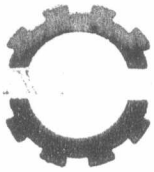


MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

IMPROVED TECHNIQUES FOR DETERMINATION OF RESIDUES AND DAMPING FROM FREQUENCY AND IMPULSE RESPONSE CURVES

KAI V. SØRENSEN, M. SC.

ABSTRACT

This paper describes the determination of modal parameters directly from the frequency and impulse response curves, and discusses the factors effecting the accuracy of the measurements. It summarizes the effects of noise, weighting functions, resolution bias and excitation methods.

A new technique, based on the Hilbert transform of the impulse response is described. The technique is used to obtain a large dynamic range of time signals on a logarithmic axis and is shown used for determination of damping from the envelope of the impulse response and for separation of modes in the time domain.

Brüel & Kjær, Denmark

INTRODUCTION

The aim of a structural analysis is to obtain a description of the structure when exposed to vibrations in order to verify a theory or to create a model, which can be treated theoretically. The tool commonly used for this is a modal analysis based on measured frequency responses, manipulated by a computer.

It is obvious that the confidence in the modal model is dependent on the accuracy with which we can measure the frequency response: of the modal parameters, mode shape, resonance frequency and damping, the last two are found directly from the frequency response and so are the residues used in the calculation of the mode shape. This paper describes some of the pitfalls in determining the parameters and some of the methods for ensuring the best accuracy.

CALCULATION OF THE FREQUENCY RESPONSE

The frequency response is found from the autospectra and cross-spectrum of the excitation and response signals, with the present technique measured, using the Fast Fourier Transform

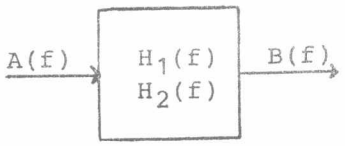


Fig. 1

$$H_1(f) = \frac{G_{AA}(f)}{G_{AB}(f)} \qquad H_2(f) = \frac{G_{BB}(f)}{G_{BA}(f)}$$

where $G_{AA}(f) = E\left[A^*(f) \cdot A(f)\right]$

$G_{BB}(f) = E\left[B^*(f) \cdot B(f)\right]$

$G_{AB}(f) = E\left[A^*(f) \cdot B(f)\right]$

$G_{BA}(f) = E\left[B^*(f) \cdot A(f)\right]$

Per definition the autospectra and cross-spectra are estimated by averaging and, assuming that the averaging times are chosen correctly, the two values, H_1 and H_2, are in theory identical. In practical systems, however, the spectra cannot be measured correctly, but will be contaminated by noise

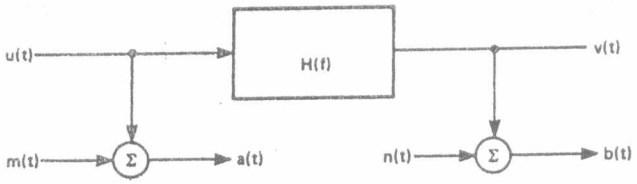


Fig. 2

The correct frequency of the system in fig. 2 is

$$H(f) = \frac{G_{UV}(f)}{G_{UU}(f)} = \frac{G_{VV}(f)}{G_{VU}(f)}$$

The measured spectra are

$$B(f) = V(f) + N(f) = H(f) \cdot U(f) + N(f)$$
$$A(f) = U(f) + M(f) = 1/H(f) \cdot V(f) + M(f)$$

Assuming that the noise signals are uncorrelated to each other and to any other signal, i.e.

$$G_{NU} = G_{NV} = G_{MV} = G_{MV} = 0$$

we have, leaving out the frequency dependency in the notation:

$$G_{BB} = G_{VV} + G_{NN} \qquad G_{AA} = G_{UU} + G_{MM}$$

$$G_{AB} = G_{AV} = G_{UB} = G_{UV}$$
$$G_{BA} = G_{BU} = G_{VA} = G_{VU}$$

The two expressions for the frequency response can now be calculated

$$H_1 = \frac{G_{AB}}{G_{BA}} = \frac{G_{UV}}{G_{UU} + G_{MM}} = \frac{H}{1 + \frac{G_{MM}}{G_{UU}}}$$

$$H_2 = \frac{G_{BB}}{G_{AA}} = \frac{G_{VV} + G_{NN}}{G_{VU}} = H + \frac{G_{NN}}{G_{VU}}$$

$$H_1 < H < H_2$$

For systems, contaminated with noise at the output only, H_1 will be the correct frequency response, whereas H_2 will give an over-estimate in magnitude.

Inversely, systems with noise at the input only will be correctly measured by H_2, whereas H_1 will give an under-estimate in magnitude. The phase will be correct in both cases, as the cross spectra are not influenced by noise.

Output noise will occur if signals are generated internally in the system or if the system is excited by sources other than those being measured, but will also occur at frequencies where the response signal is extremely small, i.e. at anti-resonances. Inversely, input noise will occur at frequencies where it is difficult to produce an excitation signal above the noise level due to a high mobility of the structure, i.e. at resonances.

In modal analysis the resonances in particular are of interest for determination of residues and damping. The method most often used for determination of the frequency response is that based on the cross spectrum and the input spectrum, i.e. H_1. However, as shown above, H_2 will normally be the better representation. In any case, a comparison of H_1 and H_2 will be of value for an estimation of the influence of noise. As all practical systems are contaminated at both input and output, none of them will be correct, but the right function will be somewhere in between.

RESOLUTION BIAS

Resolution bias is found where the bandwidth of the analyzer is too broad compared to the bandwidth of the peak or valley. A smaller bandwidth, zooming, will diminish the resolution bias, but will demand a longer analyzing time.

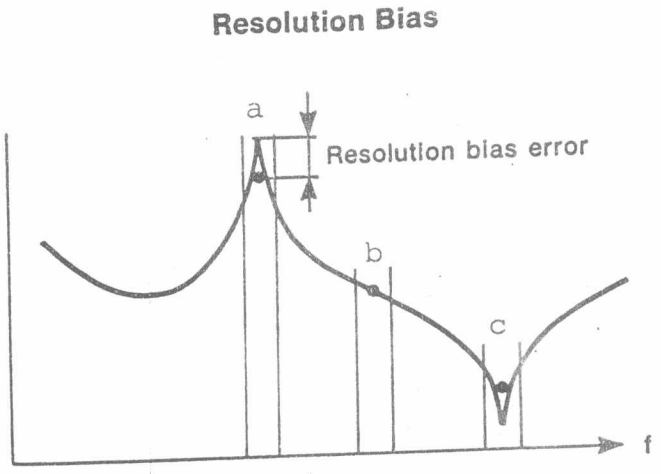
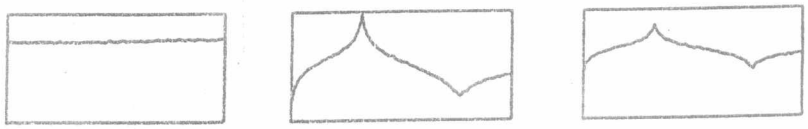


Fig. 3 Resolution Bias

a) underestimation ... b) correct average ... c) overestimation

Another improvement, perhaps less evident, can be obtained by selecting the proper method for calculating the frequency response.

Since the excitation spectrum is normally intended to be flat, the resolution bias is expected only in the response autospectrum and in the cross-spectrum. As the biases are of the same order, they will largely cancel out when dividing the two spectra, whereas the calculation based on the input spectrum will leave the bias error unchanged (fig. 4). Again, H_2 is the best approach for determination of the magnitude at resonance.



G_{AA} G_{BB} G_{AB}

$$H_1 = \frac{G_{AB}}{G_{AA}} \qquad H_2 = \frac{G_{BB}}{G_{BA}}$$

Fig. 4

ERRORS RELATED TO THE EXCITATION TECHNIQUE

The excitation technique, typically an impact by a hammer, random, pseudo-random, pulse or sine signal by a shaker, is mainly chosen from demands made to the frequency content of the spectrum, energy distribution, distribution of forces, signal to noise ratio, linearization, etc.

The sampling technique used in the FFT analysis requires that the time signal be modified by multiplication with a weighting factor in order to avoid abrupt changes of the signals between the sampling periods. Incorrect choice of weighting function or window can cause serious misinterpretation of the results.

A large choice of weighting functions is available, but the most commonly used are the Flat, the Hanning, the Transient and the Exponential windows.

The flat window is used for signals repetitive with the sample period like pseudo-random signals, whereas the Hanning window will be used with continuous, not repetitive signals, like random. These windows are mainly used with shaker excitation. The transient and exponential windows are mainly used with hammer excitation for measurement of input and output respectively. Once properly selected, the first two windows will hardly give any problems, whereas the last two require a correct position and length in relation to the input and output. A window which is not correct may cause that none or only a little of the energy of the pulse is analyzed.

Even with a correct window, different excitation techniques may cause different results, even in cases where it should not be expected due to the nature of the structure. A linear structure should show the same response for a random and a pseudo-random excitation, but nevertheless the resonance peaks are often found to be higher with pseudo-random excitation than with random. This is due to leakage of the random signal which is not found in the pseudo-random signal, as this consists of discrete frequencies corresponding to the lines of the analyzer. Zooming around the resonance will improve the result obtained from the random signal.

DETERMINATION OF DAMPING

The traditional way of determining the damping is by calculating the Quality Factor, Q, by measuring the frequencies, f_1 and f_2, at which the level of the frequency response is 3 dB below the level at the resonance frequency, ω_0:

$$Q = \frac{\omega_0}{\omega_2 - \omega_1}$$

The frequencies can be found from the magnitude curve of the frequency response, from the real part of the accelerance curve or from the Nyquist plot.

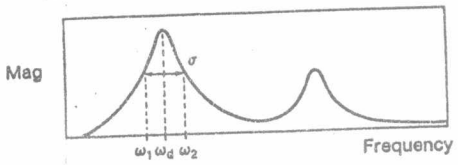
These methods are all found from the frequency response. In the time domain, the damping is calculated from the decay rate of the impulse response

$$Q = \pi \omega_0 \tau$$

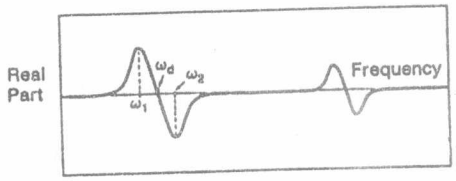
where τ is the time constant (= the time taken to decay by 8,7 dB) (fig. 5).

Measurement of Damping

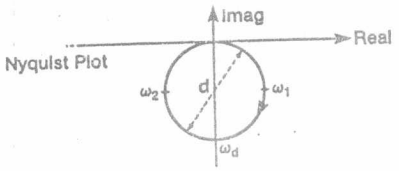
1. From the Magnitude of the Frequency Response Function



2. From the Real–part of the Frequency Response Function



3. From the Nyquist Plot of the Frequency Response Function



4. From the Impulse Response

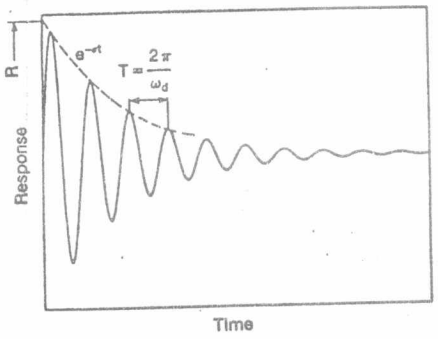


Fig. 5

DETERMINATION OF DAMPING USING THE HILBERT TRANSFORM

The decay method for determination of the damping is often better than those based on the 3 dB points due to a higher resolution. A new transform has therefore been introduced for further improvement of the accuracy of the determination: the Hilbert transform.

The Hilbert Transform of a real-valued time signal, a(t), is defined as

$$\mathcal{H}\left[a(t)\right] = \frac{1}{\pi}\int_{-\infty}^{\infty} a(t)\,\frac{1}{t-u}\,du$$

A simple non-mathematical way of describing the transform is to say that it gives all frequency components of the signal a -90° phase shift, or in the time domain that it shifts each component by 1/4 wavelength.

If the original signal is treated as the real part and the Hilbert transform as the imaginary, another time signal is obtained which can be displayed similar to the frequency domain functions. The magnitude of this signal will describe the envelope of the signal and since it is a positive quantity, it can be displayed on a logarithmic amplitude scale making a large dynamic range available in the time domain. The exponential decay of the impulse response now becomes a straight line, the slope of which is a measure of the damping.

A disadvantage of using the decay method is that when several modes are involved, the interaction between these will cause steps on the decay curve (fig. 6). This can be avoided by only looking at one mode at a time.

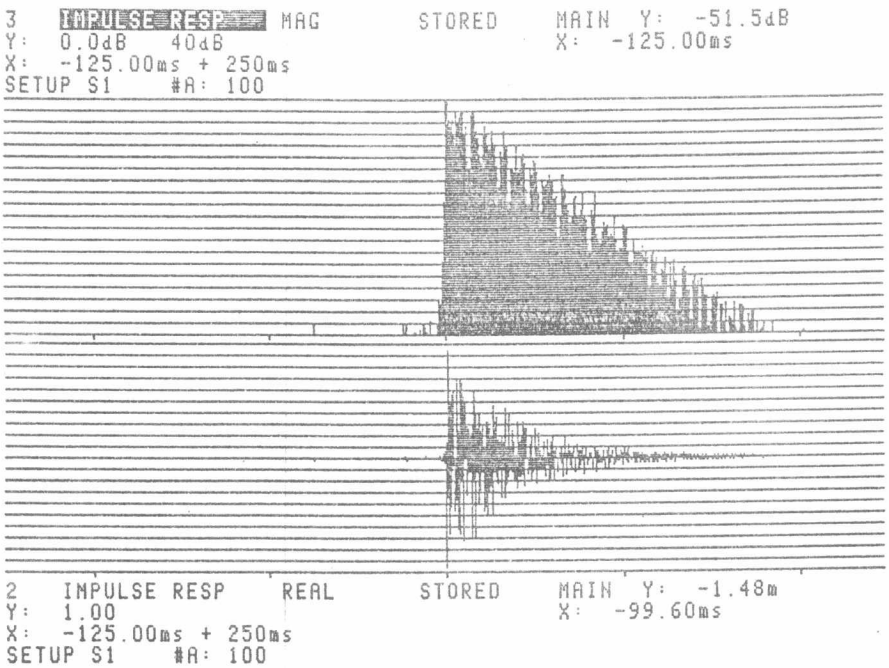


Fig. 6

Just as the time function can be weighted (Hanning, flat, etc.), the frequency response can be weighted also. A weighting function, tapered flat around the resonance and otherwise zero, will leave the frequency response of one mode alone. An inverse Fourier transform gives the impulse response, and in connection with a Hilbert transform, the slope of the magnitude is now completely linear, corresponding to a one-degree-of-freedom system, from which the damping can be measured unambiguously (fig. 7).

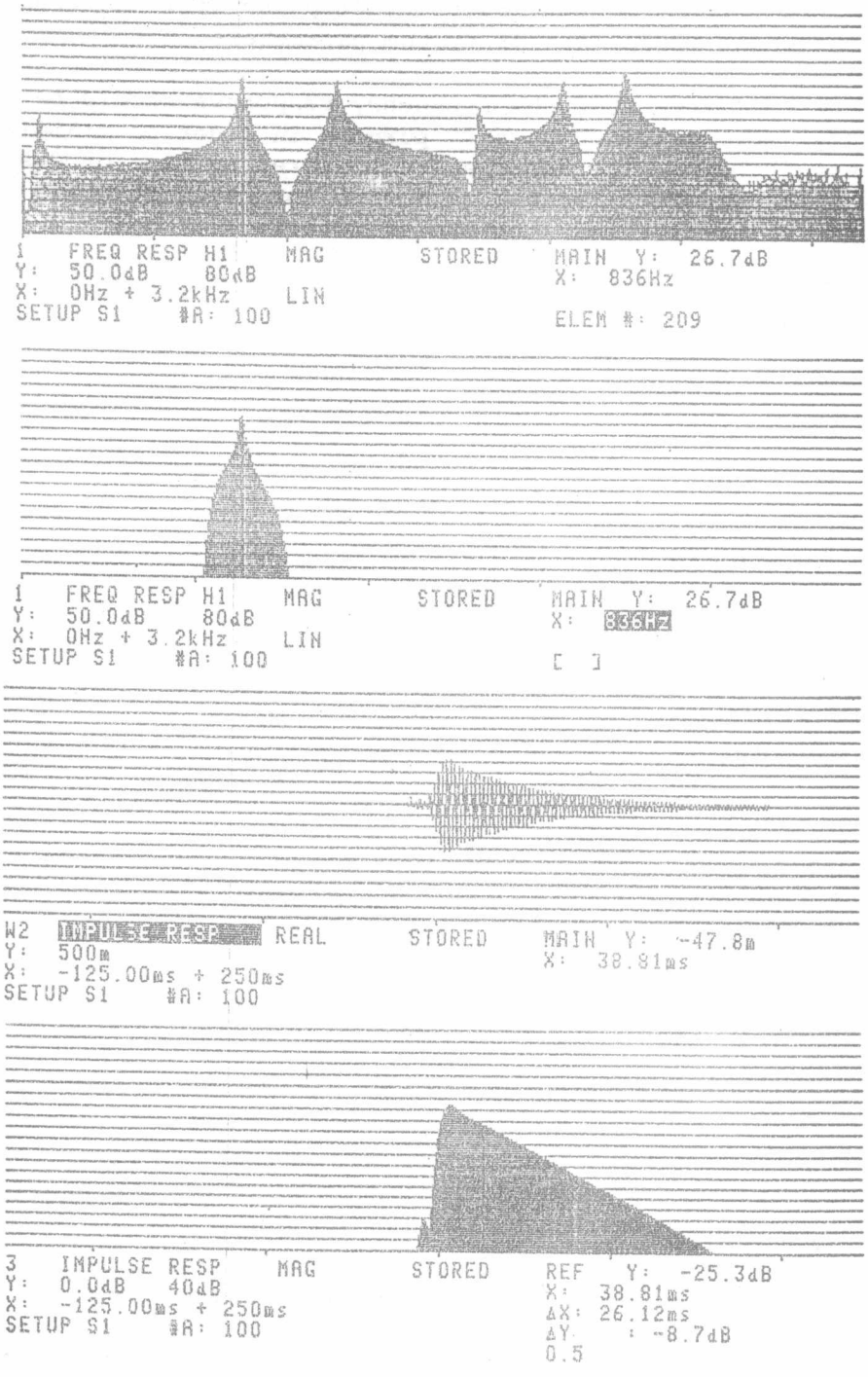


Fig. 7

CONCLUSION

This paper has described how the modal parameters, i.e. resonance frequencies, residues and damping can be found directly from the frequency response function without the use of computer programs. It also describes a number of pitfalls in the determination of the parameters giving rise to errors in the modal model if the errors are transferred to the computer program. Proper measurement of the frequency response will therefore need a careful selection and comparison of system parameters.

The conclusion is therefore that the frequency responses should be measured using an analyzer dedicated to this purpose in order to minimize the time and manpower involved. The responses are stored on tape or disk and post-processed in a computer, leaving this free for other purposes during the time used for the measurements on the structures.