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# COMPRESSION HELICAL SPRINGS, A METHOD OF DESIGN FOR A MINIMUM WEIGHT

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ABSTRACT

This paper discusses an approach for the design of compression closely coild helical springs subjected to axial load. With the presented proposed method, one can specify the significant characteristics of the spring that contains the least possible volume of material while still providing sufficient strength and stiffness. To do so, the classic design equations are developed so that they can be graphically represented in a (weight-diameter)-plane from which the global optimum design point can be determined. Also, a computation flow chart is given, as an alternative way to the graphical method. Based on the computation flow chart, a digital computer program can be written for the optimum design problem.

## INTRODUCTION

The priamary purpose of a spring is to satisfy certain functional requirements of the mechanism. These functional requirements are determined before designing the spring; from either a static or a dynamic force analysis of the entire mechanism, and usually are expressed by the specification of load versus deflection characteristics.

A typical compression spring is shown in Fig. (1). For a given material, its significant characteristics are uniquely defined by the wire diameter (d), the mean coil diameter (D), the free (unloaded) height ($h_f$), and the active number of coils ($N_a$).

The design of helical springs by the classical deterministic approach can be found in many references [1 - 3] . Conventional methods for determining the spring dimensions, that use the least possible amount of material, require a cumbersome itrative calculations before a satisfactory solution is obtained.

When load, deflection, mean coil diameter and stress are prespecified, the following is a new proposed method that determines the significant characteristics of spring (d, $h_f$, and $N_a$) that contains the least possible volume of material while still providing sufficient strength and stiffness. This method is based on the graphically representation of the relation between the weight of the spring and its wire diameter. This representation must be done for each step in the spring design, i.e for static loading and/or

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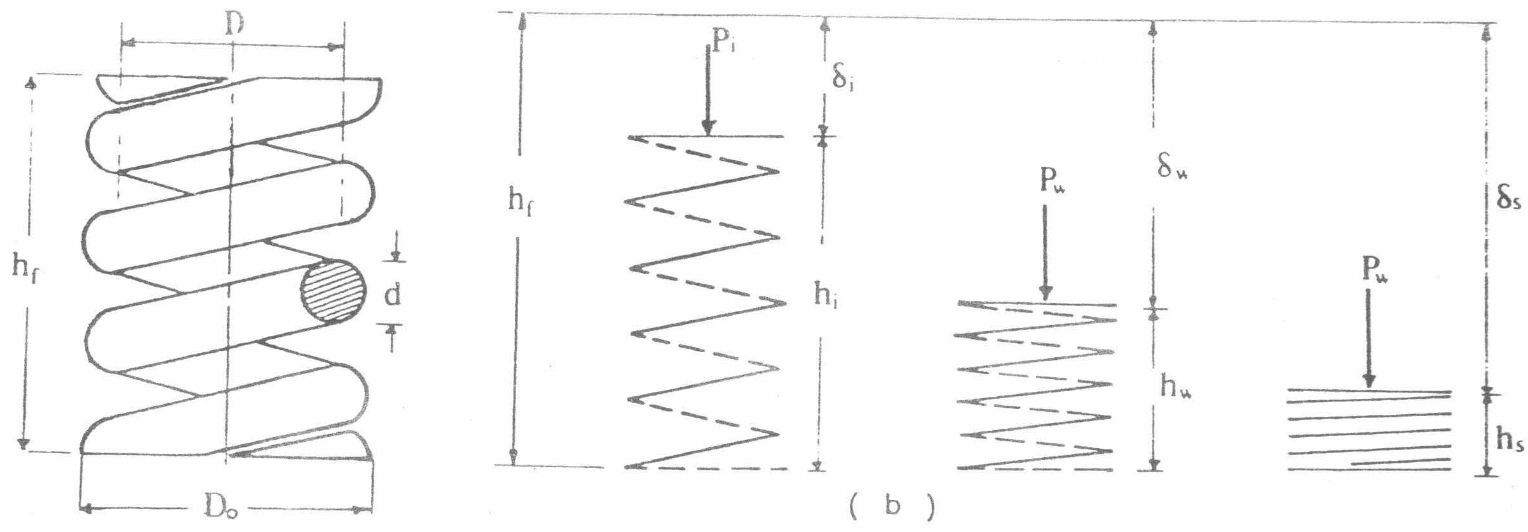


Compression helical spring

Figure ( 1 ) : a- Significant characteristics
b- Initial, Working and solid deflection

for fluctuating loads, then for buckling and practical proportions. These graphical representations may be named (weight-diameter)-plane. The optimization problem can then be initially formulated. Alternatively, a computer algorithm can be written to solve the design problem instead of using the (weight-diameter)-plane.

## BASIC BACKGROUND

To design a spring by the suggested method, one has to start by reviewing some basic design information which are pertinent to the discussion given below.

- Design for Static Loading

The load gradient (K) for a compression helical spring is given by

$$K = \frac{G \cdot d^4}{8 \cdot D^3 \cdot N_a} \tag{1}$$

Where : (G) is the modulus of rigidity for the material, (d) is the wire diameter, (D) is the mean coil diameter and ($N_a$) is the active number of coils.

The working strength in shear ($\tau$) caused by the working load ($P_w$) at solid height is given by

$$\tau \leqslant \frac{8 \cdot D \cdot P_w}{\pi \cdot d^3} K_t \tag{2}$$

Where : ($K_t$) is the Wahl correction factor that can be given by :

$$K_t = \frac{4\,(D/d) - 1}{4\,(D/d) - 4} + \frac{0.615}{(D/d)} \tag{3}$$

Equation 3, can further be simplified [4] to give

$$K_t = \frac{1.6}{(D/d)^{0.14}} \tag{4}$$

This equation is valid for design of springs having a spring index ($c = D/d$) within the range ($4 \leqslant c \leqslant 18$).

If the ends of the spring are squared and ground, the total number of coils ($N_t$) is

$$N_t = N_a + 2 \tag{5}$$

The solid height ($h_s$) is given by

$$h_s = N_t \cdot d$$

$$\text{i.e. } h_s = (N_a + 2)\, d \tag{6}$$

If a 20 percent clash allowance is selected, the solid height with a 20 percent overload will be reached. Thus, solid deflection ($\delta_s$) will be

$$\delta_s = \frac{1.2 \times P_w}{K} \tag{7}$$

The free height is given by

$$h_f = h_s + \delta_s \qquad (8)$$

- Bending and Buckling

For a compression helical spring, it is important that the working height ($h_w$) be not so great that buckling or column action will occur. An excellent approach to this problem is derived earlier [3] for springs having both ends are well seated, and squared and ground. To avoid buckling, the following equation must be satisfied [3].

$$P_w \leqslant \frac{c \cdot \pi^2 \cdot E \cdot I_{equ.}}{h_w^2 \cdot N_{cr}} \qquad (9)$$

In equation 9 , (E) is the elastic modulus, ($I_{equ.}$) is the equivalent moment of inertia of the spring coil, ($N_{cr}$) is the factor of safty and (c) is a constant that depends on the end condition of the spring (Fig. 2). The equivalent moment of inertia of the spring is taken as

$$I_{equ} = \frac{h_w \cdot d^4}{64 \cdot N_a \cdot D \cdot (1 + \frac{V}{2})} \qquad (10)$$

where (V) is the Poisson's ratio.

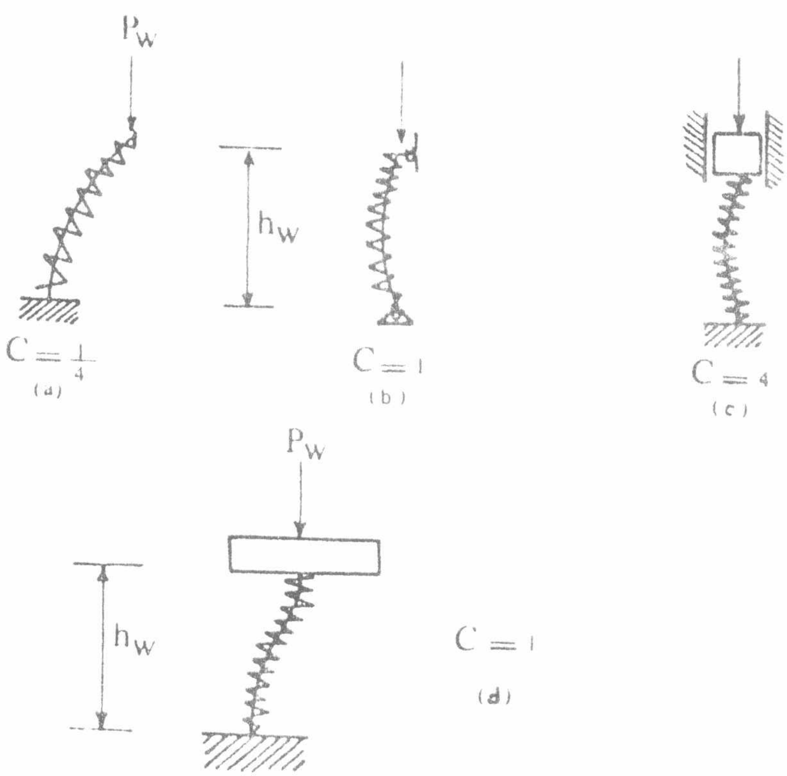


Figure (2) : Value of the constant C for various end conditions
a) Fixed-free ends b) Pinned-pinned ends
c) Clamped-clamped ends with lateral restraint
d) Clamped-clamped ends without lateral restraint.

- Practical Proportions

For a compression spring, with the condition when the initial load ($P_i$) exists, the initial height ($h_i$) can be expressed by an equation that based on the practical proportions for helical springs [5]. That is ,

$$h_i \geqslant h_{min} \approx (N_a + 2)\ d + S_a \qquad (11)$$

Where ($S_a$) is the sum of minimum spaces between individual working coils. ($S_a$) may be written as $(1 + x.d^2 \ . \ N_a)$. The value of (x) depends on the spring index [5] .

(WEIGHT-DIAMETER)-PLANE

There are many practical objectives for optimum design of helical compression springs depending on the particular problem as related to the requirements of the total mechanical device in which the spring is to function; for instance, minimization of outside diameter, maximization of lowest natural frequency for the internal longitudinal vibrations and minmization of spring weight. This paper, however, only concerned with the minimization of the spring weight. The weight of active coils for a helical spring can be very closely approximated an the basis of Pappu's second theorem, from elementry calculus, as follows ;

$$\omega = \rho \pi D N_a \left( \frac{\pi d^2}{4} \right) \qquad (12)$$

where ($\rho$) is the weight density of the material.Equation (12) can be rewritten as

$$W = N_a \ . \ d^2 \qquad (13)$$

where (W) is the weight factor that is equal to $\frac{w}{c}$ where $c = \frac{\rho \pi^2 D}{4}$

Equation (13) obviously shows that the weight factor is a function of both the wire diameter and the active number of coils. Since it is required to make (W) as a function of wire diameter only, it is necessary to find a relation between ($N_a$) and (d). Consequently, the relation between (W) and (d) in the design of the spring (on strength, buckling, and practical proportions) can be found in the following formulae, based on the above equation (1 through 11)

$$W_s = \frac{G\ d^6\ (h_i - 2\ d)}{0.75 \pi \tau (D_o - d)^{2.14}\ d^{2.28} + G\ d^5 - 8\ P_i\ (D_o - d)^3} \qquad (14)$$

$$W_b = \frac{C\ \pi^2\ E\ d^6}{64\ (D_o - d)(1 + \frac{V}{2})\ P_w \ . \ N_{cr} \ . \ h_w} \qquad (15)$$

$$W_p = \frac{(h_i - 1)\ d - 2\ d^2}{(1 + X\ d)} \qquad (16)$$

where ($W_s$ , $W_b$ , and $W_p$) are the weight factors when designing on strength, buckling, and practical proportion, respectively.

Figures (3, 4, and 5) are the graphical representation for equations (14, 15, and 16), respectively. It is found that all the springs that lie on each curve and also in its feasible solution are valid. Next, all these curves are plotted in one graph shown in Fig. (6) that called (weight-Diameter)- plane. It can be seen that an area is enveloped by the three curves. Any spring found within this area satisfies all the design requirements. It is important to realize that point A(in Fig, 6) represents a unique spring that has the least weight. Furthermore, point A is the solution of equations (14 and 15). This unique spring may or may not have a standard diameter. In case if it not a standard diameter, the next standard one should be selected. This standard diameter, and its equivalent weight factor ($W_s$), will then be substituted in equation (13) to find the least active number of coils. The other significant characteristics of the spring can be obtained by applying in equations (1, 5, 6, 7, and 8) .

The design of the spring, that mentioned above, is based on static load. The spring must then be checked for buckling and practical proportions, regardless of the type of the applied load. When designing on dynamic loading, however, the curves ($W_s$ and $W_b$) must be changed to represent that dynamic loading . This means that the practical proportion will not be changed with changing the type of the load.

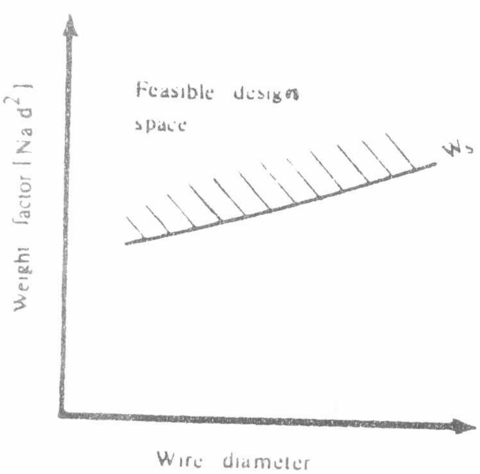


Figure (3)

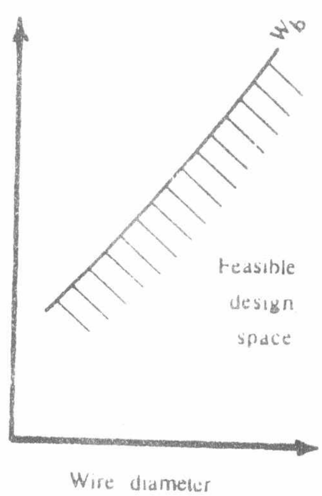


Figure (4)

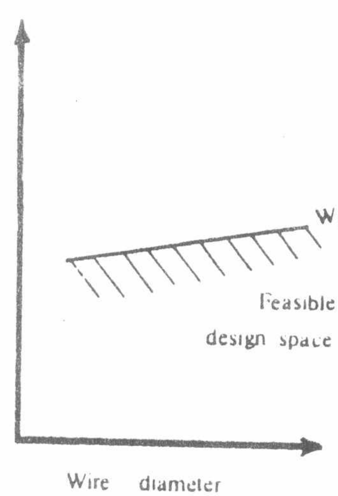


Figure (5)

Graphical representation for equations (14, 15, and 16)

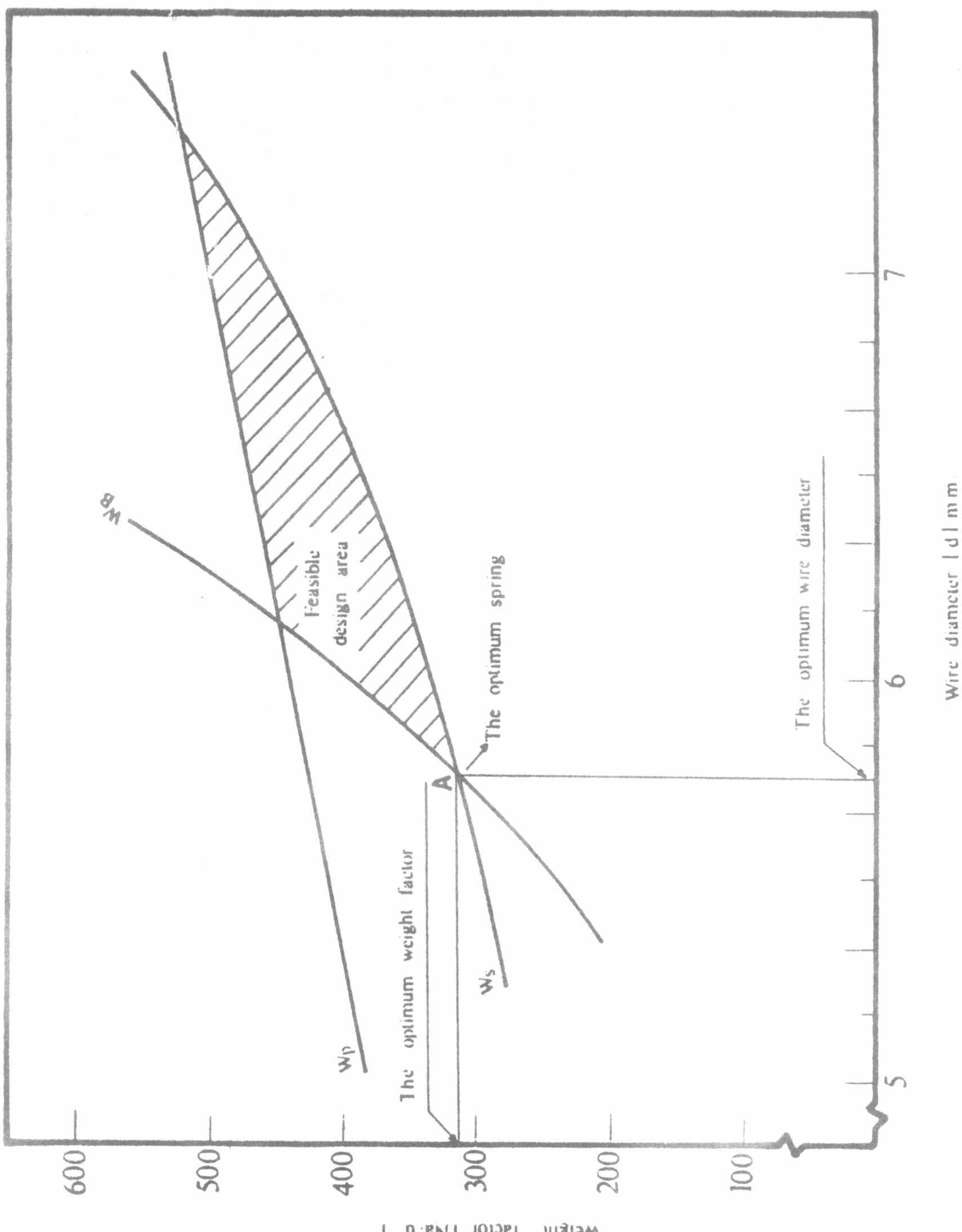


Figure (6) (Weight-diameter)-plane

## COMPUTATION FLOW CHART FOR THE OPTIMIZATION PROBLEM

Application the weight diameter plane will determine the global optimum design point (i.e., the best possible solution point in the feasible design space). Execution of the procedural steps in the weight-diameter plane is time consuming. Theorefore, a computation flow chart is an alternative solution for this drawback.

Figure 7 shows the computation flow chart which has been constructed according to the understanding of the weight diameter plane (Figure 6). The best design for minimization of the weight factor (for a specific combination of $N_a$ and d values for a certain material) can be explicity determined. This determination is made by simple calculations and comparisons based on the functions and boundaries of the weight-diameter plane. Based on the computation flow chart of figure 7, a digital computer program can be written for the optimization problem. With this, the overall optimum design can be determined automatically and ropidly in numerical applications.

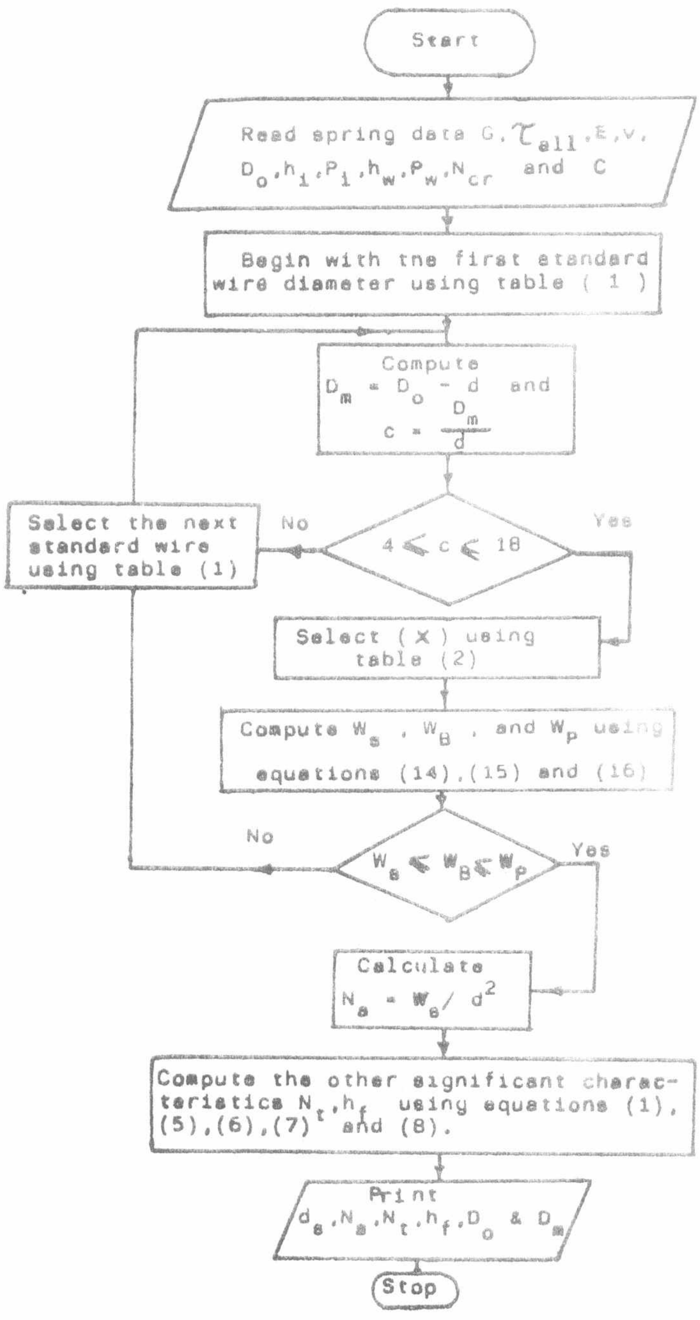


Figure (7)

Table (1) Standard diameters of spring wires for cold coiled springs, in mm, (DIN 2o76).

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| o.3 | o.7 | 1.2 | 2 | 3.2 | 5 | 8 | 12 |
| o.32 | o.75 | 1.3 | 2.1 | 3.4 | 5.3 | 8.5 | 12.5 |
| o.36 | o.8 | 1.4 | 2.25 | 3.6 | 5.6 | 9 | 13 |
| o.4 | o.85 | 1.5 | 2.4 | 3.8 | 6 | 9.5 | 14 |
| o.45 | o.9 | 1.6 | 2.5 | 4 | 6.3 | 1o | 15 |
| o.5 | o.95 | 1.7 | 2.6 | 4.25 | 6.5 | 1o.5 | 16 |
| o.56 | 1 | 1.8 | 2.8 | 4.5 | 7 | 11 | 17 |
| o.65 | 1.1 | 1.9 | 3 | 4.75 | 7.5 | 11.5 | |

| d mm | $S_a$ mm | spring index 4...6 | $\leq 6$ $\geq 8$ | $\leq 8$ $\geq 12$ | 12 |
|---|---|---|---|---|---|
| o.o7 ...o.5 | | $x = o.5\ mm^{-1}$ | o.75 | 1 | 1.5o |
| o.5<br>1 | | o.2 | o.4 | o.6 | 1 |
| 1<br>1.6 | | o.o5 | o.15 | o.25 | o.4 |
| 1.6<br>2.5 | | o.o35 | o.1 | o.2 | o.3 |
| 2.5<br>4 | $(1\ mm + xd^2\ N_a)$ | o.o2 | o.o4 | o.o6 | o.1 |
| 4<br>6.3 | | o.o15 | o.o3 | o.o45 | o.o6 |
| 6.3<br>1o | | o.o1 | o.o2 | o.o3 | o.o4 |
| 1o<br>17 | | o.oo5 | o.[illegible]1 | o.o18 | o.o2 |

Table (2) Sum of minimum spaces between individual working coils ( $S_a$ ), ( DIN 2o95 ) .

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