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NONLINEAR H_∞ CONTROL OF A QUADROTOR UNMANNED AERIAL VEHICLE

MOKHTARI[1] A., BENALLEGUE[2] A., ORLOV[3] Y. and DAACHI[4] B.

ABSTRACT

A nonlinear H_∞ output feedback controller is proposed and coupled to a PI controller to regulate an UAV in the presence of aerodynamic force and moment perturbations. The plant to be controlled is a Quadrotor helicopter described by nonlinear dynamics with plant uncertainties due to the variations of inertia moments and payload operation. A robust state estimation is considered under model uncertainties as well as external/measurement disturbances. Performance issues of the controller are illustrated in a simulation study made for an UAV prototype.

KEY WORDS

H-infinity, riccati equation, estimation

[1] *University of Sciences and Technology Oran, BP1505 El M'naour Oran Algeria.*
Email: mokhtari@lisv.uvsq.fr

[2] *Robotic Laboratory of Versailles, 10 avenue de l'Europe Velizy, France.*
Email: benalleg@lisv.uvsq.fr

[3] *Scientific Research Technology Development (CITEDI-IPN), Ave. del Parque 1310 Mesa de Otay, Tijuana, B.C., Mexico. E-mail : yorlov@cicese.mx*

[4] *Laboratoire Images, Signaux et Intelligence Artificielle (LISSI) 122-124, rue Paul Armangot 94200 Vitry sur Seine (France).*

INTRODUCTION

A Quadrotor UAV is usually required to move in different environments, following specific tasks and providing good performance under parameter variations and external disturbances. The aircraft dynamics is of 6 degree of freedom, it is nonlinear, and being affected by aerodynamic forces, it is subject to parameter uncertainties. While controlling an UAV, the following factors, relevant in practice, should be addressed in combination: i) to ensure the stability, robustness and desired dynamic properties; ii) to handle nonlinearity; iii) to be adaptive to changing parameters and environmental disturbances. Since, in addition, incomplete and imperfect state measurements are only available; the design of stabilizing controllers for Quadrotor UAV presents a challenging problem.

Various methods such as sliding mode approach [1], backstepping technique [2], input/output linearization [3] and nonlinear dynamic inversion [4] have recently been tested to control UAV's. Since these methods required perfect state measurements their practical utility remained limited. In this regard, potential applications of the nonlinear H_∞ control techniques from [5], [6], [7], [8] to UAV's seem to be attractive for handling all the above factors, including output feedback design. Such an application is studied in the present work.

The nonlinear H_∞ output feedback control of a Quadrotor UAV to be developed is inspired from [8]. A local H_∞ output feedback controller is derived by means of a certain perturbation of the differential Riccati equations, appearing in solving the H_∞ control problem for the linearized system, when these unperturbed equations have positive semi definite solutions. The local stabilizability/detectability conditions are thus ensured by the existence of the proper solutions of the unperturbed Riccati equations. This feature allows us to develop an H_∞-design procedure with no a priori-imposed stabilizability-detectability conditions on the control system. The procedure is opposed to that of [5], [6], [7] where an extra non-trivial work on verification of these conditions is required. In order to reenforce the compensation of the aerodynamical moment perturbations a PI regulator is additionally involved into the loop.

The paper is outlined as follows. The dynamic UAV model is introduced in Section 2. The H_∞ control synthesis is developed in Section 3. Simulation results are discussed in Section 4. Finally, Section 5 presents some conclusions.

DYNAMIC MODEL

A Quadrotor UAV is a rotating rigid body of six degrees of freedom. Two diagonal motors (1 and 3) are running in the same direction (anti-clockwise) whereas the others (2 and 4) in the clockwise direction to eliminate the anti-torque. Varying the rotor speeds, all with the same quantity, the lift forces affect the attitude thus enabling the vertical take-off/on. Yaw angle is obtained by speeding up the clockwise motors or slowing down dependent on the desired angle direction. Tilting around x (roll angle) axe allows the Quadrotor to move toward y direction. The sense of direction depends on the sense of angle whether it is positive or negative. Tilting around y (pitch angle) axe allows the Quadrotor to move toward x direction. The rotor is the primary source of control and propulsion for the UAV. The Euler angles orientation to the flow produces the forces and moments, controlling the altitude and position of the system. The absolute position is described by three coordinates (x_0, y_0, z_0),, and its attitude by Euler angles(ψ, θ, φ), under the conditions $(-\pi \leq \psi < \pi)$ for yaw, $(-(\pi/2) < \theta < (\pi/2))$ for pitch and $(-(\pi/2) < \varphi < (\pi/2))$ for roll.

The Newton law-based equations, describing the attitude and position of the UAV, are partitioned into kinematic equations and dynamic equations [9], [10] and they generate a MIMO nonlinear system given by

$$\begin{aligned} \dot{x} &= F(x) + G_1(x)w + G_2(x)\bar{u} \\ y_1 &= H_1(x) + K_{12}(x)\bar{u} \\ y_2 &= H_2(x) + K_{21}(x)w \end{aligned} \tag{1}$$

In the above equations the state vector $x \in \Re^{14}$ is as follows

$$x = \left(x_0, y_0, z_0, \psi, \theta, \varphi, \dot{x}_0, \dot{y}_0, \dot{z}_0, \zeta, \xi_1, \dot{\psi}, \dot{\theta}, \dot{\varphi}\right)^T \tag{2}$$

The control input $\bar{u} \in \Re^4$ is formed by means of the real control signals (u_1, u_2, u_3, u_4) through the relations

$$\begin{aligned} u_1 &= \zeta + mg \\ \dot{\zeta} &= \xi_1 \\ \dot{\xi} &= \overline{u_1} \\ u_2 &= \overline{u_2} \\ u_3 &= \overline{u_3} \\ u_4 &= \overline{u_4} \end{aligned} \tag{3}$$

(i.e., for ensuring the system controllability the scalar control signal u_1 has been delayed by double integrating whereas the others (u_2, u_3, u_4) have remained the same), $y_1 \in \Re^8$ is the controlled output, $y_2 \in \Re^4$ is the available measurement, the perturbation vector $(w_n, w_d)^T \in \Re^{10}$ consists of the measurement imperfection $w_n = (w_1, w_2, w_3, w_4)^T$ and external disturbances $w_d = \left(A_x, A_y, A_z, A_p, A_q, A_r\right)^T$ .The external disturbance w_d, that affects the UAV, is composed by aerodynamical forces disturbances $\left(A_x, A_y, A_z\right)^T$ and aerodynamical moment disturbances$\left(A_p, A_q, A_r\right)^T$. Being computed from the aerodynamical coefficients $C_i \; i = x, y, z, p, q, r$ these disturbances $A_i = \frac{1}{2}\rho_{air} C_i W^2$ appear to depend on the air density ρ_{air}, the velocity W of the UAV with respect to the air whereas C_i depends on several parameters such as the angle between airspeed and the body fixed reference system, the aerodynamical and geometric form of the wing. The other parameters, governing the state dynamics (1), are given by

$$F(x)=[f_1(x)\cdots f_{14}(x)]^T, K_{12}(x)=\begin{bmatrix}0_{4\times4} & I_{4\times4}\end{bmatrix}, K_{21}(x)=\begin{bmatrix}I_{4\times4} & 0_{4\times6}\end{bmatrix}$$

$$G_1(x)=\begin{bmatrix}0_{6\times4} & 0_{6\times3} & 0_{6\times3}\\ 0_{3\times4} & M_1 & 0_{3\times3}\\ 0_{2\times4} & 0_{2\times3} & 0_{2\times3}\\ 0_{3\times4} & 0_{3\times3} & P_1\end{bmatrix}^T, G_2(x)=\begin{bmatrix}0_{10\times4}\\ P_4\end{bmatrix}, \begin{matrix}H_1(x)=[0,0,0,0,x_0,y_0,z_0,\psi]^T\\ H_2(x)=[x_0,y_0,z_0,\psi]^T\end{matrix}$$

While the state vector being represented in the form

$$x=[x_1\cdots x_{14}]^T$$

the above relations are specified with

$$\mathrm{f}_1(\mathrm{x})=\mathrm{x}_7, \mathrm{f}_2(\mathrm{x})=\mathrm{x}_8, \mathrm{f}_3(\mathrm{x})=\mathrm{x}_9, \mathrm{f}_4(\mathrm{x})=\mathrm{x}_{12}, \mathrm{f}_5(\mathrm{x})=\mathrm{x}_{13}, \ \mathrm{f}_6(\mathrm{x})=\mathrm{x}_{14}$$

$$\mathrm{f}_7(\mathrm{x})=\mathrm{g}_1^7(\mathrm{x}_{10}+\mathrm{mg})+({}^{\mathrm{K}_1}\!/_{\mathrm{m}})\mathrm{x}_7, \ \mathrm{f}_8(\mathrm{x})=\mathrm{g}_1^8(\mathrm{x}_{10}+\mathrm{mg})+({}^{\mathrm{K}_2}\!/_{\mathrm{m}})\mathrm{x}_8$$

$$\mathrm{f}_9(\mathrm{x})=\mathrm{g}+\mathrm{g}_1^9(\mathrm{x}_{10}+\mathrm{mg})+({}^{\mathrm{K}_3}\!/_{\mathrm{m}})\mathrm{x}_9, \qquad \mathrm{f}_{10}(\mathrm{x})=\mathrm{x}_{11}, \mathrm{f}_{11}(\mathrm{x})=0$$

$$\begin{bmatrix}f_{12}(x)\\ f_{13}(x)\\ f_{14}(x)\end{bmatrix}=\begin{bmatrix}{}^{K_4}\!/_{I_y}\,x_{12}\\ {}^{K_5}\!/_{I_x}\,x_{13}\\ {}^{K_6}\!/_{I_z}\,x_{14}\end{bmatrix}, M_1={}^{1}\!/_{m}\,I_{3\times3}, P_1={}^{1}\!/_{d}\begin{bmatrix}0 & g_3^{12} & dg_4^{12}\\ 0 & g_3^{13} & dg_4^{13}\\ g_2^{14} & g_3^{14} & dg_4^{14}\end{bmatrix}, P_4=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & g_3^{12} & g_4^{12}\\ 0 & 0 & g_3^{13} & g_4^{13}\\ 0 & g_2^{14} & g_3^{14} & g_4^{14}\end{bmatrix}$$

$$g_1^7=-\frac{1}{m}\left(\cos x_6\cos x_4\sin x_5+\sin x_6\sin x_4\right);\ g_1^8=-\frac{1}{m}\left(\cos x_6\sin x_4\sin x_5-\sin x_6\cos x_4\right);$$

$$g_1^9=-\frac{1}{m}\left(\cos x_5\sin x_6\right);\ g_2^{14}=\frac{d}{I_x};\ g_3^{12}=\frac{d\sin x_6}{I_y\cos x_5};\ g_3^{13}=\frac{d\cos x_6}{I_y};\ g_3^{14}=\frac{d\sin x_5\sin x_6}{I_y\cos x_5}$$

$$g_4^{12}=\frac{\cos x_6}{I_z\cos x_5};\ g_4^{13}=-\frac{\sin x_6}{I_z};\ g_4^{14}=\frac{\sin x_5\cos x_6}{I_z\cos x_5}$$

Where g is the gravity constant ($g=9.81ms^{-2}$), d is the distance from the center of mass to the rotors. U1 is the resulting thrust of the four rotors defined as $u_1=(F_1+F_2+F_3+F_4)$. U2 is the difference of thrust between the left rotor and the right rotor defined as $u_2=d(F_4-F_2)$. U3 is the difference of thrust between the front rotor and the back rotor defined as $u_3=d(F_3-F_1)$. U4 is the difference of torque between the two clockwise turning rotors and the two counter-clockwise turning rotors defined as $u_4=C_f(F_1-F_2+F_3-F_4)$, With C_f being the force moment scaling factor and F_1,F_2,F_3 and F_4 are the thrust forces of the rotors. I_x,I_y,I_z represent the diagonal coefficients of inertia matrix of the system. $K_{1..6}$ are drag coefficients.

NONLINEAR H_∞ CONTROL SYNTHESIS

For stating the nonlinear H_∞-regulation problem we recall the following.
A causal dynamic feedback compensator

$$\overline{u} = K(y) \tag{4}$$

With internal state $\xi \in \Re^s$, is said to be a globally (locally) admissible controller if the closed-loop UAV (1), (2) is globally (uniformly) asymptotically stable when w=0.

Given a real number $\gamma > 0$, it is said that UAV (1), (2) has L_2 gain less than if the response z, resulting from w for initial state $x(t_0) = 0;\ t(0) = 0$, satisfies:

$$\int_{t0}^{t1} \|z(t)\|^2\, dt < \gamma^2 \int_{t0}^{t1} \|w(t)\|^2\, dt \tag{5}$$

for all $t_1 > t_0$ and all piecewise continuous functions w(t). The nonlinear H_∞ regulation problem is in finding a globally admissible controller (2) such that L_2-gain of the closed-loop UAV (1), (2) is less than γ. In turn, a locally admissible controller (2) is said to be a local solution of the nonlinear H_∞-control problem if there exists a neighbourhood U of the origin such that inequality (5) is satisfied for all $t_1 > t_0$ and all piece-wise continuous functions w(t) for which the state trajectory of the closed-loop UAV starting from the initial point $x(t_0) = \xi(0)$ remains in U for all $t \in [t_0, t_1]$.

In the above problem statement, we have assumed for simplicity that the origin is the desired destination of the UAV. If it is not the case one should rewrite the UAV equations in terms of the state deviation from the desired destination and then reformulate the problem for the resulting deviation equations.

Our subsequent local analysis involves the standard linear H_∞-control problem for the UAV linearization

$$\begin{aligned} \dot{x} &= A_1 x + B_1 w + B_2 \overline{u} \\ z &= C_1 x + D_{12} \overline{u} \\ y &= C_2 x + D_{21} w \end{aligned} \tag{6}$$

where

$$A_1 = \frac{\partial F}{\partial x}(0); B_1 = G_1(0);\ B_2 = G_2(0); C_1 = \frac{\partial H_1}{\partial x}(0);\ C_2 = \frac{\partial H_2}{\partial x}(0); D_{12} = K_{12}(0);\ D_{21} = K_{21}(0)$$

Such a problem is well-understood if the linear system (6) is stabilizable and detectable from u and y, respectively. Under these assumptions, the following conditions are necessary and sufficient for a solution of this problem to exist:

(A1) there exists a positive semi definite symmetric solution of the algebraic Riccati equation $PA_1 + A_1^T P + C_1^T C_1 + P\Gamma_1 P = 0$; specified with $\Gamma_1 = \gamma^{-2} B_1 B_1^T - B_2 B_2^T$ such that the matrix $A_1 - \Gamma_1 P$ is Hurwitz.

(A2) there exists a positive semi definite symmetric solution of the algebraic Riccati equation $AZ + ZA^T + B_1 B_1^T + Z\Gamma_2 Z = 0$; specified with $A = A_1 + \gamma^{-2} B_1 B_1^T P$ and $\Gamma_2 = \gamma^{-2} P B_2 B_2^T P - C_2^T C_2$;such that the matrix $A + \Gamma Z_2$ is Hurwitz.

According to [8], conditions (A1) and (A2) ensure that there exists a positive constant ε_0 such that the system of the perturbed algebraic Riccati equations

$$P_\varepsilon A_1 + A_1^T P_\varepsilon + C_1^T C_1 + P_\varepsilon \Gamma_1 P_\varepsilon = -\varepsilon I \qquad (7)$$

$$AZ_\varepsilon + Z_\varepsilon A^T + B_1 B_1^T + Z_\varepsilon \Gamma_2 Z_\varepsilon = -\varepsilon I \qquad (8)$$

has a unique positive definite symmetric solution $(P_\varepsilon, Z_\varepsilon)$ for each $\varepsilon \in (0, \varepsilon_0)$. In the sequel, equations (7), (8) are utilized to derive a local solution of the above nonlinear H_∞-regulation problem. The following result is extracted from [8].

Theorem1 Let $(P_\varepsilon, Z_\varepsilon)$ be a positive definite symmetric solution of (7), (8) under some $\gamma > 0$ and $\varepsilon > 0$. Then the output feedback

$$\begin{aligned} \dot{\xi} &= F(\xi) + \left[\gamma^{-2} G_1(\xi) G_1^T(\xi)\right] P_\varepsilon \xi - \left[G_2(\xi) G_2^T(\xi)\right] P_\varepsilon \xi + Z_\varepsilon C_2^T \left[y - H_2(\xi)\right] \\ \bar{u} &= G_2^T(\xi) P_\varepsilon \xi \end{aligned} \qquad (9)$$

is a local solution of the H_∞-regulation problem, with

$$\xi = \left(\hat{x}_0, \hat{y}_0, \hat{z}_0, \hat{\psi}, \hat{\theta}, \hat{\varphi}, \hat{\dot{x}}_0, \hat{\dot{y}}_0, \hat{\dot{z}}_0, \hat{\zeta}, \hat{\xi}_1, \hat{\dot{\psi}}, \hat{\dot{\theta}}, \hat{\dot{\varphi}}\right)^T$$

Proof. It is straightforward to verify that the following hypotheses hold:

1. The functions $F(x), G_1(x), G_2(x), H_1(x), H_2(x), K_{12}(x), K_{21}(x)$ are piecewise Continuous
2. $F(0) = 0,\ H_1(0) = 0, H_2(0) = 0$;
3. $H_1^T(x) K_{12}(x) = 0, K_{12}^T(x) K_{12}(x) = I, K_{21}(x) G_1^T(x) = 0, K_{21}(x) K_{21}^T(x) = I$

Thus, Theorem 3 from [8] turns out to be applicable to the system in question. The validity of the present theorem is then established by applying Theorem 3 of [8] to the UAV dynamics (1). The proof of Theorem 1 is completed.

In order to apply the synthesis procedure, resulting from Theorem 1, one should utilize a certain perturbation of the algebraic Riccati equations that appear in solving the standard H_∞ control problem for the linearized system. Since the local stabilizability and detectability are then ensured by the existence of a proper solution of the unperturbed algebraic Riccati equations ([8] for details) an extra work on the stabilizability/detectability is thus obviated.

Due to the nature of the nonlinear H_∞ control approach, the closed loop UAV is robust against the measurement imperfections and aerodynamical force perturbations. We must emphasize that drag forces and moments has been taken as output perturbations. However, the closed-loop system becomes extremely sensitive if it is affected by aerodynamical moment perturbations whose influence on the system dynamics (1) is amplified by a nonlinear state-dependent gain. This fact has been confirmed by simulations since the matching condition is not satisfied. To address the latter perturbations a modification of the H_∞ control law was proposed. Since A_p, A_q, and A_r affect y_0, x_0, and ψ respectively, a *PI* control law is added into the loop to enhance robustness against these disturbances and drag forces and moments. So the internal state vector is modified as follows (Fig-2):

$$\xi = \left(\hat{x}_0, \hat{y}_0, \hat{z}_0, \hat{\psi} + e_\psi, \hat{\theta} + e_x, \hat{\varphi} + e_y, \hat{\dot{x}}_0, \hat{\dot{y}}_0, \hat{\dot{z}}_0, \hat{\zeta}, \hat{\xi}_1, \hat{\dot{\psi}}, \hat{\dot{\theta}}, \hat{\dot{\varphi}} \right)^T \quad (10)$$

$$e_x = (0 - x_0)\left(K_x + \frac{1}{T_x s} \right), e_y = (0 - y_0)\left(K_y + \frac{1}{T_y s} \right), e_\psi = (0 - \psi)\left(K_\psi + \frac{1}{T_\psi s} \right) \quad (11)$$

SIMULATION RESULTS

The controller performance was studied by simulation made for the laboratory prototype of the Quadrotor UAV with the constant parameters m=1Kg, Ix=Iy=0.0996N.m/rad/s², Iz=0.1219N.m/rad/s², d=0.17m, g=9.81m/s², K1..6=0.04.

In the simulations performed with MATLAB the Quadrotor was required to move from the initial point

$$x_0(0) = 1, \dot{x}_0(0) = 0, y_0(0) = 1, \dot{y}_0(0) = 0, z_0(0) = -1, \dot{z}_0(0) = 0,$$
$$\psi(0) = 1, \dot{\psi}(0) = 0, \theta(0) = 0, \dot{\theta}(0) = 0, \varphi(0) = 0, \dot{\varphi}(0) = 0,$$ to the origin.

To design the nonlinear H_∞ controller (9) the following positive definite symmetric solutions of the Riccati equations (7),(8) were numerically found for $\gamma = 55,\ \varepsilon = 0.2$:
The parameters of the PI regulator (11) were tuned to

$$K_x = 10^{-3}, T_x = 0.3333\,\text{sec}, K_y = 10^{-3}, T_x = 0.3333\,\text{sec}, K_\psi = 10^{-2}, T_\psi = 1.6667\,\text{sec}$$

The aerodynamic force and moment perturbations were applied for:

$$Ax = 0.2 + K_1 x_7 + \Delta f; Ay = 0.2 + K_2 x_8 + \Delta f; Ax = 0.2 + K_3 x_9 + \Delta f;$$
$$Ap = 0.01 + K_4 x_{12} + \Delta a; Aq = 0.01 + K_5 x_{13} + \Delta a; Ar = 0.01 + K_6 x_{14} + \Delta a;$$

with Δf,Δa are random variations of period 60sec and magnitude 50% and 20% of the mean values of force and moment perturbations respectively.

The measurement noise signals used in all the simulation runs were uncorrelated, white, zero mean, and of 0.01 variance. To better demonstrate robustness features of the proposed synthesis various scenarios of the force perturbations were also played. Results for this case are presented in Figs(3,4,5,6). It is concluded from these figures that aerodynamical force and moment perturbations are better attenuated by the H_∞ controller when combined with *PI.*

CONCLUSION

A nonlinear H_∞ output feedback synthesis is developed to control a Quadrotor UAV, operating under uncertainty conditions. The controller consists of a nonlinear compensator and disturbance attenuator, being robust against external disturbances, measurement imperfections, and aerodynamical force perturbations. For enhancing robustness margins towards aerodynamical moment perturbations the controller is coupled to a PI controller. Theoretical results are supported by numerical simulations that demonstrate efficiency of the proposed combined PI/H_∞-controller design.

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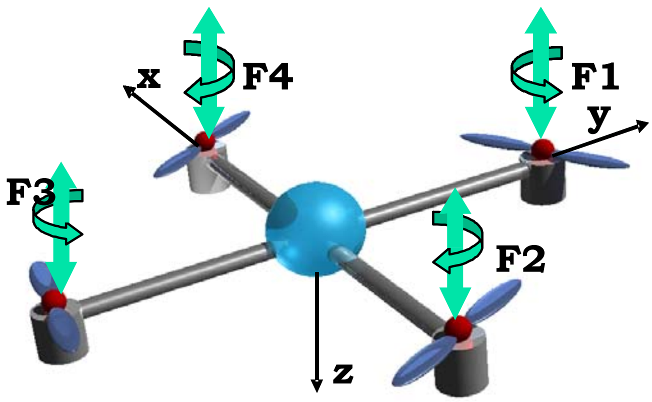

Fig.1: Quadrotor UAV

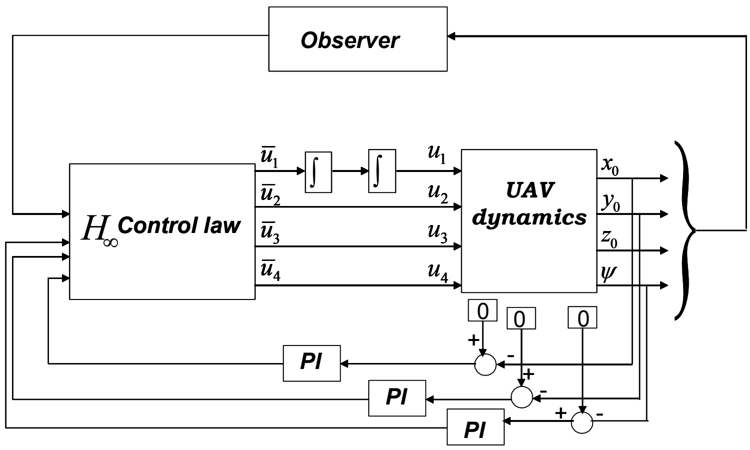

Fig.2 : Closed loop system

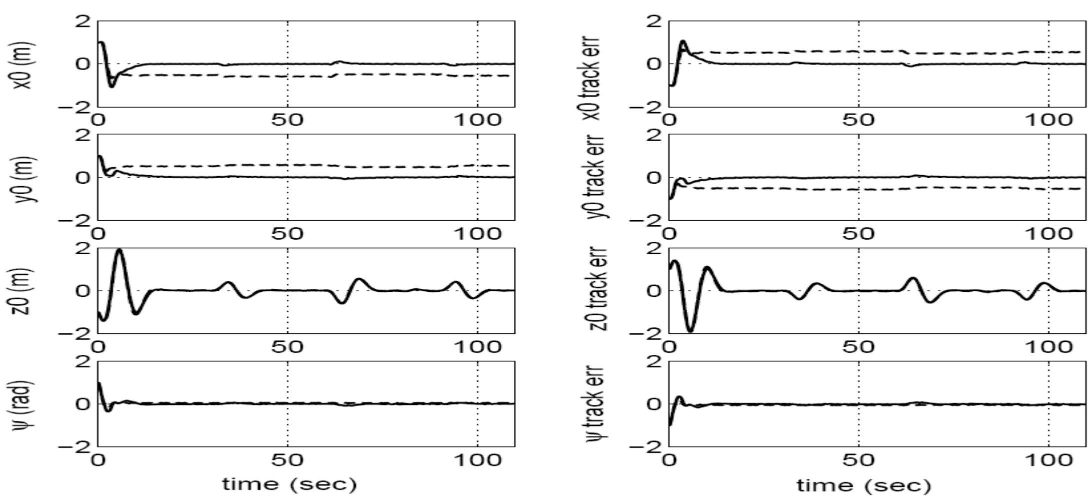

Fig.3: Output responses.

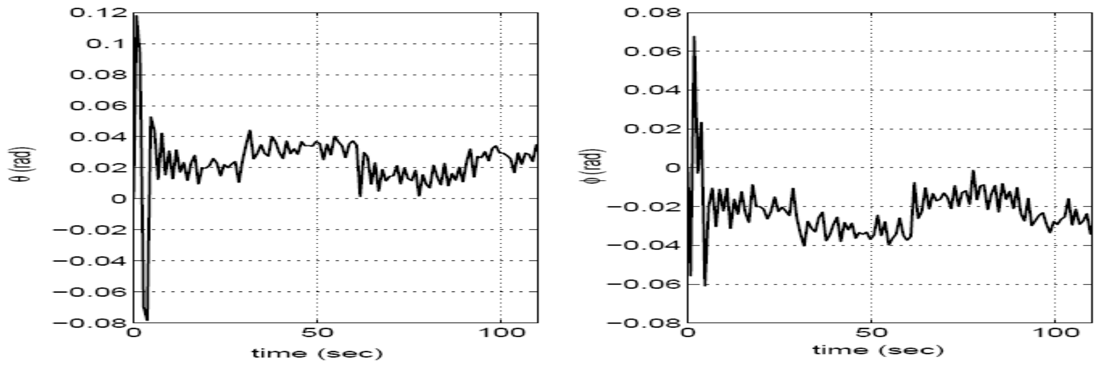

Fig.4: θ, ϕ responses.

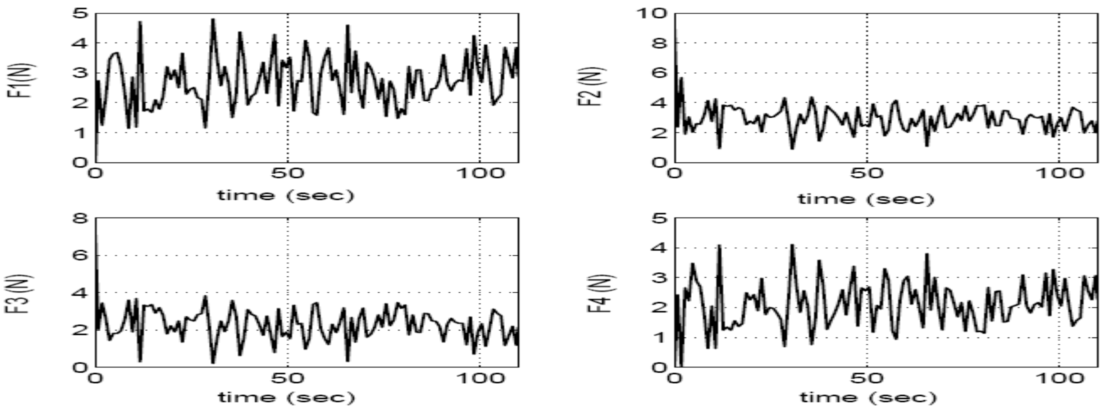

Fig-5: Force responses.

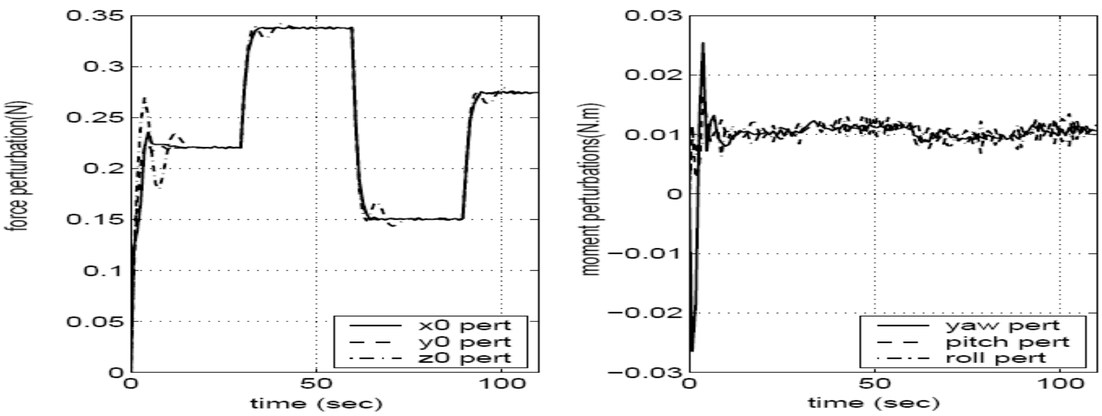

Fig-6:Force and moment disturbances