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EFFECT OF VIBRATION FREQUENCY ON DYNAMIC CONTACT STIFFNESS OF JOINTS

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ABSTRACT

In this paper, a method for the introduction of the effect of vibration frequency on dynamic contact stiffness of joints is presented. The method is based on the measurement of the natural modes of vibration of a simple mechanical system containing the joint to be investigated. The results of measurement are used, through a sensitivity technique, for the identification of a joint stiffness matrix. This matrix is assumed, at first, to be invariant, i.e, independent on the mode of vibration. However, the utilization of the identified eigensolution and the equations of sensitivity corresponding to a single mode of vibration, enable one to obtain, for this mode, a correction joint stiffness matrix.

The validity of the method has been tested by numerical simulation on a case of a plane prestressed joint between two beams.

INTRODUCTION

Structural joints not only provide the major source of energy dissipation but are also responsible in the main for a large proportion of overall dynamic deflections.

The main problem associated with structural joints, however, is the shortage of informations characterizing their stiffness and damping behaviour.

Two main directions of investigations concerning the contact stiffness of joints may be distinguished : 1) Microscopic investigations, which deals with the mechanisms of interaction between two metalic surfaces in contact under normal force [1,2,3] , and the role of surface asperities in transmitting normal and tangential forces, Estimations of the contact stiffness of joints have been made on basis of deterministic or random topographic distribution of the surface asperities [4,5]. 2) Macroscopic investigations, in which the contact stiffness of joints is determined experimentally by the measurement of the displacement at the joint interfaces produced by certain applied load [6,7,8] .

The authers [9,10,11] , however, have proposed a technique for the dynamic joint stiffness identification. This technique is based on the study of

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a mechanical system "S" consisting of two simple sub-systems "Ss^1" and "Ss^2" connected together by a joint of the type under investigation Fig.1.

An initial joint stiffness matrix is preassumed which together with the adjusted mathematical models of Ss^1 and Ss^2 provide an initial mathematical model of the whole system. The difference between the identified eigensolutions of the system "S" and the calculated eigensolutions of the initial model is used, through a sensitivity technique, for the adjustement of the joint stiffness matrix.

This matrix is assumed to be invariant i.e, independent on the mode of vibration.

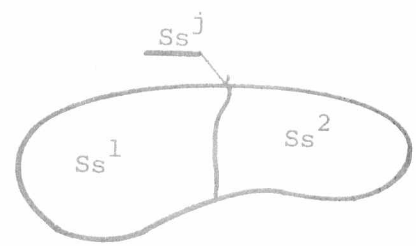

Fig.1. System with Joint "S"

In this work, however, a method for the determination of the effect of vibration frequency on the joint stiffness is presented. Strating with the invariant joint stiffness matrix, the equations of sensitivity corresponding to every mode of vibration are utilized to determine for this mode a correction joint stiffness matrix so that the modified model of the system admits, for eigensolution, the identified one corresponding to the mode in question.

DATA OF THE PROBLEM

It is assumed that an initial mathematical model of the system with joint is given by its mass matrix M and stiffness matrix K. M and K are real, symmetric, constant matrices of order NxN, positive definite and positive semidefinite respectively. These matrices are supposed to be obtained by the finite element method. They are subjected to two steps of model adjustment : 1) Adjustement of the models of Ss^1 and Ss^2 [12] using the identified eigensolutions ofa monoblock system geometrically identical to the system with joint "S" and realized from the same material, 2) Adjustement of the invariant joint stiffness matrix [9,10] .

The eigenvalue problem of the autonomus conservative system associated to "S" is given by :

$$[K - \lambda M] \; y = 0 \tag{1}$$

The first n eigensolutions are grouped in the modal submatrix Y(Nxn) and the spectral submatrix Λ (nxn). Y and Λ satisfy the orthonormality relations

$$^{t}Y \, M \, Y = E \quad ; \quad ^{t}Y \, K \, Y = \Lambda \tag{2}$$

It is also assumed that the experiments on the real system with joint "S" permit the determination of its identified eigensolutions y_r^m and λ_r^m $(1 \leq r \leq \ell)$. It is taken into account that the eigenvectors y_r^m may be partially identified. (For example, lack of informations corresponding to the rotation degrees of freedom (D.O.F.), or the internal D.O.F. inaccessible to measurement).

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MATHEMATICAL FORMULATION

The difference between the r^{th} identified eigensolution y_r^m, λ_r^m and the corresponding calculated eigensolution of the initial model y_r, λ_r is due to the noncoensidence between the invariant joint stiffness matrix and the real one expressing the physical behaviour of the joint on the r^{th} mode. Our problem now is to determine for every mode r a correction joint stiffness matrix ΔK_r^j such that the modified mathematical model of "S", given by;

$$K_r^m = K + \Delta K_r^j \quad , \qquad M^m = M \quad ; \tag{3}$$

admits y_r^m and λ_r^m as eigensolution. By consequence, for the r^{th} mode of vibration, the equation of dynamic equilibrium of the system "S" is given by :

$$\left\{ K + \Delta K_r^j - \lambda_r^m M \right\} y_r^m = 0 \tag{4}$$

It is assumed that the precision of the initial model is reasonable such that the differences between identified and calculated eigensolutions are small to enable appropriation and ensure the quasi orthonormality property. Based on this property the r^{th} identified eigenvector y_r^m may be expressed on basis of the calculated submodal matrix Y as follows [12]:

$$y_r^m = Y\, g_r \tag{5}$$

Introducing the changement of basis (5) into (4) and premultiplying by ${}^tY$;

$${}^tY \left\{ K + K_r^j - \lambda_r^m M \right\} Y\, g_r = 0 \tag{6}$$

Taking into account the conditions of orthonormality (2), the relation (6) may be rewritten as :

$${}^tY \; K_r^j Y\, g_r = \left\{ \lambda_r^m E - \Lambda \right\} g_r \tag{7}$$

Application of a Sensitivity Method

The correction joint stiffness matrix ΔK_r^j may be expressed in terms of the relative modifications of the joint stiffness parameters $\Delta p_i / p_i$; $1 \leqslant i \leqslant q$, in the form :

$$\Delta K_r^j = \sum_{i=1}^{q} p_i \left(\frac{\partial K}{\partial p_i}\right) \left(\frac{\Delta p_i}{p_i}\right) + O\left(\frac{\Delta p_i}{p_i}\right)^2 + \ldots \tag{8}$$

The matrices $(\partial K / \partial p_i)$, $1 \leqslant i \leqslant q$, are calculated from the algorithm utilized in the discretization leading to the initial model.

Considering the 1^{st} order terms in (8) the L.H.S. of (7) may be rewritten;

$${}^tY \Delta K_r^j \, Y\, g_r = \sum_{i=1}^{q} \beta_r^i \left(\frac{\Delta p_i}{p_i}\right) \tag{9}$$

where :

$$\beta_r^i (m\times 1) = p_i \; {}^tY \left(\frac{\partial K}{\partial p_i}\right) Y\, g_r \quad ; \qquad 1 \leqslant i \leqslant q \tag{10}$$

Substituting from (9) into (7) a system of linear equations in ($\Delta p_i/p_i$) as unknowns, is obtained ;

$$\sum_{i=1}^{q} \beta_r^i \left(\frac{\Delta p_i}{p_i}\right) = \left\{\lambda_r^m E - \Lambda\right\} g_r \qquad (11)$$

Equations (11) may be grouped in the following matrix form :

$$S_r Z_r = b_r \qquad (12)$$

Where ; $Z_r(qx1) = {}^t\left\{\frac{\Delta p_1}{p_1}, \ldots, \frac{\Delta p_i}{p_i}, \ldots \frac{\Delta p_q}{p_q}\right\}$ is a vector of the dimensionless joint stiffness parameters modifications;

$b_r(mx1) = \left\{\lambda_r^m E - \Lambda\right\} g_r$ is a vector characterizing the modifications of the calculated eigensolutions so that the mathematical model admits y_r^m and λ_r^m as eigensolution.

$S_r(mxq) = \left\{\beta_r^1, \ldots, \beta_r^i, \ldots, \beta_r^q\right\}$ is the sensitivity matrix which relates the modifications of the joint stiffness parameters to the modifications of the calculated eigensolutions.

The matrix equation (12) represents a system of m linear equations in q unknowns. This system is assumed to be overdetermined, i.e; $m > q$. A solution which minimizes the squre of the norm of the weighted residual vector, $\| W_r \mathcal{E}_r \|$, (where $\mathcal{E}_r = b_r - S_r Z_r$) is given by [10] ,

$$Z_r = \left\{{}^tS_r W_r^2 S_r\right\}^{-1} {}^tS_r W_r^2 b_r \qquad (13)$$

Using equation (8) the solution vector Z_r is used for the determination of the correction joint stiffness matrix ΔK_r^j corresponding to the r^{th} mode of vibration.

For the exploitation of the prescribed method it is necessary to determine the vectors g_r $(1 \leq r \leq \ell)$. From equation (5), using the pseudoinverse [10] , g_r may be written in the forms :

* When y_r^m is totally identified ;

$$g_r = \left\{{}^tY\, Y\right\}^{-1} {}^tY\, y_r^m \; ; \qquad (14)$$

* When y_r^m is partially identified ;

$$g_r = \left\{{}^t\tilde{Y}\, \tilde{Y}\right\}^{-1} {}^t\tilde{Y}\, \tilde{y}_r^m \qquad (15)$$

where [~] indicates that the elements corresponding to the nonidentified D.O.F. of y_r^m are replaced by zeros.

The precision of the representation of y_r^m on the calculated sub-basis Y increases with the number n of its vectors. However for a given value of n, the precision of calculation is not the same for the different components of g_r. Taking as notation

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$$g_r = {}^t\{g_{1r}, \ldots, g_{rr}, g_{(r+1)r}, \ldots g_{nr}\}$$

the precision of g_r components decreases as the component is more and more far from g_{rr}. Therefore it is recommended to apply the following procedure :

1) to calculate g_r using a sub-basis Y having as higher number of vectors n as possible.
2) to utilize only m equations of sensitivity (12) ($m < n$) corresponding to the components of g_r which are not very far from g_{rr}.
3) to use a weighting matrix W_r in the solution of the equations of sensitivity (13) which gives more weight for the equations corresponding to the components of g_r very near to g_{rr}.

The weighting matrix W_r is a diagonal matrix whose elements are given by one of the following two proposed forms :

$$(w_{ss})_r = 1/\{|s - r + 1|\} \quad , \text{ or} \qquad (16\text{-a})$$

$$(w_{ss})_r = 1/\{\sqrt{|s - r + 1|}\} \qquad (16\text{-b})$$

where $1 \leq s \leq m$, $1 \leq r \leq \ell$.

APPLICATION

The presented method has been applied on a case of plane pre-stressed joint between two beams of uniform cross-sections as shown in Fig.2. Plane free-free transversal vibration of this system is considered. The stiffness matrix of the joint is given by (17) under the assumption that there is no elastic coupling between the transversal and the rotational motion of the joint interfaces.

$$K^j = \begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_2 & 0 & -k_2 \\ -k_1 & 0 & k_1 & 0 \\ 0 & -k_2 & 0 & k_2 \end{bmatrix} \qquad (17)$$

k_1 and k_2 are the transversal and the rotational stiffness of the joint respectively. They represent, in this application, the joint parameters to be modified.

Validation of the Method by Numerical Simulation

The method has been tested by numerical simulation on several case studies two of them are presented hereafter. The eigensolutions y_r^m (38x1) and λ_r^m, $1 \leq r \leq 4$, simulating the results of identification are calculated for a simulated identified model. For this model the joint parameters k_{1r}^m, k_{2r}^m corresponding to the r^{th} mode are obtained by introducing a modification Δk_{1r}^i and Δk_{2r}^i on the joint parameter k_1 and k_2 corresponding to the invariant joint stiffness matrix of the initial model. The proposed method is then applied to determine the corrections Δk_{1r} and Δk_{2r} of the joint parameters on the r^{th} mode. The validity of the method is evaluated by comparing the introduced modifications Δk_{1r}^i, Δk_{2r}^i with the obtained corrections Δk_{1r}, Δk_{2r} for every one of the 1st four deformable modes.

To reduce the computation costs of solving the eigenvalue problem corresponding to the simulated identified model, the introduced modifications Δk^{i}_{1r}, Δk^{i}_{2r} are chosen to be the same over all the four tested modes of vibration, $1 \leqslant r \leqslant 4$. This does not restrict the validity of the method since the obtained corrections Δk_{1r} , Δk_{2r} are calculate separately for every mode.

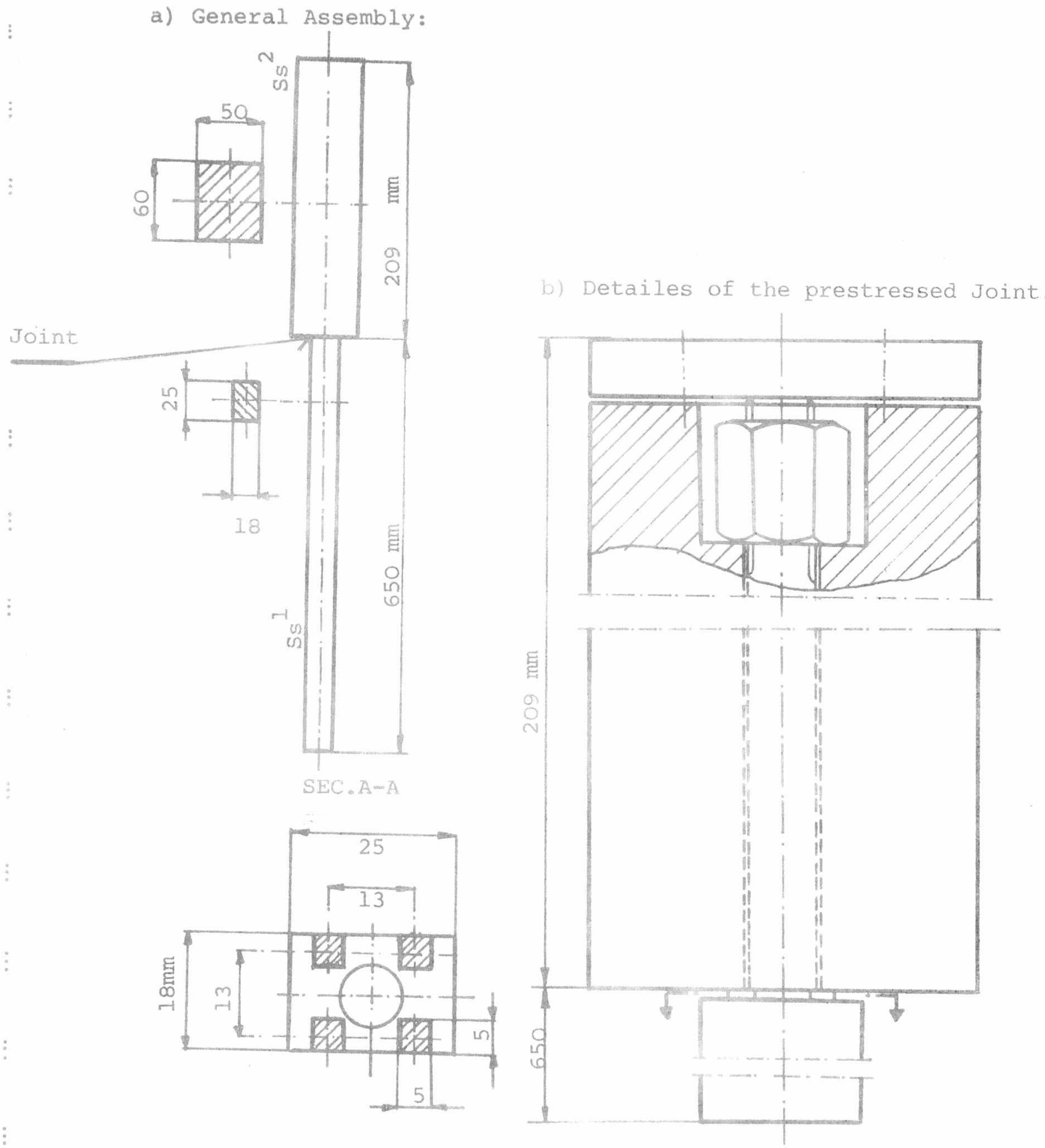

Fig. 2. Joint Assembly (Plane prestressed Joint between two Beams).

Results and Discussion

The results of numerical simulation on two case studies are given in tables 1 and 2. They represent two different levels of joint stiffness parameters. In both cases the system with joint fig.2 is devided into 17 beam elements and has 38 D.O.F. The errors of measurement are simulated by introducing random errors of maximum value $\pm$ 4 % on the simulated elements of y_r^m. The other conditions of simulation are given directly in tables 1 and 2 corresponding to every case and are denoted as follows :

- n : number of vectors of the calculated sub-basis Y.
- m : number of retained equations of sensitivity.
- NF: number of nonidentified degrees of freedom
- W : form of the weighting matrix

  - W = 1 equal weights of all sensitivity equations
  - W = 2 weighting matrix constructed by equation (16-a)
  - W = 3 weighting matrix constructed by equation (16-b)

The results of numerical simulation show that the proposed method enables the correction of the joint stiffness parameter for every one of the first four deformable modes of vibration. The absence of informations corresponding to 23 nonidentified D.O.F. (out of 38), do not affect much the quality of corrections. This is justified by the fact that representation of the identified eigensolution on basis of the calculated modal sub-matrix is quite precise.

The results show also that a more precise correction of the joint parameters may be achieved by :

1- The increase of the number n of vectors of the calculated modal sub-matrix;
2- The proper choice of the equations of sensitivity involved;
3- The solution of the weighted sensitivity equations using one of the proposed weighting formulee.

CONCLUSION

The proposed method enables the utilization of identified eigensolutions for the determination of the effect of the vibration frequency on the joint stiffness parameters. The results of numerical simulation show that the precision of the joint stiffness corrections is quite satisfactory.

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NOMENCLATURE

E	Unit matrix
K	Stiffness matrix
ℓ	Number of identified eigensolutions
M	Mass matrix
m	Number of retained equations of sensitivity
N	Number of degrees of freedom of the model
n	Number of calculated eigensolutions
W	Weighting matrix
Y	Calculatedmodal sub-matrix
y_r	r th eigenvector
λ_r	r th eigenvalue

Upper index m : quantities belonging to the identified model.

Case Study (I): Invariant Joint Stiffness parameters: $k_1 = 1.0\,10^7$ (N/m), $k_2 = 1.0\,10^4$ (N.m/rad)

Introduced Modifications : $\Delta k^i_{1r} = 2.0\,10^6$ (N/m), Δk^i_{2r} $2.0\,10^3$ (N m/rad)

Table 1.

	Conditions of Simulation				Obtained corrections on the r th mode							
					r=1		r=2		r=3		r=4	
	n	m	NF	W	Δk_1	Δk_2	Δk_1	Δk_2	Δk_1	Δk_2	Δk_1	Δk_2
1	7	7	0	1	$1.72\,10^6$	$3.22\,10^3$	$2.02\,10^6$	$2.98\,10^3$	$1.97\,10^6$	$2.33\,10^3$	$2.05\,10^6$	$6.50\,10^2$
2	7	7	23	1	$1.58\,10^6$	$1.86\,10^3$	$2.14\,10^6$	$1.88\,10^3$	$2.00\,10^6$	$1.94\,10^3$	$1.98\,10^6$	$2.18\,10^3$
3	10	5	0	1	$2.45\,10^6$	$2.13\,10^3$	$1.88\,10^6$	$2.09\,10^3$	$1.98\,10^6$	$2.01\,10^3$	$2.02\,10^6$	$1.64\,10^3$
4	10	5	0	2	$2.15\,10^6$	$2.08\,10^3$	$1.82\,10^6$	$2.06\,10^3$	$1.98\,10^6$	$1.97\,10^3$	$2.04\,10^6$	$2.22\,10^3$
5	10	5	0	3	$2.07\,10^6$	$2.03\,10^3$	$1.80\,10^6$	$2.04\,10^3$	$1.98\,10^6$	$1.97\,10^3$	$2.04\,10^6$	$2.21\,10^3$

Case Study (II) : Invariant Joint Stiffness parameters : $k_1 = 1.84\,10^8$ (N/m), $k_2 = -1.65\,10^5$ (Nm/rad)

Introduced Modifications : $\Delta k^i_{1r} = 2.10^7$ (N/m) ; $\Delta k^i_{2r} = 2.0\,10^4$ (Nm/rad)

Table 2.

	Conditions of Simulation				Obtained Corrections on the r th mode							
					r=1		r=2		r=3		r=4	
	n	m	NF	W	Δk_1	Δk_2	Δk_1	Δk_2	Δk_1	Δk_2	Δk_1	Δk_2
1	7	7	23	1	$1.15\,10^7$	$1.81\,10^4$	$3.41\,10^7$	$1.78\,10^4$	$2.02\,10^7$	$1.78\,10^4$	$1.72\,10^7$	$1.88\,10^4$
2	10	5	23	1	$1.96\,10^7$	$1.86\,10^4$	$2.78\,10^7$	$1.85\,10^4$	$2.05\,10^7$	$1.86\,10^4$	$2.04\,10^7$	$1.86\,10^4$
3	10	5	23	2	$1.85\,10^7$	$1.85\,10^4$	$3.00\,10^7$	$1.85\,10^4$	$1.82\,10^7$	$1.89\,10^4$	$1.90\,10^7$	$1.91\,10^4$
4	10	5	23	3	$1.86\,10^7$	$1.84\,10^4$	$3.25\,10^7$	$1.85\,10^4$	$1.84\,10^7$	$1.88\,10^4$	$1.92\,10^7$	$1.90\,10^4$