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RECONSTRUCTION OF DISTORTED TARGET DATA DUE TO ELECTRONIC JAMMING

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ABSTRACT

A novel approach is introduced by which the distorted signal due to electronic jamming is reconstructed. An ideal measurement trajectory in the observation space is derived. Using an LQG approach the deviation between the actual and ideal measurements are obtained. Measurements deviation covariance matrix is formed and a signal distortion detection criterion is established. The distorted signal is disregarded and a reduced order estimator is derived to reconstruct the tracking data.

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GENERAL DESCRIPTION

During the past four decades the problem of tracking and intercepting a flying target has recieved a great deal of interest. On the other hand target evasive actions also recieved the interest of many investigators. Since tracking is mainly accomplished by reflected electromagnetic wave signals, electronic devices fitted in the target may distort the signal thus resulting in wrong tracking information. Such deception techniques known as electronic counter measures (ECM). The tracking signal that is mostly sensetive to distortion is the range signal. Such errounous range data yield indicated false target position.

The target model in tracking coordinates (r, α, η) is given by;

$$\ddot{r} = \xi_1 \qquad (1a)$$

$$\ddot{\alpha} = -\dot{r}\dot{\alpha}/r + \xi_2/r \qquad (1b)$$

$$\ddot{\eta} = -\dot{r}\dot{\eta}/r + \xi_3/r \qquad (1c)$$

where r is the range, α is the azimuth angle, η is the elevation angle and ξ_i ; i=1,2,3 are random variables with variances σ_i^2 and bandwidth ω_o rad./sec. The autocorrelation function of these random processes are;

$$E[\xi_i(t)\,\xi_i(t+\tau)] = \sigma_i^2 e^{-\omega_o|\tau|}; \quad i=1,2,3 \qquad (2)$$

These coloured random processes could be generated from white gaussian random vectors ξ_i^* in the following manner;

$$\dot{\xi}_i = -\omega_o \xi_i + \xi_i^* \quad ; \ \xi_i^* \sim N(0, \sigma_i^2) \qquad (3)$$

In state space format, Eqns. (1) and (3) may be written as;

$$\dot{r}_1 = r_2 \qquad (4a)$$

$$\dot{r}_2 = \xi_1 \qquad (4b)$$

$$\dot{\alpha}_1 = \alpha_2 \qquad (4c)$$

$$\dot{\alpha}_2 = -r_2\alpha_2/r_1 + \xi_2/r_1 \qquad (4d)$$

$$\dot{\eta}_1 = \eta_2 \qquad (4e)$$

$$\dot{\eta}_2 = -r_2\eta_2/r_1 + \xi_3/r_1 \qquad (4f)$$

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$$\dot{\xi}_1 = -\omega_o \xi_1 + \xi_1^* \qquad (4g)$$

$$\dot{\xi}_2 = -\omega_o \xi_2 + \xi_2^* \qquad (4h)$$

$$\dot{\xi}_3 = -\omega_o \xi_3 + \xi_3^* \qquad (4i)$$

It is seen from Eqns.(4) that we are dealing with a nonlinear system with nine dimensional state vector. In vector form Eqns. (4) may be written as;

$$\dot{x} = f(x) \qquad (5)$$

where

$$x^T = [\, r_1 \quad r_2 \quad \alpha_1 \quad \alpha_2 \quad \eta_1 \quad \eta_2 \quad \xi_1 \quad \xi_2 \quad \xi_3 \,]$$

On the other hand the observation equation is given by;

$$z = Hx + v \qquad ; \qquad v \sim N(0,R) \qquad (6)$$

where

$$z^T = [\, r_1 \quad \alpha_1 \quad \eta_1 \,]$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

An extended Kalman filter (EKF) may thus be constructed from Eqns.(4) and (6) to estimate the state vector and thus determining the target position and velocity. The reliability of the estimated target position and velocity depends upon the observation accuracy. Errounous measurements lead to wrong estimates. Therefore in the presence of electronic counter measures (ECM) straight forward estimation of target position and velocity is not valid. Since the range measurement is the one that is seriously affected by electronic jamming,it may be dropped from the observation information. In that respect the extended Kalman filter convergence rate will be reduced.
The present work introduces a technique derived from a methodology developed by the auther (1,2) regarding sensor failure detection and isolation. Using such technique,the distorted signal will be detected and isolated. Moreover,a reduced order observer is established and utilized instead of Kalman filter thus expediting the estimation process.

MATHEMATICAL ANALYSIS

A nonlinear discrete time stochastic process may be modeled as;

$$x_{k+1} = \emptyset\,(x_k,\ u_k) + w_k \quad ; \quad w_k \sim N(0,Q_k) \qquad (7a)$$

$$z_k = h(x_k) + v_k \quad ; \quad v_k \sim N(0,R_k) \qquad (7b)$$

where x is an n-dimensional state vector, u is an m-dimensional control vector, z is a p-dimensional observation vector and $\emptyset(.)$ and $h(.)$ are non-linear functions of their arguments. W and v are white guassian process and measurements noise vectors respectively. Equation (7) may be linearized by introducing the following difference vectors, reference (3);

$$s_k = x_k - x_{k-1} \quad (8a) \quad ; \quad c_k = u_k - u_{k-1} \quad (8b)$$

$$y_k = z_k - z_{k-1} \quad (8c) \quad ; \quad \nu_k = w_k - w_{k-1} \quad (8d)$$

$$\mu_k = v_k - v_{k-1} \quad (8e)$$

The linearized form of Eqn.(7) is thus;

$$s_{k+1} = \Phi_{k-1}\, s_k + U_{k-1}\, c_k + \nu_k \ ; \ \nu_k \sim N\left[0, (Q_k + Q_{k-1})\right] \qquad (9a)$$

$$y_k = H_{k-1}\, s_k + \mu_k \ ; \ \mu_k \sim N\left[0, (R_k + R_{k-1})\right] \qquad (9b)$$

where $\Phi_k = [\emptyset_x]_k \quad ; \quad U_k = [\emptyset_u]_k \quad ; \quad H_k = [h_x]_k$

The present problem has nonredundant observations, since the number of measurements is less than the number of state variables. Introducing the observable and nonobservable state subvectors ω and λ respectively such that;

$$s_k^T = [\underbrace{\omega_k^T}_{p} \vdots \underbrace{\lambda_k^T}_{(n-p)}]$$

Based on Eqns. (9) the following equation may be derived;

$$y_{k+1} = \Psi_k^*\, y_k + \Gamma_k\, \lambda_k + U_k^*\, c_k + \mu_k^* \qquad (10)$$

where $H_k = \Big[\underbrace{N_k}_{p} \vdots \underbrace{0}_{(n-p)}\Big]\Big\}p \quad ; \quad H_k\, \Phi_{k-1} = \Big[\underbrace{\Psi_k}_{p} \vdots \underbrace{\Gamma_k}_{(n-p)}\Big]\Big\}p$

$$\Psi^*_k = \Psi_k N^{-1}_k \qquad ; \qquad U^*_k = H_k U_{k-1}$$

$$\mu^*_k = H_k \gamma_k + \mu_{k+1} - \Psi^*_k \mu_k$$

Equation (10) is an ideal measurement trajectory equation, i.e. measurements obtained by perfect sensors should satisfy it identically. Let's denote the deviation of the actual measurements from the ideal ones by $\Delta y = y - y^m$, where y is the ideal observation difference vector and y^m is the actual observation difference vector. In that respect measurements deviation propagation equation may be obtained from Eqn.(10) as;

$$y_{k+1} = \Psi^*_k \Delta y_k + \Gamma_k \lambda_k + U^*_k c_k + [\Psi^*_k y^m_k - y^m_{k+1}] + \mu^*_k \qquad (11)$$

A propagation equation for the nonobservable subvector of state may be obtained from Eqn. (9a) as;

$$\lambda_{k+1} = \Phi^{(4)}_{k-1} \lambda_k + \Phi^{*(3)}_{k-1} \Delta y_k + U^{(2)}_{k-1} c_k + \Phi^{*(3)}_{k-1} y^m_k + \gamma^{*(2)}_k \qquad (12)$$

where

$$\Phi_{k-1} = \begin{array}{c} \left[\begin{array}{c|c} \Phi^{(1)}_{k-1} & \Phi^{(2)}_{k-1} \\ \hline \Phi^{(3)}_{k-1} & \Phi^{(4)}_{k-1} \end{array}\right] \begin{array}{l} \}\, p \\ \}\, (n-p) \end{array} \\ \begin{array}{cc} \underbrace{\qquad}_{p} & \underbrace{\qquad}_{(n-p)} \end{array} \end{array} \qquad ; \qquad U_{k-1} = \begin{array}{c} \left[\begin{array}{c} U^{(1)}_{k-1} \\ \hline U^{(2)}_{k-1} \end{array}\right] \begin{array}{l} \}\, p \\ \}\, (n-p) \end{array} \\ \underbrace{\qquad}_{m} \end{array}$$

$$\gamma^T_k = [\underbrace{\gamma^{(1)T}_k}_{p} \;\vdots\; \underbrace{\gamma^{(2)T}_k}_{(n-p)}] \qquad ; \qquad \Phi^{*(3)}_{k-1} = \Phi^{(3)}_{k-1} N^{-1}_{k-1}$$

$$\gamma^{*(2)}_k = \gamma^{(2)}_k - N^{-1}_{k-1} \mu_k$$

Equations (11) and (12) are coupled propagation equations for Δy and λ. Inspection of Eqns.(10) and (11) reveals that observations deviation propagation are generated from imperfect measurements, i.e. when Eqn. (10) is not satisfied identically. Equation (11) may be written in the more convenient form

$$\Delta y_{k+1} = \Psi^*_k \Delta y_k + \epsilon_k + \mu^*_k \qquad (13a)$$

$$\epsilon_k = \Gamma_k \lambda_k + U^*_k c_k + \Psi^*_k y^m_k - y^m_{k+1} \qquad (13b)$$

The magnitude of observation deviation propagation depends upon the magnitude of vector $\in$. The permissible observation deviation for nondistorted (nonjammed) measurements may be obtained by determining the nonzero minimum of vectors Δy and $\in$. In that respect the following performance index is to be minimized subject to the propagation constraints given by Eqns. (12) and (13);

$$J = \tfrac{1}{2} E \sum_{k=0}^{N-1} (\Delta y_k^T A_k \Delta y_k + \in_k^T B_k \in_k) \tag{14}$$

where A and B are suitably chosen weighting matrices.
Following the standard approach for LQG problems we get;

$$\in_k = - B_k^{-1} S_{k+1} \Delta y_{k+1} \tag{15a}$$

$$S_k = A_k + \Psi_k^{*T} S_{k+1} (I + B_k^{-1} S_{k+1})^{-1} \Psi_k^* \quad ; \quad S_N = 0 \tag{15b}$$

Substitution of Eqn. (15a) into Eqn. (13a) yields;

$$\Delta y_{k+1} = (I + B_k^{-1} S_{k+1})^{-1} \Psi_k^* \Delta y_k + \mu_k^* \tag{16}$$

It is seen from Eqn.(16) that the permissible observations deviation are generated entirely from process and measurements noise vectors,i.e. from process model and measurements uncertainities. It is more convenient to describe the permissible observations deviation in terms of a covariance matrix. Introducing the definitions;

$$\Pi_k = E[\Delta y_k \Delta y_k^T] \quad ; \quad R_k^* = E[\mu_k^* \mu_k^{*T}]$$

The permissible observations deviation covariance matrix propagation may be obtained from Eqn. (16) as;

$$\Pi_{k+1} = (I + B_k^{-1} S_{k+1})^{-1} \Psi_k^* \Pi_k \Psi_k^{*T} (I + B_k^{-1} S_{k+1})^{-1T} + R_k^* \tag{17}$$

In deriveng Eqn.(17) it was assumed that there is no cross correlation between Δy and μ^* . On the other hand the actual observations deviation covariance matrix may be obtained from Eqn.(11) as;

$$\Pi_{k+1}^a = \Psi_k^* (\Pi_k^a \Psi_k^{*T} + \Delta \hat{y}_k^a \hat{\lambda}_k^T \Gamma_k^T + \Delta \hat{y}_k^a \xi_k^T) + \Gamma_k (\hat{\lambda}_k \Delta \hat{y}_k^{aT} \Psi_k^{*T} + \hat{\lambda}_k \hat{\lambda}_k^T \Gamma_k^T + \hat{\lambda}_k \xi_k^T) + \xi_k (\Delta \hat{y}_k^{aT} \Psi_k^{*T} + \hat{\lambda}_k^T \Gamma_k^T + \xi_k^T) + R_k^* \tag{18}$$

where $\Pi_k^a = E[\Delta y_k^a \Delta y_k^{aT}]$

6 and the estimates $\Delta \hat{y}^a$ and $\hat{\lambda}$ are obtained from Eqns. (11) and (12) as

$$\Delta \hat{y}^a_{k+1} = \Psi^*_k \Delta \hat{y}^a_k + \Gamma_k \hat{\lambda}_k + \xi_k \qquad (19a)$$

$$\hat{\lambda}_{k+1} = \Phi^{(4)}_{k-1} \hat{\lambda}_k + \Phi^{*(3)}_{k-1} \Delta \hat{y}^a_k + \eta^*_k \qquad (19b)$$

$$\xi_k = U^*_k c_k + \Psi^*_k y^m_k - y^m_{k+1} \qquad (19c)$$

$$\eta^*_k = U^{(2)}_{k-1} c_k + \Phi^{*(3)}_{k-1} y^m_k \qquad (19d)$$

The diagonal elements of the covariance matrices Π and Π^a represent the permissible and the actual deviations of the relevant measurement signals respectively. In that respect, comparison between such deviations will determine the reliability of the measurement signal. The following criterion may thus be established;

$$\Delta \Pi_k = (\Pi_k + W_k) - \Pi^a_k \qquad (20)$$

where W is a positive definite symmetric matrix that accounts for computations rounding off errors. From Eqn. (20) it is seen that for $\Delta \Pi_{ii} < 0$ this means that the i th. measurement is distorted and thus it has to be disregarded. An isolation procedure for the distorted signal is shown in detail in reference 1. Computation procedure and initialization are shown in detail in reference 2. Eqn. (19b) is a reduced order estimator for the unobserved state difference subvector λ. In that respect we do not have to use Kalman filter thus reducing a great deal the computation time.

COMPUTATIONAL PROCEDURE

The target model equations in continuous-time form as given by Eqns. (4) are first discritized with sample time τ. The differencing linearization as indicated by Eqns. (8) results in the following matrices;

$$N_k = I_3 \quad ; \quad U_k = 0 = U^*_k \quad ; \quad U^{(1)} = 0 \quad ; \quad U^{(2)} = 0$$

$$\Psi_k = \Psi^*_k = I_3 \quad ; \quad \mu^*_k = v_{k+1} - 2v_k + v_{k-1} \quad ; \quad \nu^{(1)} = 0$$

$$\nu^{(2)T}_k = [0 \quad 0 \quad 0 \quad \tau(s^*_{1k} - s^*_{1(k-1)}) \quad \tau(s^*_{2k} - s^*_{2(k-1)}) \quad \tau(s^*_{3k} - s^*_{3(k-1)})]$$

$$\Gamma = \begin{bmatrix} \tau & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau & 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Phi^{(1)} = I_3 \quad ; \quad \Phi^{(2)} = \Gamma$$

$$\Phi^{(3)} = \begin{bmatrix} 0 & 0 & 0 \\ \tau(r_2\alpha_2 - \varsigma_2)/r_1^2 & 0 & 0 \\ \tau(r_2\eta_2 - \varsigma_3)/r_1^2 & 0 & 0 \end{bmatrix}$$

$$\Phi^{(4)} = \begin{bmatrix} 1 & 0 & 0 & \tau & 0 & 0 \\ -\tau\alpha_2/r_1 & (1-\tau r_2/r_1) & 0 & 0 & \tau/r_1 & 0 \\ -\tau\eta_2/r_1 & 0 & (1-\tau r_2/r_1) & 0 & 0 & \tau/r_1 \\ 0 & 0 & 0 & (1-\tau\omega_0) & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-\tau\omega_0) & 0 \\ 0 & 0 & 0 & 0 & 0 & (1-\tau\omega_0) \end{bmatrix}$$

$\Phi^{*(3)} = \Phi^{(3)}$; I_3 is a (3x3) identity matrix

Inspection of the matrices involved shows that their elements are functions of some of the state variables evaluated at previous steps. Those state variables are namely $(r, \dot{r}, \dot{\alpha}, \dot{\eta}, \varsigma_2, \varsigma_3)$. It is seen that the most influential state variable that affects the tracking accuracy is the range. The robustness of a control system depends upon the accuracy of the coefficient matrices elements. To increase the degree of robustness of the present scheme the estimated values of the state variables appearing in the coefficient matrices are used. Inspection of Eqns. (17)-(19) and the expressions for matrices Γ, $\Phi^{(3)}$ and $\Phi^{(4)}$ reveals that the estimation accuracy of the tracking parameters increases as the sample time decreases and the range increases. To initialize the computation procedure an initial fix from a two tracking units (TTU) for the range measurement is utilized, Fig. 1. Denoting the tracking parameters from tracking units 1 and 2 respectively by $(r_1^{(1)}, \alpha_1^{(1)}, \eta_1^{(1)})$ and $(r_1^{(2)}, \alpha_1^{(2)}, \eta_1^{(2)})$. The following relations may be obtained;

$$r_1^{(1)} \sin \alpha_1^{(1)} = r_1^{(2)} \sin \alpha_1^{(2)} \qquad (21a)$$

$$r_1^{(2)} \cos \alpha_1^{(2)} \cos \eta_1^{(2)} - r_1^{(1)} \cos \alpha_1^{(1)} \cos \eta_1^{(1)} = d_e \qquad (21b)$$

$$r_1^{(1)} \cos \alpha_1^{(1)} \sin \eta_1^{(1)} + r_1^{(2)} \cos \alpha_1^{(2)} \sin \eta_1^{(2)} = d_n \qquad (21c)$$

where d_e and d_n are the eastward and northward distances between the tracking stations respectively. Considering tracking unit number 1 to be the main tracking station, the following relation may be obtained from Eqns.(21);

$$r_1^{(1)} = d_n / [\cos \alpha_1^{(1)} \sin \eta_1^{(1)} + (\sin \alpha_1^{(1)} / \sin \alpha_1^{(2)}) \cos \alpha_1^{(2)} \sin \eta_1^{(2)} \qquad (22)$$

Equation (22) is the one to be used for initializing the range. The block diagram of the estimation process for tracking information in the presence of electronic counter measures (ECM) is shown in Fig. 2.

CONCLUSION

A new methodology is presented for determining the tracking parameters of a target in the presence of electronic counter measures (electronic jamming). The concept introduced is based on establishing a permissible signal deviation defined by a covariance matrix. Signal distortion detection criterion is then derived which determines the distorted signal. The distorted signal is disregarded and the tracking parameters are estimated by a derived simplified reduced order estimator. The resulting algorithm has been demonstrated successfuly by simulating different signal distortion patterns on a digital computer.

REFERENCES

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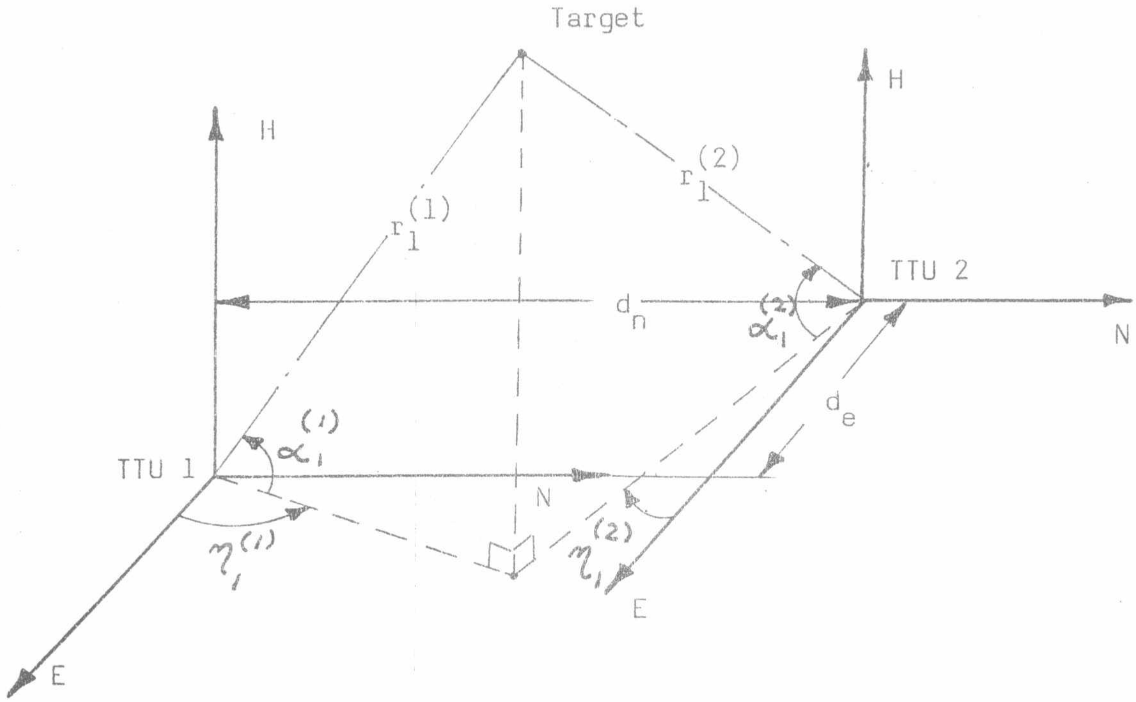

Fig. 1 Geometry of Two Tracking Units

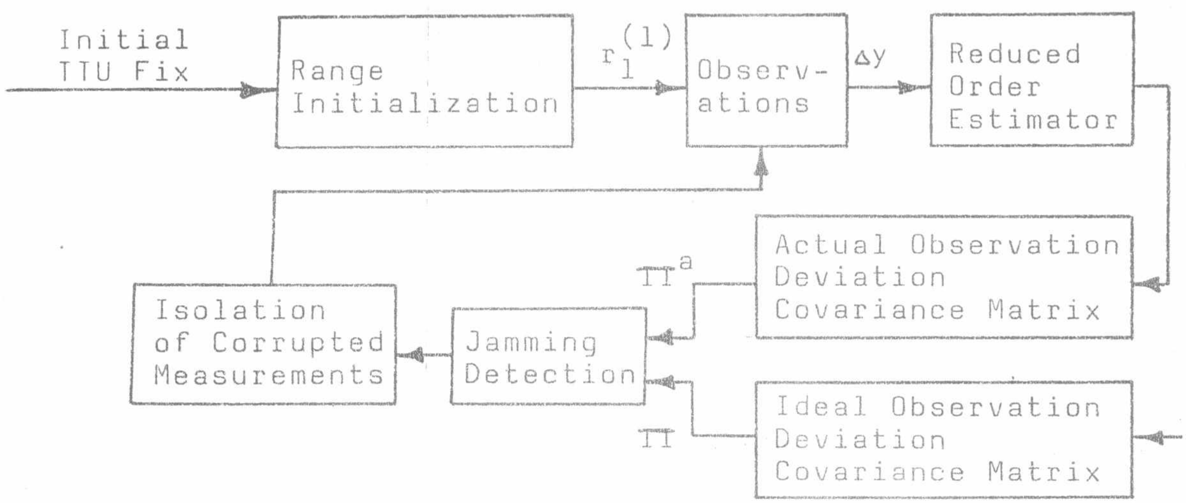

Fig. 2 Estimation of Tracking Information in the Presence of ECM