ON THE DYNAMIC ANALYSIS OF A CAM MECHANISM WITH BEARING CLEARANCES

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ABSTRACT

The paper develops a procedure for the dynamic analysis of a cam mechanism with bearing clearances. The analysis mainly relies on determining the clearance angles β_{ij} and its first and second derivatives. The governing equations of each clearance angle are developed on the condition that the variations in displacements and velocities due to clearance effect are very small and could be ignored. Also, the equations of these clearance angles are developed such that the common normal at the point of contact at each clearance connection coincides with the direction of the clearance eccentricity vector which has the same direction of the normal reaction force at the point of contact. A straight sided cam operating a roller follower is considered to illustrate the procedure.

INTRODUCTION

Bearing clearances due to manufacturing tolerances and subsequent wear have undesired effects on the dynamic behavior of machinery rotating at high speed. Although, the problem has a complexity, there is a need for a systematic procedure to estimate quantitatively the accelerations and bearing forces as affected by these bearing clearances.

Chen [1]^{**} presented a method to obtain the dynamic response of the cam-driven systems numerically. Wiederrich and Roth [2] presented methods for dynamic synthesis of cams using finite trigonometric series. Koster [3] investigated the dynamic behavior of a follower driven by a flexible shaft. Chen and Polvanish [4-5] investigated the dynamic response of the cam-driven mechanism based on a linear and non-linear lumped system models of single and double freedom. The dynamic synthesis of modeled cam systems with one and two degrees of freedom is reported by Matthew and Tesar [6-8,10] to validate a number of fundamental design rules of thumb pertaining to classification, motion specifications, motion distortion etc. Koster [9] provided a digital simulation program based on a model with four degrees of freedom.

The dynamic behavior of a cam-actuated system is reported by Chen [11], in

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which the follower is coupled with a non-linear pneumatic mechanism of hysteric type and contains a compressible fluid on a plunger. Sankar and Osman [12] investigated the dynamic accuracy in cam production using hybrid profiling mechanism in turning with a single point cutting tool. The optimum elastodynamic synthesis of a cam-follower train is reported by Rao [13], in which, he found the retainer spring constant (stiffness) and the mass of follower train required to give minimum flexibility error and also to prevent separation between cam and follower. Szakallas and Savage [14] used dimensional analysis and found two ratios which characterize the level of vibration in the cam-driven system.

In this work, the dynamic analysis of a cam mechanism with bearing clearances is carried out. The clearance angles β_{ij} are obtained using three conditions as follows :

- 1) The variation in displacements due to clearance effects are negligible.
- 2) The variations in velocities due to clearance effects are negligible.
- and 3) The common normal at the point of contact coincides with the direction of the bearing eccentricity vector $\vec{A}_{ij}$ and also coincides with the direction of the bearing reaction force at the point of contact.

The estimated clearance angles together with its first and second derivatives are systematically used to evaluate kinematic and dynamic quantities of the cam mechanism.

A straight sided cam operating a roller follower with two bearing clearances is used to illustrate the procedure.

ANALYTICAL METHOD

In this section a system of equations is developed which are valid for all contact conditions at any cam rotational angle θ_2 in the period of the cam action. A simultaneous contact at joints with clearance is assumed and used in the dynamic analysis of the cam mechanism. This is a reasonable assumption, since a separation at any joint will last only an infinitesimal small period of time ($\Delta t \rightarrow 0$). Therefore, the solution during the separation period ($\Delta t \rightarrow 0$) is obtained graphically by joining the results of the two contact points before and after impact.

Figure 1 shows a straight sided cam operating a roller follower with two bearing clearances which is considered in this analysis. One, 2, 3, and 4 are link numbers corresponding to the frame, cam, roller and follower respectively.

Referring to figures 1 and 2, the angles α and γ and be expressed as,

$$\gamma = \cos^{-1}[(r_b - r_n)/d]$$

$$\alpha = \tan^{-1}[d \sin \gamma / (r + r_b)] \quad (1)$$

Three cases A, B and C are considered through the analysis.

Case A : Contact with Flate Flank During Upward Motion of the Follower

Figure 2 shows the contact at any instant during this period, where,

$$(\pi/2 - \gamma) \leq \theta_2 \leq (\pi/2 - \gamma + \alpha). \quad (2)$$

From figure 2, the distances r_1 and q which is the projection of vector $\vec{D}$ along the X direction and their derivatives can be written as,

$$r_1 = (r + r_b) (1 - \cot \gamma \tan \theta_2') \quad (3)$$

$$\dot{r}_1 = -(r + r_b) \omega_2 \cot \gamma \sec^2 \theta_2'$$

$$\ddot{r}_1 = -2(r + r_b) \omega_2^2 \cot \gamma \sec^2 \theta_2' \tan \theta_2' \quad (4)$$

$$q = D \cos \theta_2 = \Delta_{43} \cos \beta_{43} - r_1 \cos(\gamma + \theta_2) - \Delta_{12} \cos \beta_{12}$$

$$\dot{q} = -\Delta_{43} \dot{\beta}_{43} \sin \beta_{43} - \dot{r}_1 \cos(\gamma + \theta_2) + r_1 \omega_2 \sin(\gamma + \theta_2) + \Delta_{12} \dot{\beta}_{12} \sin \beta_{12}$$

$$\begin{aligned} \ddot{q} = & -\Delta_{43} (\ddot{\beta}_{43} \sin \beta_{43} + \dot{\beta}_{43}^2 \cos \beta_{43}) + \Delta_{12} (\ddot{\beta}_{12} \sin \beta_{12} + \dot{\beta}_{12}^2 \cos \beta_{12}) \\ & - [(\ddot{r}_1 - r_1 \omega_2^2) \cos(\gamma + \theta_2) - 2\dot{r}_1 \omega_2 \sin(\gamma + \theta_2)] \end{aligned} \quad (5)$$

where

$$\theta_2' = \theta_2 - \left(\frac{\pi}{2} - \gamma\right)$$

The follower displacement, velocity and acceleration Y_4 , $\dot{Y}_4$, $\ddot{Y}_4$ can be expressed as

$$Y_4 = \Delta_{12} \sin \beta_{12} + q \tan \theta_2 + r_1 \sin(\gamma + \theta_2) - \Delta_{43} \sin \beta_{43} - (r + r_b)$$

$$\begin{aligned} \dot{Y}_4 = & \Delta_{12} \dot{\beta}_{12} \cos \beta_{12} + \dot{q} \tan \theta_2 + q \omega_2 \sec^2 \theta_2 + \dot{r}_1 \sin(\gamma + \theta_2) \\ & + r_1 \omega_2 \cos(\gamma + \theta_2) - \Delta_{43} \dot{\beta}_{43} \cos \beta_{43} \end{aligned}$$

$$\begin{aligned} \ddot{Y}_4 = & \Delta_{12} (\ddot{\beta}_{12} \cos \beta_{12} - \dot{\beta}_{12}^2 \sin \beta_{12}) - \Delta_{43} (\ddot{\beta}_{43} \cos \beta_{43} - \dot{\beta}_{43}^2 \sin \beta_{43}) \\ & + (\ddot{q} \tan \theta_2 + 2 \dot{q} \omega_2 \sec^2 \theta_2 + 2 q \omega_2^2 \sec^2 \theta_2 \tan \theta_2) \\ & + [(\ddot{r}_1 - r_1 \omega_2^2) \sin(\gamma + \theta_2) + 2\dot{r}_1 \omega_2 \cos(\gamma + \theta_2)] \end{aligned} \quad (6)$$

Using D'Alembert principle, the dynamic equations of the cam assembly, roller and follower are given by,

$$\begin{aligned}
 R_{12}^x - R_{32} \cos(\gamma + \theta_2) &= m_2 \ddot{X}_2 \\
 -R_{12}^y - R_{32} \sin(\gamma + \theta_2) &= m_2 \ddot{Y}_2 \\
 -R_{43}^x - R_{32} \cos(\gamma + \theta_2) &= m_3 \ddot{X}_3 \\
 R_{43}^y - R_{32} \sin(\gamma + \theta_2) &= m_3 \ddot{Y}_3 \\
 m_4 \ddot{Y}_4 + C \dot{Y}_4 + K Y_4 &= R_{43}^y - F_0 \\
 T_2 = m_2 S (\ddot{Y}_2 \cos \theta_2 - \ddot{X}_2 \sin \theta_2) &+ (q \sin \gamma / \cos \theta_2) R_{32}
 \end{aligned} \tag{7}$$

where:

$$\begin{aligned}
 \ddot{X}_2 &= -S \omega_2^2 \cos \theta_2 - \Delta_{12} (\ddot{\beta}_{12} \sin \beta_{12} + \dot{\beta}_{12}^2 \cos \beta_{12}) \\
 \ddot{Y}_2 &= -S \omega_2^2 \sin \theta_2 + \Delta_{12} (\ddot{\beta}_{12} \cos \beta_{12} - \dot{\beta}_{12}^2 \sin \beta_{12}) \\
 \ddot{X}_3 &= -\Delta_{43} (\ddot{\beta}_{43} \sin \beta_{43} + \dot{\beta}_{43}^2 \cos \beta_{43}) \\
 \ddot{Y}_3 &= \ddot{Y}_4 + \Delta_{43} (\ddot{\beta}_{43} \cos \beta_{43} - \dot{\beta}_{43}^2 \sin \beta_{43})
 \end{aligned} \tag{8}$$

Equation (8) are the same for the three cases of contact A, B and C.

If the clearance angles (β_{12}, β_{43}) and its derivatives $(\dot{\beta}_{12}, \dot{\beta}_{43}), (\ddot{\beta}_{12}, \ddot{\beta}_{43})$ are known, equations (2) to (8) can be used to evaluate displacement, velocity, acceleration of follower $(Y_4, \dot{Y}_4, \ddot{Y}_4)$, the dynamic reactions (R_{12}, R_{32}, R_{43}) and the torque T_2 .

Clearance Angles

The follower displacement Y_4 is given by equation (6). From figure 2, the follower velocity $\dot{Y}_4$ can be written as,

$$\dot{Y}_4 = \omega_2 \cdot \overline{O_{12} O_{42}} \tag{9}$$

where

$$\overline{O_{12} O_{42}} = - [(\Delta_{12} \sin \beta_{12} + q \tan \theta_2) / \tan(\gamma + \theta_2)] - r_1 \cos(\gamma + \theta_2) + \Delta_{43} \cos \beta_{43} \quad (10)$$

Since, the variations of Y_4 and $\dot{Y}_4$ due to clearance effect are negligible, these conditions lead to,

$$\begin{aligned} f_1 &= \sin(\beta_{12} - \theta_2) - \Delta \sin(\beta_{43} - \theta_2) = 0 \\ f_2 &= \sin(\beta_{12} - \theta_2) - \Delta f(\theta_2) \cos \beta_{43} = 0 \end{aligned} \quad (11)$$

where,

$$\Delta = |\vec{\Delta}_{43}| / |\vec{\Delta}_{12}|$$

$$\text{and } f(\theta_2) = -\sin \theta_2 + \cos \theta_2 \tan(\gamma + \theta_2)$$

Equations (11) are solved for the clearance angles (β_{12}, β_{43}) using Newton Raphson's method. The derivatives of equations (11) give the angular velocities of clearance angles $(\dot{\beta}_{12}, \dot{\beta}_{43})$ as,

$$\begin{aligned} \dot{\beta}_{43} &= \omega_2 [f'(\theta_2) \cos \beta_{43} + \cos(\beta_{43} - \theta_2)] / [f(\theta_2) \sin \beta_{43} + \cos(\beta_{43} - \theta_2)] \\ \dot{\beta}_{12} &= \Delta \xi \dot{\beta}_{43} + (1 - \Delta \xi) \omega_2 \end{aligned} \quad (12)$$

where,

$$\xi = \cos(\beta_{43} - \theta_2) / \cos(\beta_{12} - \theta_2)$$

and

$$f'(\theta_2) = -\cos \theta_2 - \sin \theta_2 \tan(\gamma + \theta_2) + \cos \theta_2 \sec^2(\gamma + \theta_2)$$

Equation (12) is used to evaluate $(\dot{\beta}_{12}, \dot{\beta}_{43})$. Since the directions of the clearance vectors $\vec{\Delta}_{12}$ and $\vec{\Delta}_{43}$ coincide with the directions of the reactions $\vec{R}_{12}$ and $\vec{R}_{43}$, these conditions lead to,

$$\begin{aligned} R_{12}^x \sin \beta_{12} &= R_{12}^y \cos \beta_{12} \\ R_{43}^x \sin \beta_{43} &= R_{43}^y \cos \beta_{43} \end{aligned} \quad (13)^*$$

*Equation (13) is the same for the three cases A, B and C.

Solving equations (7) for the reactions $R_{12}^X, R_{12}^Y, R_{43}^X, R_{43}^Y$ in terms of the accelerations ($\ddot{X}_2, \ddot{Y}_2, \ddot{X}_3, \ddot{Y}_3$) and substituting into the results the values of these accelerations using equations (6) and (8) and substituting the resulting values of these reactions into equations (13) and re-arranging leads to ,

$$a_1 \ddot{\beta}_{12} + a_2 \ddot{\beta}_{43} = a_3$$

$$b_1 \ddot{\beta}_{12} + b_2 \ddot{\beta}_{43} = b_3 \quad (14)$$

The angular accelerations of the clearance angles (β_{12}, β_{43}) are obtained from equation (14) as ,

$$\ddot{\beta}_{12} = (a_3 b_2 - a_2 b_3) / (a_1 b_2 - a_2 b_1) \quad (15)$$

$$\ddot{\beta}_{43} = (a_1 b_3 - a_3 b_1) / (a_1 b_2 - a_2 b_1) \quad (16)$$

Once, the values of (β_{12}, β_{43}), ($\dot{\beta}_{12}, \dot{\beta}_{43}$) and ($\ddot{\beta}_{12}, \ddot{\beta}_{43}$) are evaluated, equations (2) to (8) are used to evaluate follower motion, reaction forces and torque.

Case B: Contact with Cam Noze

Figure 3 shows the contact at any instant during this period, where

$$\left(\frac{\pi}{2} - \gamma + \alpha\right) \leq \theta_2 \leq \left(\frac{\pi}{2} + \gamma - \alpha\right) \quad (17)$$

From figure 3 the angle λ and its derivatives can be written as,

$$\lambda = \sin^{-1}[(\Delta_{12} \cos \beta_{12} + d \cos \theta_2 - \Delta_{43} \cos \beta_{43}) / (r + r_n)] \quad (18)$$

$$\left. \begin{aligned} \dot{\lambda} &= (-\Delta_{12} \dot{\beta}_{12} \sin \beta_{12} - d \dot{\omega}_2 \sin \theta_2 + \Delta_{43} \dot{\beta}_{43} \sin \beta_{43}) / (r + r_n) \cos \lambda \\ \ddot{\lambda} &= \{ [\Delta_{43} (\ddot{\beta}_{43} \sin \beta_{43} + \dot{\beta}_{43}^2 \cos \beta_{43}) - \Delta_{12} (\ddot{\beta}_{12} \sin \beta_{12} + \dot{\beta}_{12}^2 \cos \beta_{12}) - d \dot{\omega}_2^2 \cos \theta_2] / (r + r_n) \cos \lambda \} + \dot{\lambda}^2 \tan \lambda \end{aligned} \right\} \quad (19)$$

The follower displacement, velocity and accelerations are,

$$Y_4 = \Delta_{12} \sin \beta_{12} + d \sin \theta_2 + (r + r_n) \cos \lambda - \Delta_{43} \sin \beta_{43} - (r + r_b)$$

$$\dot{Y}_4 = \Delta_{12} \dot{\beta}_{12} \cos \beta_{12} + d \dot{\omega}_2 \cos \theta_2 - (r + r_n) \dot{\lambda} \sin \lambda - \Delta_{43} \dot{\beta}_{43} \cos \beta_{43}$$

$$\ddot{Y}_4 = \Delta_{12}(\ddot{\beta}_{12} \cos \beta_{12} - \dot{\beta}_{12}^2 \sin \beta_{12}) - \Delta_{43}(\ddot{\beta}_{43} \cos \beta_{43} - \dot{\beta}_{43}^2 \sin \beta_{43}) - d\omega_2^2 \sin \theta_2 - (r+r_n)(\ddot{\lambda} \sin \lambda + \dot{\lambda}^2 \cos \lambda) \quad (20)$$

Using D'Alembert principle, the dynamic equations of cam, roller and follower are,

$$\begin{aligned} R_{12}^X + R_{32} \sin \lambda &= m_2 \ddot{X}_2 \\ -R_{12}^Y - R_{32} \cos \lambda &= m_2 \ddot{Y}_2 \\ -R_{43}^X + R_{32} \sin \lambda &= m_3 \ddot{X}_3 \\ R_{43}^Y - R_{32} \cos \lambda &= m_3 \ddot{Y}_3 \\ m_4 \ddot{Y}_4 + C\dot{Y}_4 + KY_4 &= R_{43}^Y - F_0 \\ T_2 &= m_2 S(\ddot{Y}_2 \cos \theta_2 - \ddot{X}_2 \sin \theta_2) + R_{32} d \cos(\theta_2 - \lambda) \end{aligned} \quad (21)$$

Clearance Angles

The follower displacement Y_4 is given by equation (20). From figure 3, the follower velocity $\dot{Y}_4$ can be written as ,

$$\dot{Y}_4 = \omega_2 \cdot \overline{O_{12} O_{42}} \quad (22)$$

where,

$$\begin{aligned} \overline{O_{12} O_{42}} &= (\Delta_{12} \sin \beta_{12} + d \sin \theta_2) \tan \lambda + (r+r_n) \sin \lambda \\ &+ \Delta_{43} \cos \beta_{43} \end{aligned} \quad (23)$$

Since the variations of Y_4 and $\dot{Y}_4$ due to clearance effect are negligible, these conditions lead to,

$$\begin{aligned} f_1 &= (\sin \beta_{12} - \xi_1 \cos \beta_{12}) - \Delta(\sin \beta_{43} - \xi_1 \cos \beta_{43}) = 0 \\ f_2 &= \xi_1 \sin \beta_{12} + (1 + \xi_2) \cos \beta_{12} - \Delta \xi_2 \cos \beta_{43} = 0 \end{aligned} \quad (24)$$

$$\text{where, } \xi_1 = \beta / \sqrt{1 - \beta^2}$$

$$\xi_2 = n \sin \theta_2 / (1 - \beta_2^2)^{3/2}$$

$$\begin{aligned} \beta &= n \cos \theta_2 \\ \text{and } n &= d / (r + r_n) \end{aligned}$$

Equations (24) are solved for the clearance angles (β_{12}, β_{43}) using Newton Raphson's method. The derivatives of equation (24) gives,

$$\underline{A} \cdot \dot{\beta} = \dot{v} \quad (25)$$

where,

$$\underline{A} = \begin{bmatrix} (\cos\beta_{12} + \xi_1 \sin\beta_{12}) & -\Delta (\cos\beta_{43} + \xi_1 \sin\beta_{43}) \\ \xi_1 \cos\beta_{12} - (1 + \xi_2) \sin\beta_{12} & \Delta \xi_2 \sin\beta_{43} \end{bmatrix}$$

$$\dot{\underline{\beta}}^T = \{\dot{\beta}_{12}, \dot{\beta}_{43}\}$$

$$\dot{V}^T = \omega_2 \{ \omega_2 \xi_1' (\cos\beta_{12} - \Delta \cos\beta_{43}), \xi_2' (\Delta \cos\beta_{43} - \cos\beta_{12}) - \xi_1' \sin\beta_{12} \}$$

$$\xi' = \beta' / (1 - \beta^2)^{3/2}$$

$$\xi' = [\beta / (1 - \beta^2)^{5/2}] (1 - \beta^2 - 3n^2 \sin^2 \theta_2)$$

$$\text{and } \beta' = n \sin \theta_2$$

Equation (25) is used to evaluate $(\dot{\beta}_{12}, \dot{\beta}_{43})$.

Solving equations (21) for the reactions $R_{12}^X, R_{12}^Y, R_{43}^X, R_{43}^Y$ in terms of the accelerations $(\ddot{X}_2, \ddot{Y}_2, \ddot{X}_3, \ddot{Y}_3, \ddot{Y}_4)$ and substituting into the results the values of these accelerations using equations (20) and (8) and substituting the resulting values of these reactions into equations (13) and re-arranging leads to,

$$\alpha_1 \ddot{\beta}_{12} + \alpha_2 \ddot{\beta}_{43} = \alpha_3$$

$$\gamma_1 \ddot{\beta}_{12} + \gamma_2 \ddot{\beta}_{43} = \gamma_3 \quad (26)$$

$\ddot{\beta}_{12}$ and $\ddot{\beta}_{43}$ are obtained as,

$$\ddot{\beta}_{12} = (\alpha_3 \gamma_2 - \alpha_2 \gamma_3) / (\alpha_1 \gamma_2 - \alpha_2 \gamma_1) \quad (27)$$

$$\ddot{\beta}_{43} = (\alpha_1 \gamma_3 - \alpha_3 \gamma_1) / (\alpha_1 \gamma_2 - \alpha_2 \gamma_1) \quad (28)$$

Once, the values of (β_{12}, β_{43}) , $(\dot{\beta}_{12}, \dot{\beta}_{43})$ and $(\ddot{\beta}_{12}, \ddot{\beta}_{43})$ are evaluated, equations (17) to (21) and equation (8) are used to evaluate follower motion, reaction forces and torque.

Case C: Contact with the Flat Flank During Downward Motion of the Follower

Figure 4 shows the contact at any instant during this period, where,

$$(\pi/2 + \gamma - \alpha) < \theta_2 < (\pi/2 + \gamma) \quad (29)$$

Similar to section (A), the following equations can be written.

$$r_1 = (r + r_b)(1 - \cot \gamma \tan \theta_2'') \quad (30)$$

$$\left. \begin{aligned} \dot{r}_1 &= (r + r_b) \omega_2 \cot \gamma \sec^2 \theta_2'' \\ \ddot{r}_1 &= -2(r + r_b) \omega_2^2 \cot \gamma \sec^2 \theta_2'' \tan \theta_2'' \end{aligned} \right\} \quad (31)$$

$$\begin{aligned} q &= -D \cos \theta_2 = -\Delta_{43} \cos \beta_{43} + r_1 \cos(\theta_2 - \gamma) + \Delta_{12} \cos \beta_{12} \\ \dot{q} &= \Delta_{43} \dot{\beta}_{43} \sin \beta_{43} + \dot{r}_1 \cos(\theta_2 - \gamma) - r_1 \omega_2 \sin(\theta_2 - \gamma) - \Delta_{12} \dot{\beta}_{12} \sin \beta_{12} \\ \ddot{q} &= \Delta_{43} (\ddot{\beta}_{43} \sin \beta_{43} + \dot{\beta}_{43}^2 \cos \beta_{43}) - \Delta_{12} (\ddot{\beta}_{12} \sin \beta_{12} + \dot{\beta}_{12}^2 \cos \beta_{12}) \\ &\quad + [(\ddot{r}_1 - r_1 \omega_2^2) \cos(\theta_2 - \gamma) - 2\dot{r}_1 \omega_2 \sin(\theta_2 - \gamma)] \end{aligned} \quad (32)$$

where,

$$\begin{aligned} \theta_2'' &= (\gamma - \theta_2 + \pi/2) \\ Y_4 &= \Delta_{12} \sin \beta_{12} - q \tan \theta_2 + r_1 \sin(\theta_2 - \gamma) - \Delta_{43} \sin \beta_{43} - (r + r_b) \\ \dot{Y}_4 &= \Delta_{12} \dot{\beta}_{12} \cos \beta_{12} - \dot{q} \tan \theta_2 - q \omega_2 \sec^2 \theta_2 + \dot{r}_1 \sin(\theta_2 - \gamma) \\ &\quad + r_1 \omega_2 \cos(\theta_2 - \gamma) - \Delta_{43} \dot{\beta}_{43} \cos \beta_{43} \\ \ddot{Y}_4 &= \Delta_{12} (\ddot{\beta}_{12} \cos \beta_{12} - \dot{\beta}_{12}^2 \sin \beta_{12}) - \Delta_{43} (\ddot{\beta}_{43} \cos \beta_{43} - \dot{\beta}_{43}^2 \sin \beta_{43}) \\ &\quad - (\ddot{q} \tan \theta_2 + 2\dot{q} \omega_2 \sec^2 \theta_2 + 2q \omega_2^2 \sec^2 \theta_2 \tan \theta_2) \\ &\quad + [(\ddot{r}_1 - r_1 \omega_2^2) \sin(\theta_2 - \gamma) + 2\dot{r}_1 \omega_2 \cos(\theta_2 - \gamma)] \end{aligned} \quad (33)$$

$$R_{12}^x - R_{32} \cos(\theta_2 - \gamma) = m_2 \ddot{X}_2$$

$$-R_{12}^y - R_{32} \sin(\theta_2 - \gamma) = m_2 \ddot{Y}_2$$

$$-R_{43}^x - R_{32} \cos(\theta_2 - \gamma) = m_3 \ddot{X}_3$$

$$R_{43}^y - R_{32} \sin(\theta_2 - \gamma) = m_3 \ddot{Y}_3$$

$$m_4 \ddot{Y}_4 + C \dot{Y}_4 + K Y_4 = R_{43}^y - F_0$$

$$T_2 = m_2 S (\ddot{Y}_2 \cos \theta_2 - \ddot{X}_2 \sin \theta_2) + (q \sin \gamma / \cos \theta_2) R_{32} \quad (34)$$

Clearance Angles

The follower displacement Y_4 is given by equation (33). From figure 4, the follower-velocity $\dot{Y}_4$ can be written as,

$$\dot{Y}_4 = \omega_2 \cdot \overline{O_{12} O_{42}} \quad (35)$$

where,

$$\begin{aligned} \overline{O_{12} O_{42}} &= [(\Delta_{12} \sin \beta_{12} - q \tan \theta_2) / \tan(\theta_2 - \gamma)] + r_1 \cos(\theta_2 - \gamma) \\ &\quad - \Delta_{43} \cos \beta_{43} \end{aligned} \quad (36)$$

Since, the variations of Y_4 and $\dot{Y}_4$ due to clearance effect are negligible, these conditions lead to,

$$f_1 = \sin(\beta_{12} - \theta_2) - \Delta \sin(\beta_{43} - \theta_2) = 0$$

$$f_2 = \sin(\beta_{12} - \theta_2) - \Delta f(\theta_2) \cos \beta_{43} = 0 \quad (37)$$

where, $f(\theta_2) = -\sin \theta_2 + \cos \theta_2 \tan(\theta_2 - \gamma)$

Equations (37) are solved for the clearance angles (β_{12}, β_{43}) using Newton Raphson's method. The derivatives of equation (37) give,

$$\dot{\beta}_{43} = \omega_2 [f'(\theta_2) \cos \beta_{43} + \cos(\beta_{43} - \theta_2)] / [f(\theta_2) \sin \beta_{43} + \cos(\beta_{43} - \theta_2)]$$

$$\dot{\beta}_{12} = \Delta \xi \dot{\beta}_{43} + (1 - \Delta \xi) \omega_2 \quad (38)$$

where, $f'(\theta_2) = -\cos \theta_2 - \sin \theta_2 \tan(\theta_2 - \gamma) + \cos \theta_2 \sec^2(\theta_2 - \gamma)$

Equations (38) are used to evaluate ($\dot{\beta}_{12}, \dot{\beta}_{43}$). Solving equations (34) for the reactions $R_{12}^x, R_{12}^y, R_{43}^x, R_{43}^y$ in terms the accelerations ($\ddot{x}_2, \ddot{y}_2, \ddot{x}_3, \ddot{y}_3, \ddot{y}_4$) and substituting into the results the values of these accelerations using equations (33) and (8) and substituting the resulting values of these reactions into equations (13) and rearranging leads to,

$$\mu_1 \ddot{\beta}_{12} + \mu_2 \ddot{\beta}_{43} = \mu_3$$

$$\delta_1 \ddot{\beta}_{12} + \delta_2 \ddot{\beta}_{43} = \delta_3 \quad (39)$$

($\ddot{\beta}_{12}, \ddot{\beta}_{43}$) are obtained as,

$$\ddot{\beta}_{12} = (\mu_3 \delta_2 - \mu_2 \delta_3) / (\mu_1 \delta_2 - \mu_2 \delta_1) \quad (40)$$

$$\ddot{\beta}_{43} = (\mu_1 \delta_3 - \mu_3 \delta_1) / (\mu_1 \delta_2 - \mu_2 \delta_1) \quad (41)$$

Once, the values of (β_{12}, β_{43}), ($\dot{\beta}_{12}, \dot{\beta}_{43}$) and ($\ddot{\beta}_{12}, \ddot{\beta}_{43}$) are evaluated, equations (29) to (34) and equation (8) are used to evaluate follower motion, reaction forces and torque.

A Fortran IV computer program is written and used to evaluate the kinematic and dynamic quantities for a cam mechanism with two bearing clearances. Figure 5 shows a flow chart of the computation procedure.

RESULTS AND CONCLUSIONS

The following input data for a cam mechanism is used .

$$r_b = 0.03388 \text{ m.}, r_n = 0.0275 \text{ m.}, r = 0.01375 \text{ m.}, d = 0.01914 \text{ m.}$$

$$S = 0.03364 \text{ m.}, \Delta_{12} = 100. \mu\text{m.}, \Delta_{43} = 60. \mu\text{m.}$$

$$m_2 = 1.8 \text{ kg.}, m_3 = 0.0894 \text{ kg.}, m_4 = 1.139 \text{ kg.}$$

$$F_0 = 24.2293 \text{ N.}, K = 7787.79 \text{ N/m}, C = 9.7812 \text{ N.s./m.}$$

The input speed to the driving cam is assumed 60.rad./s. anticlockwise.

Figures 6 to 12 show the computer analysis results of the cam mechanism. The results are compared with the case when clearance is ignored. There are considerable changes in accelerations, bearing reaction forces and torques due to the effect of bearing clearances. The changes of bearing reaction forces and torques are mainly due to the changes in accelerations.

Figures 6 and 7 show the variations of clearance angles (β_{12}, β_{43}) during the angle of action of the cam. There are changes in β_{12} in the range of $0 < \theta_2 < 20^\circ$ and at $\theta_2 = 120^\circ$. The changes of β_{43} occur at $\theta_2 = 20^\circ$ and $\theta_2 = 120^\circ$. Figures 8 to 12 demonstrate the magnifications of accelerations, dynamic forces and torques when clearance effect is taken into account. At $\theta_2 = 50^\circ$ there is an instant of separation since, the reaction forces change from positive to zero and vice-versa.

It is important to note that the magnifications of accelerations forces and torques in the cam mechanism as affected by revolute clearances essentially depend on clearance sizes and operating speed of the cam shaft. Cam and follower dimensions have a small effect compared to that of clearance size or operating speed.

The results indicated in this paper represent an upper bound of the bearing reaction forces due to the existence of bearing clearance, since the stiffness and damping of oil films and a part of the system elasticity are not included in the analysis. These effects tend to reduce the values of these bearing reaction forces.

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NOMENCLATURE

C	damping coefficient affecting the follower motion
d	distance between the center of base and noze circles
D	distance between base circle center and the intersection of the common normal at the point of contact with the cam centerline.
K	stiffness of the spring attached to the follower
m_2	mass of camshaft assembly
m_3	mass of roller
m_4	mass of follower and its attachments
F_0	initial force of the spring attached to the follower
$f_i, i=1,2$	constraint functions of the clearance angles β_{12} and β_{43}
r	radius of roller
r_b	radius of base circle
r_n	radius of noze circle
r_1	distance between roller center and cam centerline measured along the normal to the flat flank
$\bar{R}_{12}^x$	reaction of the main bearing (1) on the cam (2) in (X) direction
$\bar{R}_{12}^y$	reaction of the main bearing (1) on the cam (2) in (Y) direction
$\bar{R}_{32}$	reaction of the roller (3) on the cam (2)
$\bar{R}_{43}^x$	reaction of the follower (4) on the roller (3) in (X) direction
$\bar{R}_{43}^y$	reaction of the follower (4) on the roller (3) in (Y) direction
q	projection of the vector $\vec{D}$ along (X) direction
S	distance between base circle center and center of gravity G_2 of the camshaft assembly
T_2	required input torque on the camshaft
$\ddot{x}_2, \ddot{y}_2$	acceleration components of the center G_2
$\ddot{x}_3, \ddot{y}_3$	acceleration components of mass center G_3 of the roller
y_4	follower displacement
$\dot{y}_4$	follower velocity
$\ddot{y}_4$	follower acceleration
α	cam rotational angle during contact along the flat flank

β_{12}, β_{43}	clearance angles at the main bearing and the roller bearing
$\dot{\beta}_{12}, \dot{\beta}_{43}$	angular velocities of clearance angles
$\ddot{\beta}_{12}, \ddot{\beta}_{43}$	angular accelerations of clearance angles
2γ	cam angle of action
$\vec{\Delta}_{12}$	clearance vector at the main bearing
$\vec{\Delta}_{43}$	clearance vector at the roller bearing
Δ	the ratio of $ \vec{\Delta}_{43} / \vec{\Delta}_{12} $
λ	angle between (Y) direction and the vector joining point G_3 with the center of the noze circle
θ_2	cam rotation measured from the (X) axis
ω_2	angular velocity of the cam

Figure 5: A Flow Chart of the Computational Procedure

Fig. (8)

Fig. (9)

Fig. (6)

Fig. (7)

Fig. (10)

Fig. (11)

Figure 12: Required Torque on the Camshaft vs. Camshaft Angle