ANALYSIS OF ELASTIC AND VISCOUS PROPERTIES OF LINEAR VISCOELASTIC BODIES

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ABSTRACT

The use of plastics in manufacture is of great industrial importance today. In order to make the best economic use of plastics, their mechanical properties must be properly determined |1|.

In this paper some analysis of elastic and viscous properties of linear viscoeleastic bodies are made. A simulant of a solid propellant rocket fuel is subjected to this study. Gottenberg(2) made some experiments to find the relaxation function which represent the mechanical behaviour of this simulant of a solid propellant rocket fuel. A procedure is proposed here to find the mechanical constant of each element contained in a mathematical model for a linear viscoelastic material using experimental data. This model consisting of some suitable combination of springs, and viscous dashpots. The mechanical functions entering the constitutive relations of the theory of viscoelasticity are determined by experimental and theoritical means. Some experiments are made to find the mechanical behaviour of an available viscoelastic material (PVC).

NOMENCLATURE

$\Theta(t)$:Twist angle as a function of time.
P(D),Q(D):Functions of the operator $D=\partial/\partial t$.
P(s),Q(s):Laplace transform of the functions P(D) & Q(D).
t :Current time.
τ :variable time $0 \leq \tau \leq t$.
E :Spring constant.
$1/\eta$:viscosity coefficient of the dashpot.
h(t) :step function
$\delta(t)$:Dirac delta function.
T, w :Torque, weight.
R :Wheel radius.
r :specimen radius.
r_m :mean radius of the PVC specimen.
L :length of the P.V.C specimen.
b :Wall thickness of the PVC specimen.

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INTRODUCTION

The classical theory of elasticity deals with mechanical properties of perfectly elastic solid, for which in accordance with Hook's law stress is always directly proportional to strain but independent of the rate of strain. The theory of hydrodynamics deals with properties of perfectly viscous liquids, for which in accordance with Newton's law the stress is always directly proportional to rate of strain but independent of the strain itself. There are two important types of deviations. First, the strain (in a solid) or the rate of strain (in a liquid) may not be directly proportional to the stress but may depend on stress in a more complicated manner. Such stress expressions are familiar when the elastic limit is exceeded for a solid. Second, the stress may depend on both the strain and the rate of strain together, as well as higher time derivatives of the strain. Such time dependence evidently reflect a behaviour which combines liquid like and solid like charactersitics and they are therefore called viscoelastic.

In practice the most frequently used methods of specifications are namely, the step function constant stress (or creep)experiment,the constant strain (stress relaxation) experiment and the sinusoidial (dynamic) experiment, to measure relaxation modulus or compliance. It cannot be said that any one of the pervious methods of representation is more fundamental than the others, in principle they are equivalent to each other, and the relations between the different types of measurement (at a given temperature) are results of the linear viscoelastic behaviour and are given by linear viscoelasticity theory |4,5|. In practice, information which may be obtained easily in one type of test can be obtained only with difficulty in another type of test.

STRESS-STRAIN RELATIONS

For a linear viscoelastic material in which shear strain dominate the deformation so that the dilatational strain can be neglected and the theory of nicompressible flow is valid the general isotropic linear viscoelastic law has the form

$$P(D)\ s_{ij}(t) = Q(D)\ e_{ij}(t) \qquad (1)$$

where P(D) and Q(D) are linear operators of the form

$$P(D) = \sum_{k=0}^{N} p_k D^k \text{ and } Q(D) = \sum_{k=0}^{N} q_k D^k \qquad (2)$$

and D is the time derivative $\partial/\partial t$. The coefficients p_k and q_k and the numbers N are in general different for each operator, although certain restrictions on the N's values are required to determine observed physical characteristies, s_{ij} and e_{ij} are respectively the stress and strain deviators defined in the usual way

$$s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{mm}\delta_{ij},\quad e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_{mm}\delta_{ij} \qquad (3)$$

where σ_{ij} and ε_{ij} are the stress and infinitesimal strain tensors and

$$\delta_{ij} = 1 \text{ if } i=j \text{ and } \delta_{ij} = 0 \text{ if } i \neq j$$

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Taking the laplace transform* (L.T.) of (2) yields

$$\bar{P}(s)\ \bar{S}_{ij}(s) - \frac{1}{s}\sum_{k=1}^{N} P_k \sum_{r=1}^{N} s^r S_{ij}^{(k-r)}(o) =$$

$$\bar{Q}(s)\ \bar{e}_{ij}(s) - \frac{1}{s}\sum_{k=1}^{N} q_k \sum_{r=1}^{N} s^r e_{ij}^{(k-r)}(o) \qquad (4)$$

If the intial condition of the problem has the form

$S_{ij}(o) = e_{ij}(o) = 0$ then equation (4)

becomes

$$\bar{P}(s)\ \bar{S}_{ij}(s) = \bar{Q}(s)\ \bar{e}_{ij}(s) \qquad (5)$$

The relaxation integral form of the stress-strain relation for isotropic linear viscoelastic material is written as

$$S_{ij}(t) = \int_0^t G(t-\tau)\ \frac{\partial e_{ij}(\tau)}{\partial \tau}\ d\tau \qquad (6)$$

where G(t) is the relaxation modulus.
Taking the laplace transform of (6) we get

$$\bar{S}_{ij}(s) = s\ \bar{G}(s)\ \bar{e}_{ij}(s) \qquad (7)$$

Comparing equations (5) and (7) we obtain

$$\bar{G}(s) = \frac{1}{s}\ \frac{\bar{Q}(s)}{\bar{P}(s)} \qquad (8)$$

The compliance integral form of the stress-strain relation for isotropic linear viscoelastic material has the form

$$e_{ij}(t) = \int_0^t J(t-\tau)\ \frac{\partial S_{ij}(\tau)}{\partial \tau}\ d\tau \qquad (9)$$

where J(t) is the shear compliance function.
Taking laplace transform of eqn.(9) we get

$$\bar{e}_{ij}(s) = s\ \bar{J}(s)\ S_{ij}(s)$$

comparing this result by equation (5) then we get

$$\bar{J}(s) = \frac{1}{s}\ \frac{\bar{P}(s)}{\bar{Q}(s)} \qquad (10)$$

Comparison of equations (8) and (10) gives the relation between the shear modulus G(t)and creep compliance J(t) in linear viscoelasticity as follows

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* L.T is a transform which converts a function f(t) of real variable t into a function $\bar{f}(s)$ of complex variable s=p+i by the formula

$$\mathcal{L}\, f(t) = \bar{f}(s) = \int_0^\infty f(t)\ e^{-st}\ dt.$$

The function f(t) which is continuous in the range (o,∞) except for some isolated points and has finite increase is called the original. The function f(s) is called the L.T. of the function f(t).

$$\bar{J}(s) = 1 / \{S^2 \bar{G}(s)\} \qquad (11)$$

When a material exhibits linear viscoelastic behaviour its mechanical properties can be duplicated by a model consisting of some suitable combination of springs which obey Hook's law and viscous dashpots which obey Newtons law.

The behaviour of the Hookean spring and Newtonian dashpot as well as that of their parallel (Voigt) and series (Maxwell) arrangment was already known. Maxwellarrangement of the two elements in series corresponds to an unlimited deformation under load similar to that occuring in an uncross linked polymer e.g. polyisobutylene . On the other hand , the Voigt arrangement providing for a maximum displacament limited by the elastic deformation of the spring,corresponding physically to a cross linked polymer (e.g. polyurthane).

A model which prosses most of the general features of viscoelastic material is the typical four parameter model, Fig.1. It is a series connection of Maxwell and Voigt models. It is obvious that,we can obtain the mechanical properties of the three parameter model by letting $\eta_1 = \infty$ in the four parameter model. Voigt model can be obtained if we put $\eta_1 = \infty$ and $E_1=0$. Maxwell model is obtained if we consider $\eta_2 = \infty$, $E_2 = 0$ in the four parameter model.

Although the accuracy over large time intervals increases with the increase of the number of elements in the model but the mathematical formulae and computations may be unmanagable. Then we consider now the four parameter model Fig.1 and we compute the shear relaxation modulus. The four parameter model is the simplest model that includes the three basic types of behaviour: instantaneous elasticity , delayed elasticity and viscousflow. The differential equation which describe the considered four parameter model is obtained in the form

$$\left[E_1E_2 + \left(\frac{E_1}{\eta_1} + \frac{E_1}{\eta_2} + \frac{E_2}{\eta_1}\right)D + \left(\frac{1}{\eta_1\eta_2}\right)D^2\right] S_{ij}(t) = \left[\left(\frac{E_1E_2}{\eta_1}\right)D + \left(\frac{E_1}{\eta_1\eta_2}\right)D^2\right] e_{ij}(t) \qquad (12)$$

Examination of the four parameter model.

Let us see how accurate this model represents the behaviour of some actual material. The selected example for actual material is the simulant of solid fuel tested by Gottenberg |2| . It is a polyurthane matrix containing salt crystals and aluminium powder. At first we find analytically the expression of the relaxation modulus of the considered four parameter model and the check whether it fits the experimentally |2| obtained curve G(t) or not. Comparison between equations (1) and (12) gives

$$P(D)=E_1E_2 + (E_1/\eta_1 + E_1/\eta_2 + E_2/\eta_1)D + (1/\eta_1\eta_2)D^2$$

$$Q(D)=(E_1E_2/\eta_1)D + (E_1/\eta_1\eta_2)D^2$$

then $\bar{P}(s)$ and $\bar{Q}(s)$ are expressed as follows

$$\bar{P}(s)=E_1E_2 + (E_1/\eta_1 + E_1/\eta_2 + E_2/\eta_1)s + (1/\eta_1\eta_2)s^2$$

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$$\bar{Q}(s)=(E_1E_2/\eta_1)s+(E_1/\eta_1\eta_2)s^2$$

Using equation (8) we get the expression of the Laplace transform of the shear modulus $\bar{G}(s)$ as follows

$$\bar{Q}(s)=\frac{(E_1E_2/\eta_1)s+(E_1/\eta_1\eta_2)\,s^2}{s\;E_1E_2+(E_1/\eta_1+E_1/\eta_2+E_2/\eta_1)s+(1/\eta_1\eta_2)s^2}$$

Let us denote

$$a=E_1E_2/\eta_1\,,\quad b=E_1/\eta_1\eta_2\,,\quad c=E_1E_2\,,$$

$$f=1/\eta_1\eta_2 \quad\text{and}\quad d=(E_1/\eta_1+E_1/\eta_2+E_2/\eta_1)$$

Then $\bar{G}(s)$ can be written as

$$\bar{G}(s)=(a+bs)\,/\,(c+ds+fs^2) \qquad (13)$$

We use the Heavfside's expansion formula |3| to find the original of (13), i.e. to get G(t) we obtain

$$G(t)=\frac{a+b(-d+\sqrt{d^2-4fc})/2f}{c+d(-d+\sqrt{d^2-4fc})/f+3(-d+\sqrt{a^2-4fc})^2/4f}\;e^{\left[(-d+\sqrt{d^2-4fc})/2f\right]t}$$

$$+\frac{a+b(-d-\sqrt{d^2-4fc})/2f}{c+d(-d-\sqrt{d^2-4fc})/f+3(-d-\sqrt{d^2-4fc})^2/4f}\;e^{\left[(-d-\sqrt{d^2-4fc})/2f\right]t} \qquad (14)$$

Let us denote the coefficient of the first term by B and the power of the exponent in this term by a_2 and the coefficient of the last term by C and the power of the exponent in this term by a_3, then equation (14) takes the form

$$G(t)=B\,e^{a_2t}+C\,e^{a_3t} \qquad (15)$$

We shall assume that the expression(15)of G(t)Characterizes the behaviour of the selected above material |2|. In this case equation (15) will be the equation of the curve G(t) versus t obtained experimentally by Gottenberg |2|. These experimental results (Fig.2) are used to get the values of the constants B,C, a_2 and a_3.

Four equations are necessary for determination of these four constants. These equations are obtained by substitution in equation (15) by four points t_i, $G_i(t)$where i=1,2,3,4. The selected points are(10^{-4}, 156.4 10^5), (10^{-3}, 110.4 10^5), (10^{-2}, 91.4 10^5) and (1, 52.73 10^5). Substitution by thes values in (15) we get the for algebric equations for determination of B, q_2, C and a_3

$$\begin{aligned}156.4\;10^5 &= B\,e^{10^{-4}a_2}+C\,e^{10^{-4}a_3}\\ 110.4\;10^5 &= B\,e^{10^{-3}a_2}+C\,e^{10^{-3}a_3}\\ 91.4\;10^5 &= B\,e^{10^{-2}a_2}+C\,e^{10^{-2}a_3}\\ 52.73\,10^5 &= B\,e^{a_2}+C\,e^{a_3}\end{aligned} \qquad (16)$$

Solving these four equations by trial and error, the values of constants are obtained as follows

$$B = 0.6\ 10^7 \quad , \quad a_2 = -\ 0.01 \quad , \quad C = 1.4\ 10^7 \text{ and } d = -10$$

Solvability of the system of equations (16)means that the assumption that the curve G(t) versus (t) obtained experimentally fits equation (15) is right. Then equation (15) takes the form

$$G(t) = (0.6\ e^{-0.01\ t} + 1.4\ e^{-10\ t})\ 10^7 \quad N/m^2 \qquad (17)$$

for the selected above material.
It may be noticed that the formula (15) representing the characteristics of the four parameter model is similar to that obtained by fitting the experimental curves which represent the mechanical behaviour of the considered simulant of the rocket fuel.

Analysis of Elastic and Viscous properties to analyse the elastic and viscous properties of the choosed material, we determine the roll of each element of the four element model representing the mechanical properties of the used material to make a quantitative analysis of the elastic and viscous properties of the considered viscoelastic solid propellant fuel. For this purpose the modulus of elasticity of the two springs E_1 and E_2 and the viscosity coefficients $1/\eta_1$ and $1/\eta_2$ are calculated. This needs the solution of the following equations.

$$\frac{\frac{E_1E_2}{\eta_1} - \frac{E_1}{2}\left(\frac{E_1}{\eta_1} + \frac{E_1}{\eta_2} + \frac{E_2}{\eta_1}\right) \pm \left[\left(\frac{E_1}{\eta_1} + \frac{E_1}{\eta_2} + \frac{E_2}{\eta_1}\right)^2 - \frac{4E_1E_2}{\eta_1\eta_2}\right]^2}{E_1E_2 - \frac{1}{\eta_1\eta_2}\left(\frac{E_1}{\eta_1}+\frac{E_1}{\eta_2}+\frac{E_2}{\eta_1}\right)\left(\frac{E_1}{\eta_1}+\frac{E_2}{\eta_1}+\frac{E_1}{\eta_2}+\left[\left(\frac{E_1}{\eta_1}+\frac{E_1}{\eta_2}+\frac{E_2}{\eta_1}\right)^2 - \frac{4E_1E_2}{\eta_1\eta_2}\right]\right)^{\frac{1}{2}} + \frac{3}{4\eta_1\eta_2}\left(\frac{E_1}{\eta_1}+\frac{E_1}{\eta_2}+\frac{E_2}{\eta_1} \pm \pm\left[\left(\frac{E_1}{\eta_1}+\frac{E_1}{\eta_2}+\frac{E_2}{\eta_1}\right)^2 - \frac{4E_1E_2}{\eta_1\eta_2}\right)\right]^{\frac{1}{2}}}$$

$$= 0.6 \times 10^7 \quad \text{for positive sign.} \qquad (18)$$

$$= 1.4 \times 10^7 \quad \text{for negative sign.} \qquad (19)$$

$$\left(-\frac{\eta_1\eta_2}{2}\right)\left[\left(\frac{E_1}{\eta_1}+\frac{E_1}{\eta_2}+\frac{E_2}{\eta_1}\right) \pm \left(\left(\frac{E_1}{\eta_1}+\frac{E_1}{\eta_2}+\frac{E_2}{\eta_1}\right)^2 - \frac{4E_1E_2}{\eta_1\eta_2}\right)^{\frac{1}{2}}\right] =$$

$$= 0.01 \quad \text{for positive sign.} \qquad (20)$$

$$= 10 \quad \text{for negative sign.} \qquad (21)$$

Trial and error gives the following values for η_1, η_2, E_1 and E_2 as:

$$E_1 = 4 \times 10^9\ N/m^2, \qquad \eta_1 = 10^{-3}\ (N\text{-}s/m^2)^{-1}$$

$$E_2 = 7 \times 10^9\ N/m^2, \qquad \eta_2 = 8.8\text{x}10^{-13}\ (N\text{-}s/m^2)^{-1}$$

Form these values it is clear that the delayed elastic behaviour of this model is more rigid than the direct elastic behaviour (7/4 times). It is noticed also that the direct viscosity is more than the constrained viscosity inducing recoverable viscous strain in the material (8.8 times).

EXPERIMENTAL WORK

There are many different but equivalent means of measuring the mechanical properties which enter the stress strain constitutive relations. The most fundamental describitions of mechanical properties are probably those given by relaxation function and creep function. These can be determined by direct experimental observations.

1 Creep Test:

In this test the specimen, which initially is supposed to be undisturbed is subjected to a load which is applied as quickly as possible and varies in such a manner as to keep the stress at a constant value after load application. The resulting deformation or strain in the specimen is measured as a function of time elapsed since the load was first applied. If the instant of load application is chosen as the origin of the time scale, the stress in the specimen has the form

$$\sigma(t) = \sigma_0 h(t) \tag{22}$$

where σ_0 is a constant depending on the magnitude of the applied load and the dimensions of the specimen, h(t) denotes the Heaviside step function $h(t)=0$ for $t < 0$ and $h(t)=1$ for $t \geq 0$. Since $(\partial h(t)/\partial t)=\delta(t)$ where $\delta(t)$ is the Dirac delta function.
The explicit linear form for a non-ageing viscoelastic material is

$$\varepsilon(t) = \int_0^t J(t-\tau)\,\frac{\partial\sigma(\tau)}{\partial\tau}\,d\tau \tag{23}$$

Subjecte equation(23) to step stress(22), then

$$\varepsilon(t) = \sigma_0\, J(t) \tag{24}$$

Thus, the creep function J(t) is readily determined by dividing the measured strain response by known constant σ_0 .

2 Description of the creep testing apparatus:

The testing apparatus is shown in Fig.2. The test specimen(thin walled PVC cylinder) is gripped from each end by a jaw which tightened by a split clamping ring. To prevent the formation of the thin walled PVC cylinder due to clamping, a plug was fitted inside the specimen at each end.
One of the two jaws clamping the specimen ends is fixed to the stand by two bolts. The other end is coupled to a spindle by means of a chuck. The torque is applied to the specimen through the spindle which is mounted on two single raw ball bearing mounted in the stand. The torque is obtained by a weight suspended at one of a flexible wire wounded around a wheel.The wheel is fixed on the spindle by means of threaded joint.
Two pointers are fixed on the split clamping rings, to measure the relative rotation of one cross section with respect to the other. The two stands which are rigidly fixed together by two rods are fixed on the bench by four bolts.

3 Angle measurements:

It is noticed that the creep, indicating by the increase in the twist angle of the PVC pipe is very small and cannot be accurately measured by the ordinary scale. For accurate measurement of the rotation angle, th pointer of the scale is connected to a displacement transducer. This transducer changes the displacement into an electric signal which is transmitted through a shielded cable to a 6-channel amplifier to amplify it and then to an U.V.R. The installation and electric set-up of the displacement transducer are shown in Fig.3a, Fig.3b shows a photogtaph for the creep test apparatus.

4 Test specimen:

A long PVC thin walled tube with 3.6cm outside diameter, 0.35cm thickness and 55.5cm length are used. This specimen was machined from PVC rods with diameter 5cm, and reduced to 3.6cm to avoid the outer layers. Because the outer layers may change their properties due to the sudden cooling after extrusion of the PVC rods during their production. A long specimen was used to avoid the effect of the clamping at the ends. During turning of the specimen, a small depth of cut and sharp tools are used to prevent overheating of the outer surface of the specimen which change its properties. And also during drilling of the specimen, the feed is also made small and the drilling process is made intermittent to prevent over-heating of the inner surface of the specimen.

5 Experimental procedure:

The specimen is mounted in the apparatus as mentioned above and the displacement transducer fitted in place and connected to the amplifier and U.V.R. The torque is applied by putting the required weight on the weights carrier. Then, the pointer rotates, moving the displacement transducer to record the change in the twist angle $\theta(t)$ on a senstive paper by the U.V.R.

The torque T exerted on the specimen is calculated as follows $T=W.R.$(kg.cm), where W the weight suspended in kgs.,and R the radius of the wheel in cms. $T=10x7.5=82.125$ kg.cm. The shear stress is $\tau_0=Txr/J_0$. (kg/cm ^{2}), where r the tube radius (cms) and J_0 is the polar moment of inertia of the thin walled tube = $2\pi r^3 b$(cm^4), (b is the thickness of tube cm.).

$$J_o = 2\,\pi(1.8)^3 \times 0.35 = 12.825 \text{ cm}^4 \quad \text{then,}$$

$$\tau_o = 82.125 \times 1.8/12.825 = 11.526 \quad \text{kg/cm}^2.$$

using equation (24) the creep compliance is expressed as

$$J(t) = \gamma(t)/\tau_o . \qquad (25)$$

The small shear strain $\gamma(t)$ is calculated from the measured twist angle $\theta(t)$ as follows

$$\gamma(t) = r_m \times \theta(t)/L = 1.18 \times 10^{-3}\ \theta(t). \qquad (26)$$

From the measured values of $\theta(t)$ the corresponding values of $\gamma(t)$ are calculated using equation(26). From these values and using equation (25) the creep compliances are calculated. The resulting creep compliance J(t) are plotted in Fig.(4).
It is noticed from Fig.(4) that, the creep compliance increase gradually

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with time and after 120 minutes it has almost a constant value. The working period of a solid propellant fuel starts from t=0 up to t=5 or even 20 minute. The properties of the viscoelastic material will be determined for the period from 0 to 120 minute.

The first part of the curve J(t) from t=0 to t=1 is denoted by $J_1(t)$ and can be fitted with good enough approximation by a first order polynomial in the form

$$J_1(t)=4.9\times10^{-5}+0.3\times10^{-5}t,$$ making the Laplace transform gives

$$J_1(s)=\frac{4.9\times10^{-5}}{s}+\frac{0.3\times10^{-5}}{s^2},$$ multiplication by s^2 gives

$$s^2J_1(s) = 0.3\times10^{-5} + 4.9\times10^{-5}\,s.$$

using equation (11) the Laplace transform of the relaxation function is

$$G_1(s) = \frac{1}{4.9\times10^{-5}(s+0.06)},$$ making the Laplace inverse gives

$$G_1(t) = 20408.16\ e^{-0.06\ t}.$$

The second part of the curve J(t) from t=1 up to t=120 is denoted by $J_2(t)$ and can be fitted with good enough approximation by a first order polynomial in the form

$$J_2(t) = 5.195\times10^{-5} + 7.07\times10^{-9}\,t.$$

by the same procedure, the relaxation function of the second part has the form

$$G_2(t) = 19249.278\ e^{-9.817\times10^{-4}\,t}.$$

Then, the relaxation function G(t) for the time range from t=0 to t=120 minute, can be expressed as

$$G(t)=20408.16\ e^{-0.06\ t}\,|h(t)-h(t-1.0)|+19230.77\ e^{-1.359\times10^{-4}t}h(t-1.0)$$

$$= A_1\ e^{c_1t}\ |h(t)-h(t-1.0)|+A_2\ e^{c_2t}h(t-1.0)(kg/cm^2)^{-1} \quad (27)$$

6 Experimental considerations:

When a particular form of the experiment is suggested for determining material properties, attention must obviously be given to the condition under which the experiment can be performed. In practise, the suitablity of a particular mode of deformation of interest, and by the range of values over which functions are required.
The creep test requires application to the specimen of a homogeneous stress field having the time dependent for, $\tau(t) = \tau_0 h(t)$, and the applied load must be continuously adjusted as the area of cross section varies. Since, the dimensions of the specimen do not change during this test, the distinction between nominal and true stress does not arise, and the torque input and twist data are related to the stress and strain components.
To ensure that the material remains in the linear range during test, the magnitude of the applied stress τ_0 must be kept within certain limit.
In engineering applications the time scales of possible interest can vary

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from a few microseconds to several years.The particular application which is investigated will determine the time scale required in the corresponding creep test. The time scale for the discussed here problem depends on the total working time of a rocket with solid propellant fuel.

DISCUSSION AND CONCLUSION

From the above analysis, we can say that the four parameter model represent to enough approximation the viscoelastic behaviour for a typical solid propellant rocket fuel. Also, we can determine the roll of each element of the four elements model representing the mechanical properties of the used material to make a quantitative analysis of the elastic and viscous properties of the considered viscoelastic solid propellant rocket fuel. For this purpose the modulus of elasticity E_1 and E_2 and the viscosity coefficients $1/\eta_1$ and $1/\eta_2$ are calculated. This done by putting the expression (12) of a_1, a_2, A and B equal to their values in eqn.(2), then

$$E_1 = 4\times10^7 \quad N/m^2 , \qquad \eta_1 = 10^{-3} \quad (N\text{-}S/m^2)^{-1}$$

$$E_2 = 7\times10^7 \quad N/m^2 , \qquad \eta_2 = 8.8\times10^{-13} \quad (N\text{-}S/m^2)^{-1}$$

From these values it is clear that the delayed elastic behaviour of this model is more rigid than the direct elastic behaviour (7/4 times). It is noticed also that the direct viscosity is more than the constrained viscosity inducing recoverable viscous strain in the material (8.8 times). The general features of the creep function for real material(PVC) is shown in Fig.(4),there is a relatively rapid increase in J(t) for small values of t, that is, after application of the load, due to the instanteous elasticity of the material. As t increase the slope of the curve decrease and as t reaches about 120 minutes the slope is approximatly zero.

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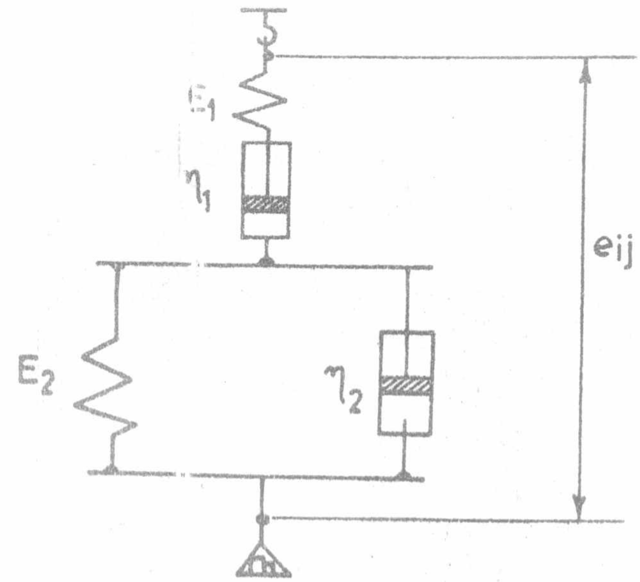
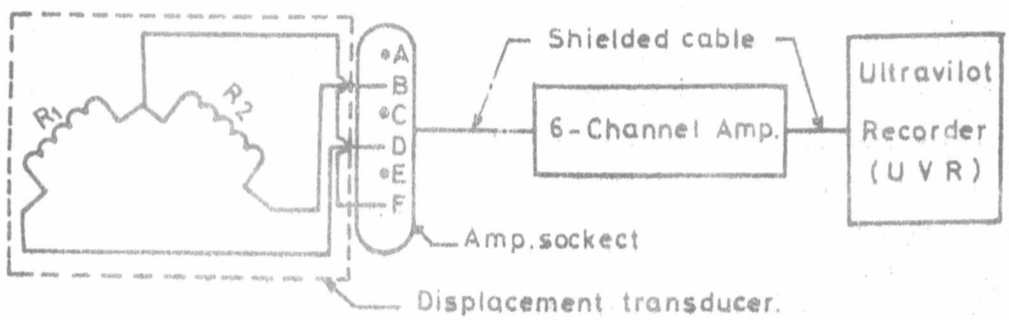
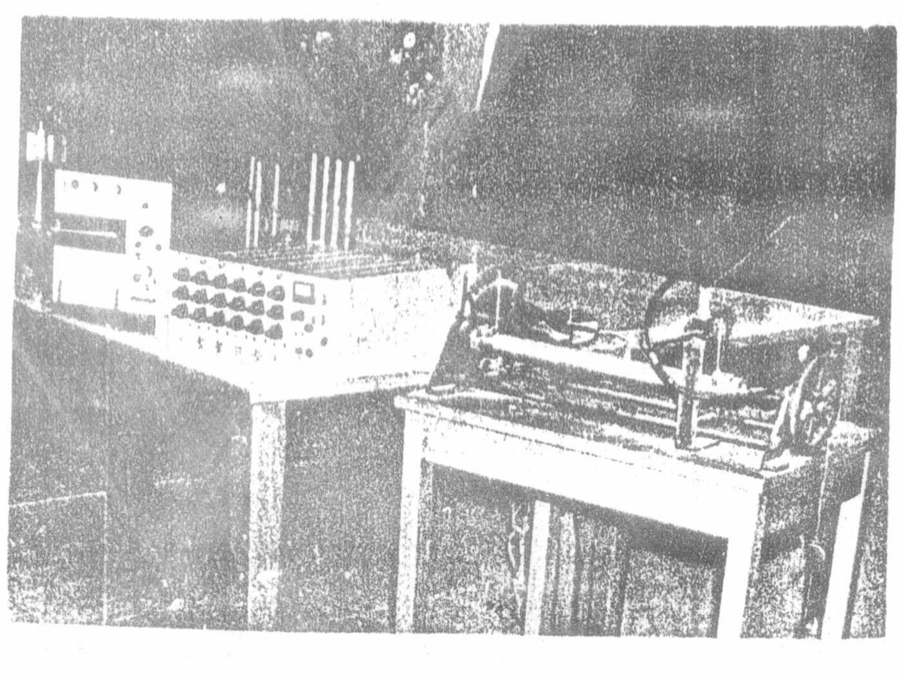


Fig. 1- Four parameter model.



(a)



(b)

Fig. 3:a- Electrical setup of the displacement transducer
b- Photograph of the creep testing arrangement

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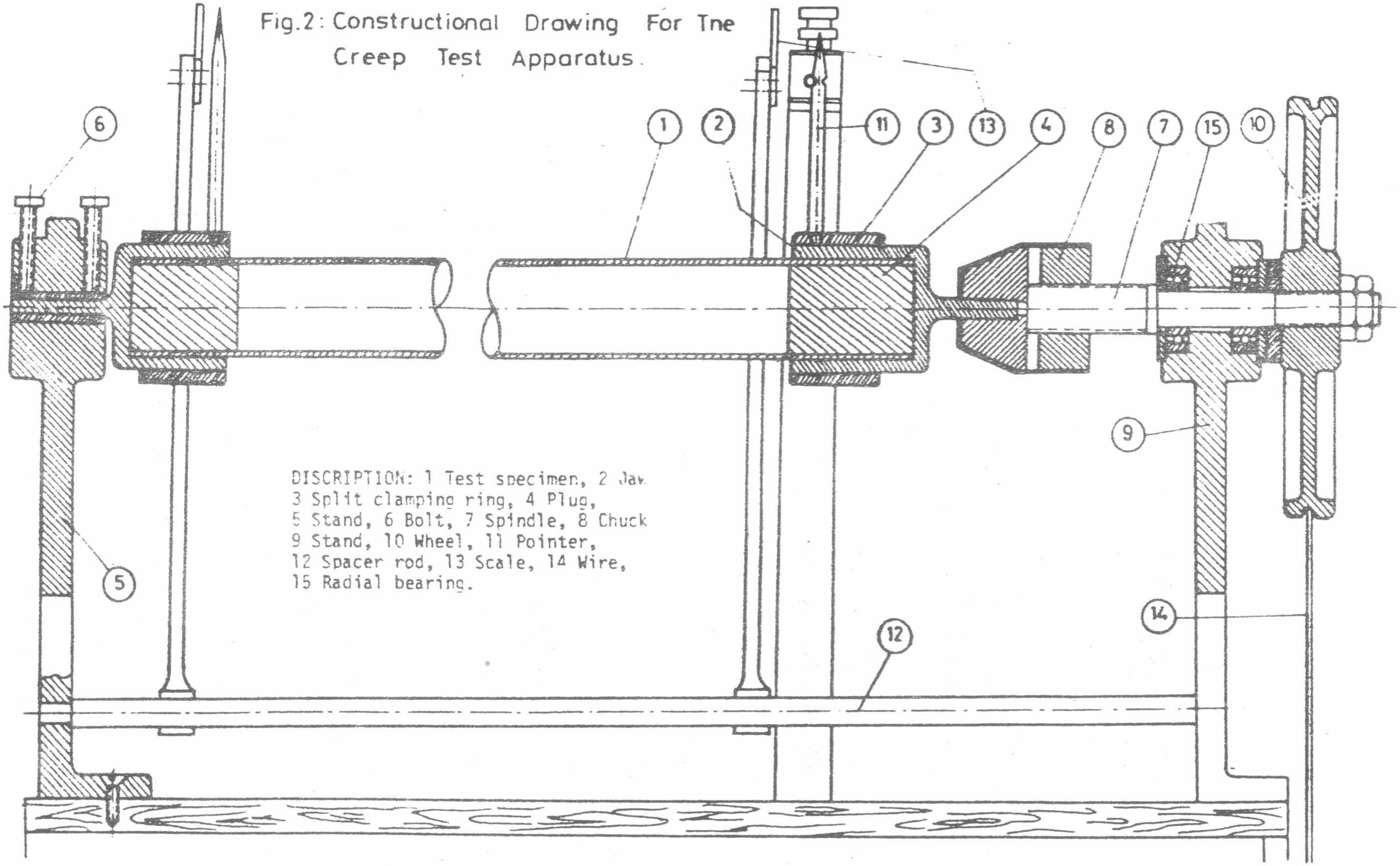
Fig.2: Constructional Drawing For Tne Creep Test Apparatus.



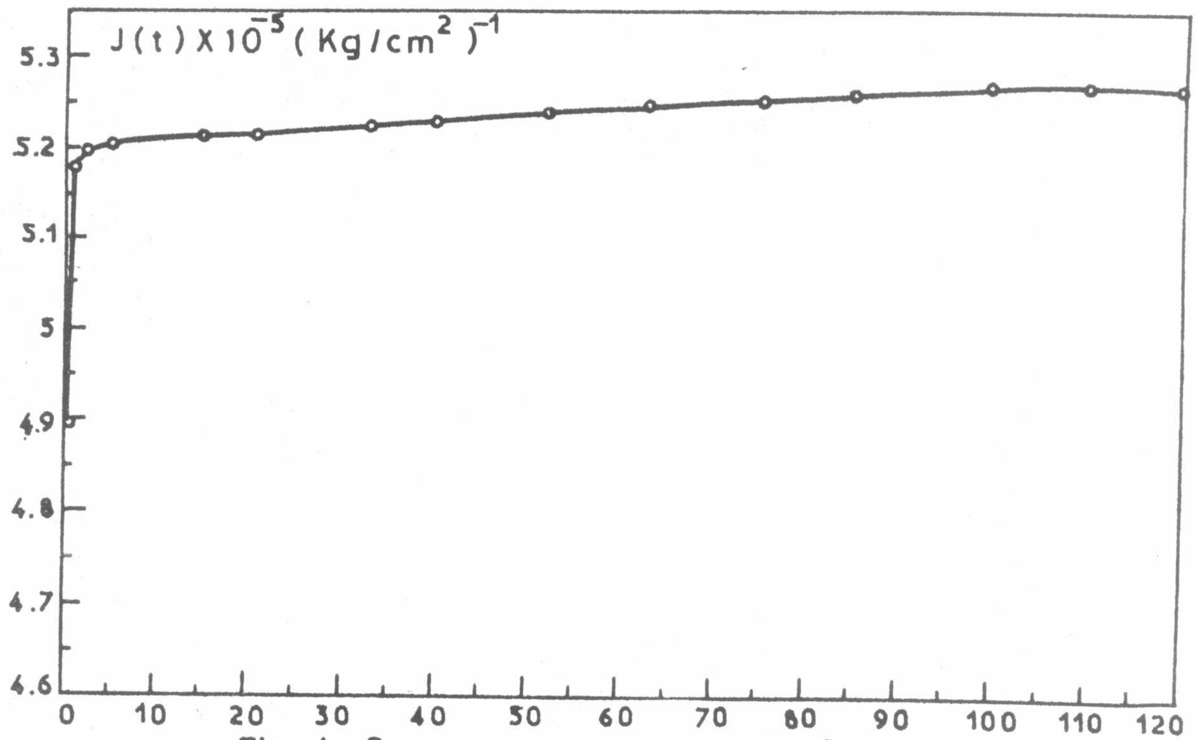


Fig. 4 : Creep compliance for PVC.

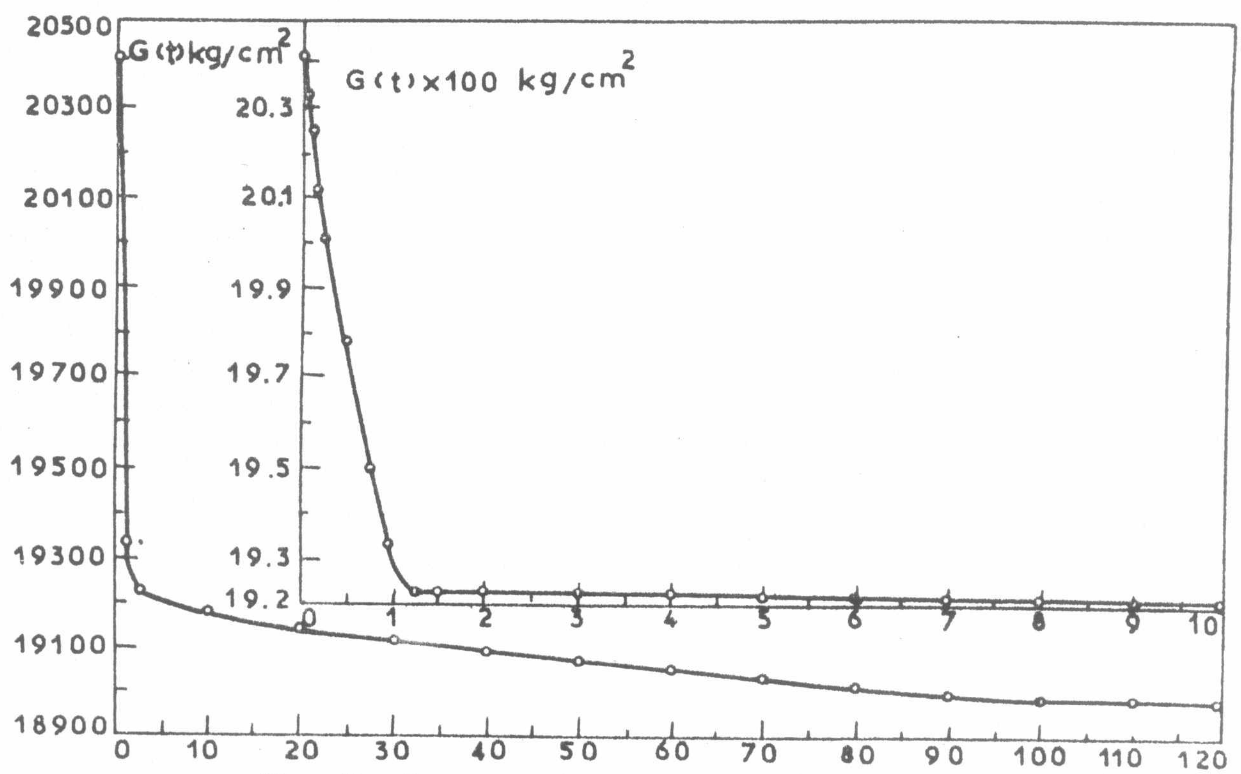


Fig. 5 : Relaxation for PVC.