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NONLINEAR RESPONSE ANALYSIS OF PLATES UNDER RANDOM ACOUSTIC EXCITATION

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ABSTRACT

This paper describes a perturbation technique to perform the nonlinear response analysis of plate structure under random acoustic excitation. In the analysis, use is made of triangular plate element together with a nonlinear plate stiffness element which is dependent on the modal response of the structure. The nonlinear plate stiffness element represents the coupling term between the membrane and bending deformation. Applying these elements and the associated consistent mass matrices, the equivalent linear eigen matrix of the complete plate is organized. The eigen solution and the following modal spectral computation completes the iteration cycle. A flow diagram and a numerical example are included which illustrate the application of the method to practical problems.

INTRODUCTION

In random acoustic excitation, the structure usually exhibits a nonlinear behavior. This is evident from the fact that the response spectrum shapes change with the intensity of the excitation even though the pressure input spectrum shape is kept unchanged. The nonlinear condition may be caused by a number of factors including material nonlinearity, interaction of internal stresses and large deformation (geometrical nonlinearity). The nonlinearity referred to in this paper is classified as large deformation. A number of nonlinear acoustic techniques are known theoretically, among which is perturbation method called the equivalent linearization technique. Its basic theory states that in a single degree of freedom system, the statistical data of the response may be determined by an equivalent linear system where the linear damping and spring constant are determined by minimizing the response square error. The linearization is accomplished with the assumption that the external random input is stationary, Gaussian and has zero expectation.

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$$U_o = \frac{Eh}{2(1-\nu^2)} \iint_A \Big\{ u_{,x}\,(u_{,x}+\nu v_{,y}) + v_{,y}(v_{,y}+\nu u_{,x}) + \frac{1-\nu}{2}(v_{,x}+u_{,y})^2 + \frac{h^2}{12}\Big[w_{,xx}(w_{,xx}+\nu w_{,yy}) + w_{,yy}(w_{,yy}+\nu w_{,xx}) + 2(1-\nu)w_{,xy}^2\Big]\Big\}\, dx\, dy \qquad (8)$$

$$U_1 = \frac{Eh}{2(1-\nu^2)} \iint_A \Big[u_{,x}(w_{,x}^2+\nu w_{,y}^2) + v_{,y}(w_{,y}^2+\nu w_{,x}^2) + (1-\nu)(u_{,y}+v_{,x})w_{,x}w_{,y}\Big]dx\,dy \qquad (9)$$

Where the nonlinear strain energy U_1 represents the coupling term between membrane and bending.

Consider a typical triangular plate element, five deflection parameters are used at each node, namely, the inplane deflections, the transverse deflection and its first derivatives. These make a total of 15 degrees of freedom for each element. The nodal displacements may be assembled into a column vector $\{d\}$ whose transpose is

$$\{d\}^T = \begin{Bmatrix} d_u \\ d_w \end{Bmatrix}^T = [u_1, v_1, u_2, v_2, u_3, v_3; w_1, w_{,x1}, w_{,y1}, w_2, w_{,x2}, w_{,y2}, w_3, w_{,x3}, w_{,y3}] \qquad (10)$$

In Eq. (10), the subscripts 1-3 denote the nodes 1-3, respectively. The deformation patterns within a triangular element are taken as :-

$$u(x, y) = A_1 + A_2 x + A_3 y \qquad (11)$$

$$v(x, y) = A_4 + A_5 x + A_6 y \qquad (12)$$

$$w(x, y) = B_1 + B_2 x + B_3 y + B_4 x^2 + B_5 xy + B_6 y^2 + B_7 x^3 + B_8(x^2y + xy^2) + B_9 y^3 \qquad (13)$$

The inplane displacement matrix $\{d_u\}$ may be expressed as

$$\{d_u\} = [T_u]\{A\} \quad \text{or} \quad \{A\} = [T_u]^{-1}\{d_u\} \qquad (14)$$

where $\{A\}^T = [A_1, A_2, \ldots, A_6]$

A similar relation between $\{d_w\}$ and $\{B\}$ can be written as

$$\{d_w\} = [T_w]\{B\} \quad \text{or} \quad \{B\} = [T_w]^{-1}\{d_w\} \qquad (15)$$

where $\{B\}^T = [B_1, B_2, \ldots, B_9]$

and $[T_w]$ is a 9x9 nonsingnlar transformation matrix.

The corresponding element force-displacement relationships in the global coordinate system is simply:

$$\begin{Bmatrix} F_u \\ F_w \end{Bmatrix} = \begin{Bmatrix} R_u^T \; f_u \\ R_w^T \; f_w \end{Bmatrix} = \left[\begin{array}{c|c} R_u^T \; Kaa \; R_u & R_u^T \; K'ab \; R_w \\ R_w^T \; K'_{ab} \; R_u & R_w^T (K_{bb} + K'_{bb}) R_w \end{array} \right] \begin{Bmatrix} q_u \\ q_w \end{Bmatrix} \qquad (22)$$

The consistent mass matrix for an element is used as given in ref. (5)

LINEARIZED MODAL ANALYSIS AND ACOUSTIC RESPONSE

The linearized eigenvalue solutions for the natural modes are obtained from Eq. (7) by dropping the $[C]$ and $\{F\}$ matrices

$$[K^e][\Omega] = [M][\Omega][\omega_o^2] \qquad (23)$$

Based on the modal data, the complete Eq. (7) is now used to determine the modal response of the plate under random acoustic excitation. For this purpose, the plate deformation is represented as ;-

$$\{q\} = [\Omega]^T \{\xi\} \qquad (24)$$

In order to simplify the problem, the generalized damping matrix is assumed to be a diagonal matrix

$$[\Omega]^T [C][\Omega] = 2[\gamma][\omega_o] \qquad (25)$$

where γ_i represents the damping ratio of the ith mode
The insertion of Eqs (24) and (25) into Eq. (7) leads to the matrix equation for the generalized coordinates $\xi(t)$

$$\{\ddot{\xi}\} + 2[\gamma][\omega_o]\{\dot{\xi}\} + [\omega_o^2]\{\xi\} = [\Omega]^T \{F(t)\} \qquad (26)$$

where $\{F(t)\}$ is a column matrix of the random pressure function which is assumed ergodic. Thepower spectrum density of the modal amplitudes $\{\xi\}$ maybe determined as :-

$$[\phi_\xi(\omega)] = [Z(\omega)]^{-1}[\phi(\omega)][Z^*(\omega)]^{-1} \qquad (27)$$

where $[Z(\omega)]$ is the modal impedance matrix and $[\phi(\omega)]$ represents the PSD matrix of the generalized force. For the most general case, the pressure input is both timewise and spacewise random. For the case where the pressure distribution is timewise random and spatially finite, the generalized. PSD force is

$$[\phi] = [\Omega]^T [A][\phi_p][A][\Omega] \qquad (28)$$

where $[A]$ is diagonal matrix whose elements represent the areas associated with the nodes. If the timewise random pressure is of the raindrop type with uniform intensity, then $[\phi_p]$ in Eq. (28) is

Proceeding in this manner, the nonlinear stiffness matrix is computed based on the rms values of the component matrices as shown below;-

$$[K^e] = [K_{ww}] + [K'_{ww}(e_u)] + [K'_{uw}(e_w)]^T [K_{uu}]^{-1} [K'_{uw}(e_w)] \qquad (36)$$

$$\{e_u\} = -[K_{uu}]^{-1} [K'_{uw}(e_w)] \{e_w\} \qquad (37)$$

where

$$\{e_w\} = [Q] \{(\bar{\xi}^2)^{1/2}\} \qquad (38)$$

Based on the preceding formulation, the nonlinear finite element problem is handled iteratively, the flow chart showing the method is given in Fig. (1).

NUMERICAL RESULTS

Consider the case of a rectangular plate with one edge builtin and the other three edges free subjected to raindrop type and uniform intensity random acoustic loading, ref. (6). The dimensions and properties of the plate are given as;

a = 50 (cm) ; b = 30 (cm) ; h = 2 (mm)

Material: duralumin

Three acoustic load level are considered:-

ϕ_o = 137.9 x 10^{-3} ; 68.95 x 10^{-1} ; 151.33 x 10^{-1} $(N/m^2)^2$/Hz.

The grid for the finite element analysis appears in Fig.(2). The eigenfrequency data for the linear and nonlinear analyses using the first three modes of the plate corresponding to various load intensities are summarized in Table (1).

$_o(N/m^2)^2$/Hz	Iteration Number	Modes		
		1	2	3
Linear Case	0	30.905	80.958	90.660
Nonlinear Case ϕ_o=137.9 x 10^{-3}	1	30.943	80.969	90.667
	2	30.942	80.971	90.660
Nonlinear Case ϕ_o= 68.95 x 10^{-1}	1	32.523	81.538	91.014
	2	32.221	81.495	90.956
	3	32.266	81.508	90.965
	4	32.260	81.506	90.963
	5	32.261	81.507	90.964
Nonlinear Case ϕ_o= 151.33x10^{-1}	1	34.327	82.224	91.433
	2	33.317	81.982	91.221
	3	33.559	82.062	91.270
	4	33.482	82.070	91.268
	5	33.433	82.080	91.273

Table (1): Iterated eigenfrequencies in Hz.

5) Shebl M.G., "Stresses and deflection in circular plates due to dynamic loading", Ph.D Thesis, University of Helwan, 1982.

6) El NOMROSSY M.M. and Kholoussy M.I., " Acoustic response analysis of plates using the finite element method", Proceedings of the 11 th International Congress in Acoustics, Vol.5, Paris, 19-27 July 1983.

NOMENCLATURE

a,b	length of a side of the rectangular plate
{A} , {B}	coefficient matrices of the assumed deformation patterns
[C]	damping matrix
$[C'_{cs}]$	intermediate stiffness matrices
$\{d\},\{d_u\},\{d_w\}$	nodal displacement matrices referred to element coordinate system.
$\{e_u\},\{e_w\}$	equivalent linearized nodal displacement matrices
E	Young's modulus
$\{f\},\{f_u\},\{f_w\}$	nodal force matrices referred to element coordinate system
$\{F\},\{F_u\},\{F_w\}$	nodal force matrices referred to global coordinate system
h	plate thickness
[K],[K']	linear and nonlinear stiffness matrices
$[K^e]$	equivalent linearized stiffness matrices
[M]	mass matrix
$\{q\},\{q_u\},\{q_w\}$	nodal displacement matrices referred to global coordinate system
[Q]	matrix with eigenvectors in successive columns
$[R],[R_u],[R_w]$	Coordinate transformation matrices
t	time
T	kinetic energy
$[T_u],[T_w]$	transformation matrices
u , v	inplane deformation of the plate
U	strain energy
U_o	linear strain energy
U_1	nonlinear strain energy
u , v	inplane deformation of the plate
W	work done
w	transverse deformation of the plate
x , y, z	element coordinate system
[z]	modal impedance matrix
$\{\xi\}$	modal amplitude matrix
$[\phi]$	modal force PSD matix
ϕ_o	acoustic pressure level
$[\phi_p]$	acoustic pressure PSD matrix
$[\phi_\xi]$	modal amplitude PSD matrix

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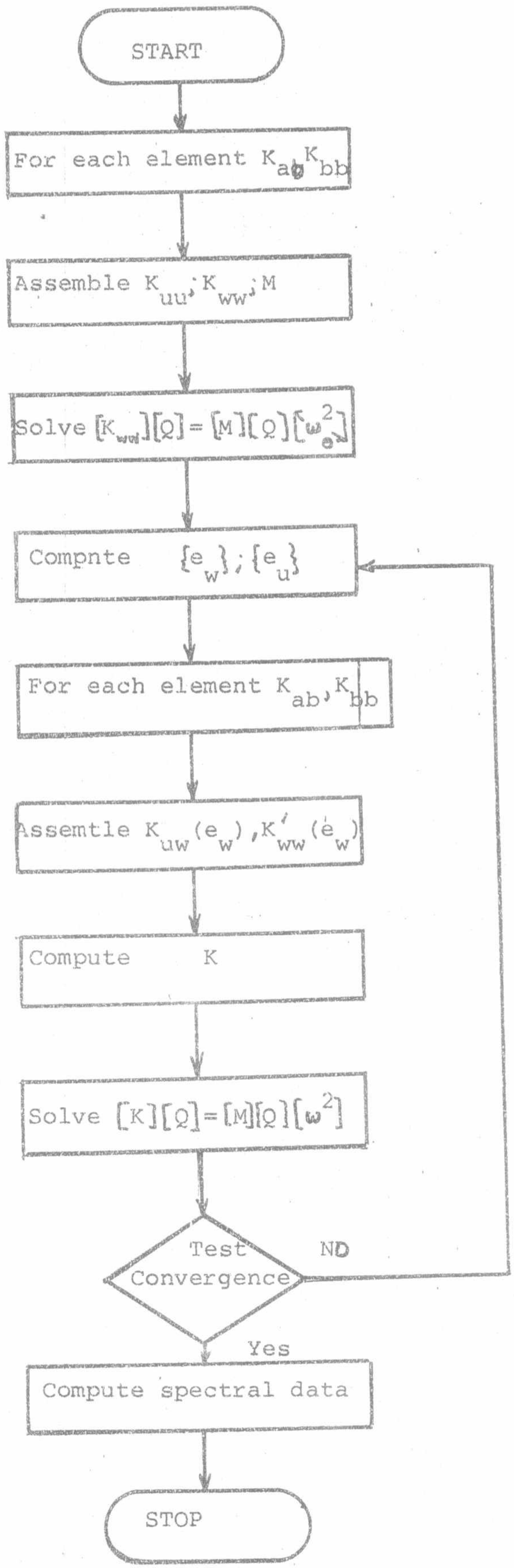

Fig.1: Flow chart of finite element computer program for nonlinear dynamic analysis of plate.