C_3 \quad 1$

and $0 < \lambda_s < 1$

Then our problem of difference equation is

$$V_{ss} - 2a\, V_{sg} + b^2\, V_n = C\, V_s + d\, V_{sc}$$

where (V_{sc}) is the applicable dependent variable of the complex system's constraints, and (a, b & c) are real constants with the boundary conditions

V_s is a function of $\lambda_{ss}\, V_{ss} + f(n)$

V_{ss} is a function of $\lambda_{sg}\, V_{sg} + f'(n)$

V_{sg} is a function of $\lambda_n\, V_n + f''(n)$

which satisfy $(0 \leq \lambda_s \leq \frac{1}{|b|}$ or $|b|)$

and additional arbitrary initial conditions at (n=o) and (n=1) or at (n=o) if (d=o), is stable if the roots of the equation

$$R^2 + 2/c\ (a-b\cos\alpha)\ R + d = 0$$

cannot exceed unity in absolute value for any real value of (α), and neither ($R = \pm 1$) may be a repeated root of the equation. If the first condition is satisfied and the other is violated, our problem difference equation may be linearly unstable, i.e., errors may grow at a rate which is directly proportional to the elapsed time of the system under service. Then we can extend our problem of difference equation for complex systems as

$$(a_3\, V_{ssc'} + b_3\, V_{sgc'} + C_3\, V_{nc'}) + (a_2\, V_{ss} + b_2\, V_{sg} + C_2\, V_n) + (a_1\, V_{ssc} + b_1\, V_{sgc} + C_1\, V_{nc}) = 0$$

where the abbreviation (c') and (c) is the constraints above and under the applicable dependent variable, and

$$C_1 = p^2\, a_1\ ,\quad C_2 = p^2\, a_2 \quad \text{and} \quad C_3 = p^2\, a_3$$

where (p) is real with the boundary conditions mentioned before, which satisfy $(0 \leq \lambda_s \leq \frac{1}{|p|}$ or $|p|)$
and additional arbitrary initial conditions at two consecutive values of (n) or at one value of (n) if ($a_1 = b_1 = c_1 = 0$) is stable if the roots of the equation

$$(b_3 + 2pa_3\ \cos\alpha)\ R^2 + (b_2 + 2pa_2\ \cos\alpha)\ R + (b_1 + 2pa_1\ \cos\alpha) = 0$$

cannot exceed unity in absolute value for any real value of α, and neither ($R = \pm 1$) may be a repeated root of the equation. If the first condition is satisfied and the other is violated, our problem difference equation may be linearly unstable.

The convergence criteria are demonstrated by comparison of the exact solutions of the governing partial differential equation and of the finite difference equivalent. The stability criteria are obtained as a result of an examination of the form of the exact solution of the finite difference equation when it is regarded as an error propagating equation.

The stated stability criteria apply only to difference equations with constant coefficients. The coefficients in our problem depend upon the complexity of the system, so the equation broken up into a series of equations, each equation being applied to the subsystems, subgroups and components. In the system of equation, the truly variable coefficients were replaced by the actual failure rate. Then the stability criteria had been applied to each equation in turn, then the system equations had been used to solve our problem equation. The improved finite difference equation is then

$$\frac{\Delta x^2}{\Delta t}(V_s - V_{sg}) = (V_{ssc'} - 2\,V_{sgc'} + V_{nc'}) + (V_{ss} - 2\,V_{sg} + V_n)$$

where (Δx^2) is the degree of complexity of the system and is a function of the sample size and (Δt) is the increment of service life and is a function of failure rate (λ), by using a Taylor series expansion about V_s gives the value of $(\partial^2 V/\partial x^2)$ at a specified service time.

$$\left(\frac{\partial^2 V}{\partial x^2}\right)_s \simeq \frac{V_{ssc'} - 2V_{sgc'} + V_{nc'}}{\Delta x^2}$$

substitution of the finite difference equivalents

$$\left(\frac{\partial^2 V}{\partial x^2}\right)_s \simeq \frac{1}{2}\left[\left(\frac{\partial^2 V}{\partial x^2}\right)_{sg} + \left(\frac{\partial^2 V}{\partial x^2}\right)_{sgc'}\right]$$

In order to determine the stability criteria to be applied to the improved finite difference equation, it is rearranged in a form which can be compared directly with the extended problem difference equation.

$$\left(V_{ssc'} - 2\left(\frac{\Delta x^2}{\Delta t} + 1\right) V_{sgc'} + V_{nc'}\right) + \left(V_{ss} + 2\left(\frac{\Delta x^2}{\Delta t} - 1\right) V_{sg} + V_n\right) = 0$$

then $(a_1 = b_1 = c_1 = 0)$, $(a_2 = c_2 = a_3 = c_3 = 1)$

$$b_2 = 2\left(\frac{\Delta x^2}{\Delta t} - 1\right) \qquad b_3 = -2\left(\frac{\Delta x^2}{\Delta t} + 1\right)$$

then the conditions of extended problem difference equation is satisfied by $(p = 1)$, and its root is

$$\left(2\cos\alpha - 2\left(\frac{\Delta x^2}{\Delta t} + 1\right)\right) R^2 + \left(2\cos\alpha + 2\left(\frac{\Delta x^2}{\Delta t} - 1\right)\right) R = 0$$

$$R = \frac{\cos\alpha + \left(\frac{\Delta x^2}{\Delta t} - 1\right)}{\cos\alpha - \left(\frac{\Delta x^2}{\Delta t} + 1\right)} = -\left(1 + \frac{2\,\frac{\Delta x^2}{\Delta t}}{\cos\alpha - \left(\frac{\Delta x^2}{\Delta t} + 1\right)}\right)$$

4. ANALYTICAL CONSIDERATIONS

Since (α) is a multiple valued function of (p), each set corresponds to a separate branch of the function, and only one set should be used in carrying out our problem difference equation, i.e. the positive set.

A suggested values of samples had been carried out from which it was possible to estimate the required numbers of observations based on special statistical samples sizes with special distribution, which allowed more accuracy than required. A special computerized mathematical and statistical treatments of our research problems formulation for relations required for the new sampling and control charts techniques were made. The results from our formulized approaches were firstly tested by the analysis of variance technique, and Kolmogorov's criterion, as a test of their significances to verify the truth of our hypotheses that the samples in Table (1) are random and taken from a continuous distribution of finite elements to ensure a good fitness of all variables. The calculations for formulated approaches distributions and universe line by the method of correlation and regression were calculated, Figures (1) which shows the relation between, sample size, failure rate and reduction of life time detection based on the testing duration is more than 25% of the expected life service time. A computer flow charts and computer programs were constructed based on Runge Kutta method for that purpose. After these statistical treatments of the variables of the formulized problems, it could be concluded from the comparison of all functions that the distributions found from the data of the computer calculations rather accurately corresponds to the data and curves given which are the inverse functions of the theoretical distribution based on the formulated approach.

5. PRACTICAL APPLICATION AND CONCLUDING REMARKS

If a sample of several units are placed on life test, what is the plan for any lot size, to detect service life and hence the effectiveness of reliability. Solution using graph figure (1) or the table (1), taking into consideration that the life test must be carried at least 25% of the expected or designed service life without any failed unit. If any unit failed before the 25% of the expected or designed service life, the test must be cancelled.

6. REFERENCES

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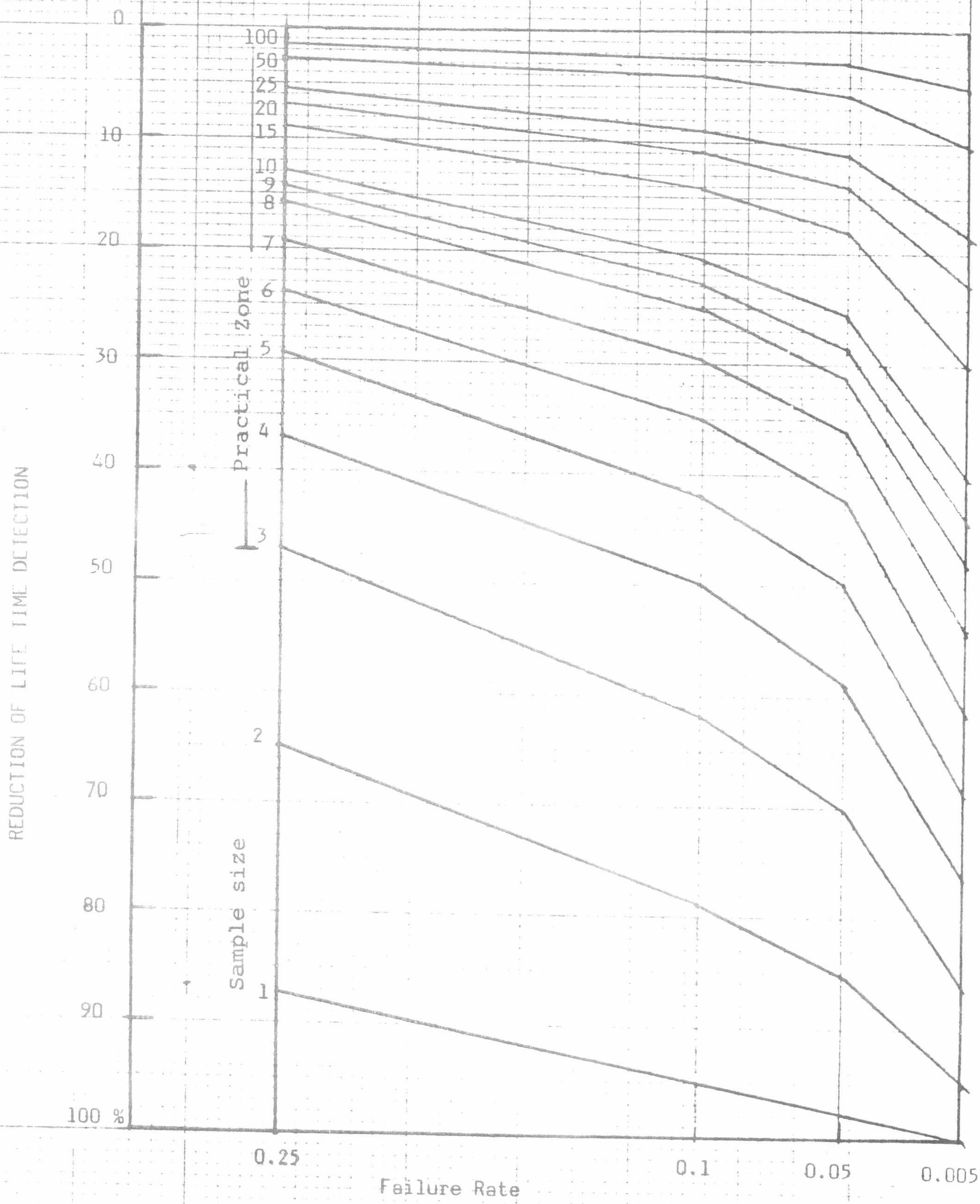
STATISTICAL SIMULATION DISTRIBUTIONS FOR SEVERAL SAMPLE SIZES BASED ON THE TESTING DURATION IS MORE THAN 25% OF THE EXPECTED LIFE SERVICE TIME.

Figure No. 1



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REDUCTION OF LIFE TIME DETECTION

n \ λ	0.005	0.010	0.025	0.05	0.10	0.25
1	96.5	96.1	95.7	94.9	93.6	92.4
2	94.5	93.0	92.9	90.3	89.6	88.3
3	91.2	90.5	88.9	87.4	86.3	83.9
4	89.8	87.6	85.9	84.9	82.9	79.7
5	76.3	73.9	70.8	66.2	63.8	58.4
6	63.1	59.8	54.2	49.3	44.6	37.8
7	50.3	45.6	38.7	32.9	26.3	16.4
8	48.5	43.8	37.0	31.3	25.1	16.0
9	44.5	40.1	33.7	28.4	22.6	14.3
10	41.2	37.0	30.9	25.9	20.6	13.0
15	29.8	26.5	21.9	18.2	14.3	8.90
20	23.3	20.6	16.9	14.0	10.9	6.70
25	19.1	16.9	13.8	11.3	8.8	5.40
50	10.1	8.8	7.2	5.9	4.6	2.80
100	5.2	4.6	3.7	3.0	2.3	1.40

Table No. 1