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VIBRATION BEHAVIOR OF THE ROTOR WITH TRANSVERE CRACK WITH SOME COMMENTS ON CRACK DETECTION

METWALLY* H. M., MORSI** E. and BADAWY*** M.H

ABSTRACT

The present work deals with crack behavior, detection, analysis, determination and diagnosis of cracked rotors. Crack detection is very important for any rotating machine. It can save all the rotating parts like blades, impellers, fans and bearings.

Crack behavior arises up non-linear differential equations that include the most influence of stiffness change. The nonlinear differential equations are solved by a computer program to calculate reduction in stiffness due to crack. The theoretical solution and calculations of stiffness reduction for rotor suspension systems was carried out.

Programming software to predict the crack spectrum time wave form, run-up and cost-down amplitude verse rotating speed, and the difference of natural frequency due to crack behavior unbalance amount, eccentricity and angles.

The experimental work is helpful to assure the correctness of theoretical work, so constructing a test rig that simulates our problem has to be done. The experimental analysis is used to predict the cracked shaft and the change in crack stiffness. Run-up and cost down method is used for directly predicting the cracked shaft by monitoring the shift of frequency for critical speed.

The conclusion of the results show that, crack affects the natural frequency of the shaft and its stiffness which in turn has an effect on all vibration characteristics. The theoretical results show sub-harmonics and harmonics of rotating shafts according to speed, how the crack affects the natural frequency and the great effect of changing the phase angle and how it can be used for correction the crack effects.

KEY WORDS

Vibration, transverse cracked, detection.

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* Professor of Mechanical Engineer, Alexandria University, Egypt.
** Assistant Professor of Mechanical Engineer, Alexandria University, Egypt.
*** Mechanical engineer, Alexandria University, Egypt.

NOMENCLATURE

M	mass
ko	the ideal stiffness for non cracked shaft
ζ	rotating coordinate in y direction
η	rotating coordinate in z direction
k_ζ (kzo)	stiffness in the ζ direction
k_η (keo)	the stiffness in the η direction
$k_{\zeta\eta}$ (kez)	the cross coupled stiffness.
1X, 2X , 3X,…..Xn	first , second ,third rotating speed for cracked shaft.
1X0, 2X0 , 3X0,…..nX0	first , second ,third rotating speed for ideal shaft.
Q_ζ , Q_η	the acting forces along the ζ and η axes on the cross-section containing the crack
w	rotating speed (rad/sec)
c	damping coefficient.
d	shaft diameter.
L	shaft length
g	the gravity acceleration
F	force

INTRODUCTION

Cracked shafts are most commonly detected by observing excessive vibration at rotating speed. Yet, the cracks have not been detected as early as they could be. All known tests currently used to detect cracks, such as ultrasonic and visual inspection, etc, should be continued. These tests are in addition to the methods we are discussing today, but do not involve disassembly of the machine train. Cracked shafts is an important problem has not addressed in much detailed as mass unbalance, misalignment, bearing fault and etc.. The high levels of vibration readings is caused by the presence of a crack and found, in general that the vibration amplitude is dependent on the depth and the shape of the crack and the position of the crack on the relation to the shaft mode shape.

It is therefore important to be able to diagnose cracks early on in order to carry out any necessary maintenance, before more damage than is necessary is caused by this excess vibration. The presence of a crack causes asymmetry in the stiffness of the rotor, which is difficult to detect under normal running circumstances using costmary monitoring techniques.

Another difficulty is encountered when examining a breathing crack which is the crack that opens and closes as the shaft rotates. The shaft stiffness properties actually vary during the course of the revolution, as the crack opens and closes.

ANALYTICAL MODELING

The primary effect of the presence of a crack in a rotating shaft is clearly a local reduction in stiffness.

So that studying the change in the stiffness due to crack is mandatory before solving the equation of motion.

STIFFNESS ESTIMATION FROM FRACTURE MECHANICS

In the bent shaft state while rotating, as illustrated in Fig. 1, the force, Q_ζ and Q_η, acting along the ζ and η axes on the cross-section containing the crack, induce deflections of the solid shaft. The additional deflections due to the crack are estimated by using fracture mechanics concepts. In this study a crack is assumed to be located at the mid-span of the shaft. When the forces Q_ζ and Q_η exist, the stress may have different values along the crack front. [7]. Figure 2. shows the stiffness ratio due to crack by using MATLAB program to solve equations.

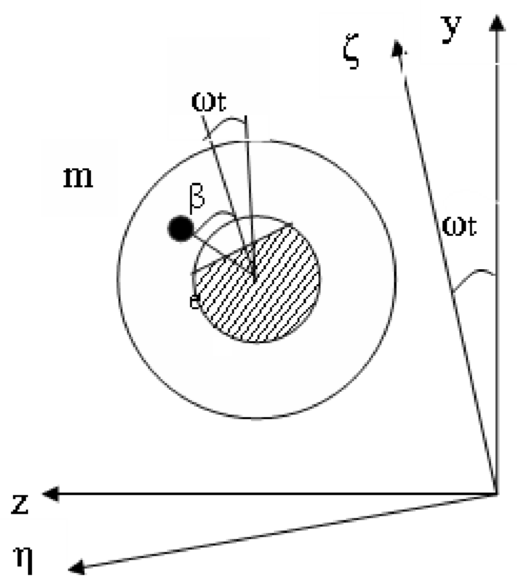

Fig. 1. coordinate system.

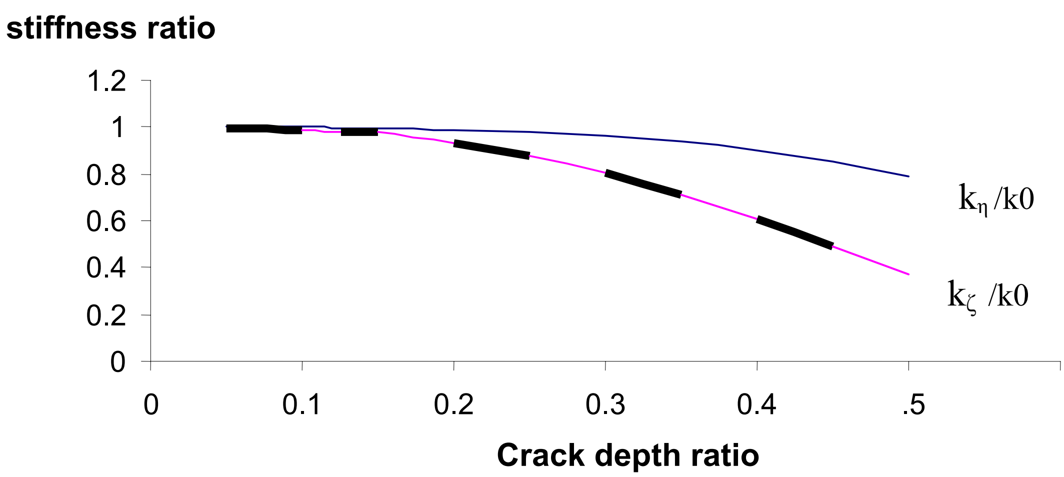

Fig. 2. Stiffness change ratio due to crack corresponding to ideal shaft stiffness ko .

EQUATION OF MOTION

Let us assume an ideal situation in which a crack is a switching crack, can completely open or closed, depending upon the sign of the shaft deflection in the crack direction ζ .Since the switching crack is not allowed to be partially open or closed. Then the equations of motion (eqn. 1) for a simple rotor with a switching crack can be written as [6].

$$m[\ddot{\zeta} - 2\omega\dot{\eta} - \omega^2\zeta] + c[\dot{\zeta} - \omega\eta] + k_\zeta \zeta = -mg * \cos(\omega\tau) + me\omega^2 * \cos(\beta),$$

$$m[\ddot{\eta} + 2\omega\dot{\zeta} - \omega^2\eta] + c[\dot{\eta} - \omega\zeta] + k_\eta \eta = mg * \sin(\omega\tau) + me\omega^2 * \sin(\beta) \qquad (1)$$

Making Laplace transform on Eqn. (1)

$$[m^2 s^4 + 2\,mcs^3 + [2\,m^2\omega^2 + c^2 + m\,(k_\zeta + k_\eta)s^2 + [c(k_\zeta + k_\eta) + 2mc\omega^2]\,s + m^2\omega^4 - m\omega^2 k_\eta\, s - m\,k_\zeta\,\omega^2 + k_\zeta k_\eta + \omega c^2\,]\zeta = \frac{s}{s^2 + \omega^2}\,[-ms^3 + 3m\omega^2 s - k_\eta\, s - cs^2 + c\omega^2] \qquad (2)$$

$$[m^2 s^4 + 2\,mcs^3 + [2\,m^2\omega^2 + c + m\,(k_\zeta + k_\eta)s^2 + [c(k_\eta + k_\zeta) + 2mc\omega^2]\,s + m^2\omega^4 - m\omega^2(\,k_\eta + k_\zeta) + k_\zeta k_\eta + \omega^2 c^2]\,\eta = \frac{s}{s^2 + \omega^2}\,[3m\omega s^3 + 2\omega cs + \omega\,k_\zeta - m\omega^3] \qquad (3)$$

Then using Matlab program to solve the Eqns (2) and (3).

ANALYSIS TECHNIQUES IN TIME DOMAIN

A part from regarding the time signals to get a first feeling of the vibration behavior of the system, the calculation of the statistical value give first hints to faulty operating conditions. Figure 3 shows a time wave form for our crack model generated by using a Matlab program .It is very hard to diagnose many problems by using time wave form.

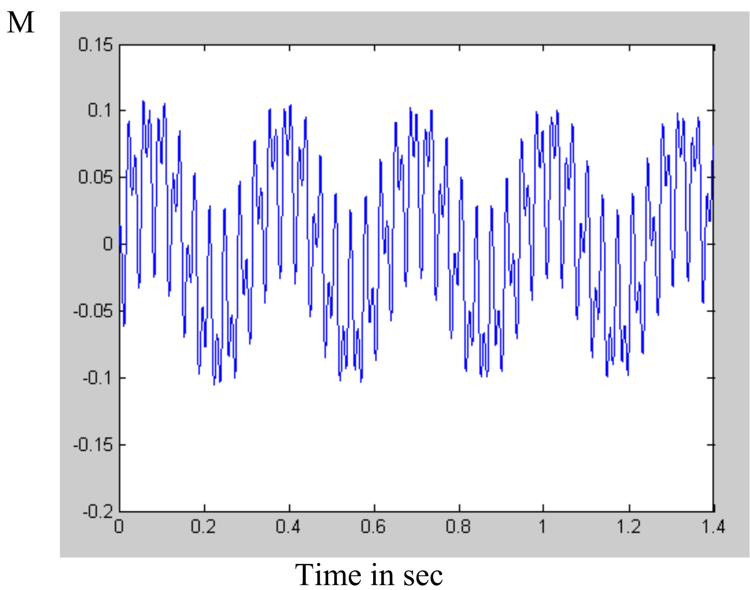

Fig. 3. Time wave form

ANALYSIS TECHNIQUES IN FREQUENCY DOMAIN

The method of frequency analysis is based on the **FOURIER** expansions.

$$f(t) = X_0 + X_1 \sin(\omega t + \phi_1) + X_2 \sin(\omega t + \phi_2) + X_3 \sin(\omega t + \phi_3) + ... + X_n \sin(\omega t + \phi_n) \quad (4)$$

Figure 4 shows the corresponding frequency spectrum of the periodical time form.

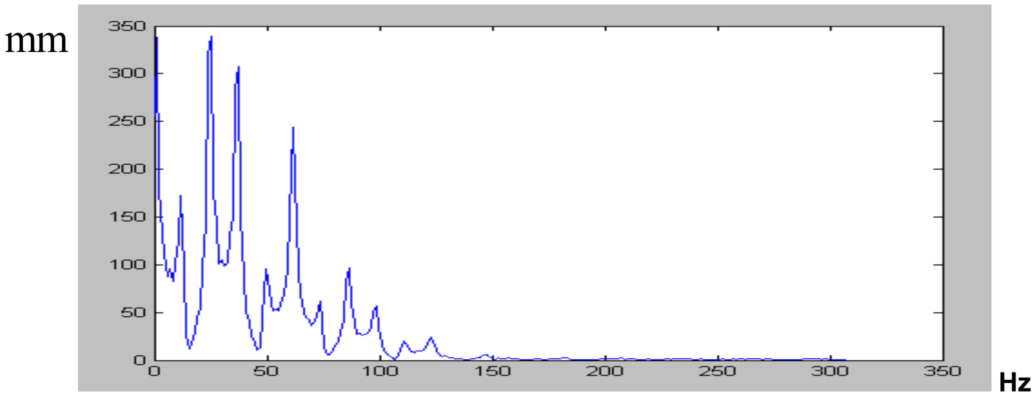

Fig. 4. FFT (amplitude against frequency) for rotating speed w=170 rad/s, crack depth 0.5 of shaft diameter.

RUN-UP AND COAST –DOWN

Run -up and coast-down (fig. 5) is the most way for watching crack phenomena. The coast-down and run-up diagram for crack depth to diameter ratio =0.3 is shown in Fig. 5.

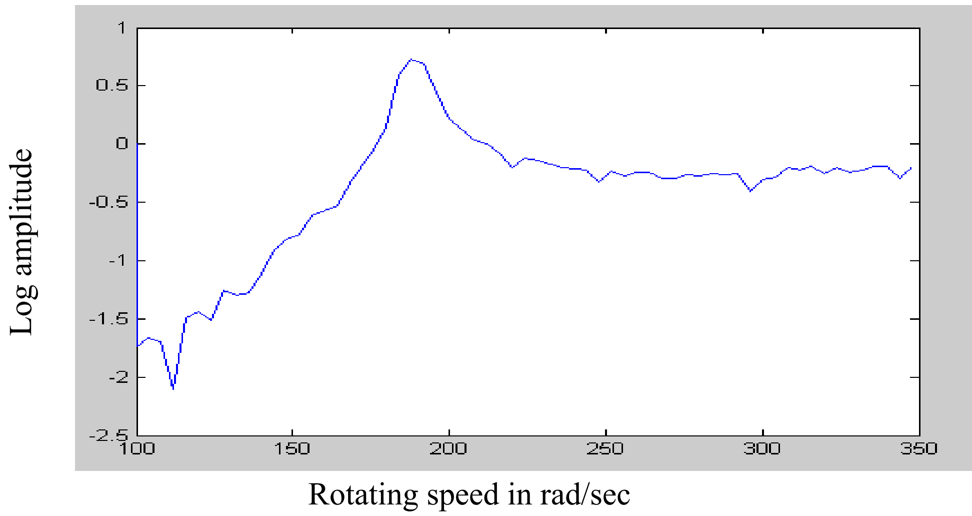

Fig. 5. Run-up and Coast-down.

One of the most predominant features of a cracked rotor is the change in the natural frequency. The vibration of a cracked rotor due to unbalance and gravity consists of many harmonics.

THE VARIATION IN RESONANCE SPEED

As the crack depth increases the shaft stiffness decreases and thus an increase in vibration amplitude, the degree of asymmetry in stiffness becomes larger, resulting in a change in the response spectrum and amplitude.

Table 1. Resonance Speed against Crack to diameter ratio.

Crack depth/diameter	0	0.05	0.1	0.12 5	0.15	0.2	02.5	0.3	0.35	0.4	0.45	0.5
W/w0	1	1	.998	.998	.998	.984	.963	.932	.911	.869	.822	.781

Table 1 show that as crack depth increases, the vibration natural frequency ratio decreases.

THE EFFECT OF THE UNBALANCE ANGLE ON THE CRACK

The coupling between the gravitation and unbalance affect in the vibration of a cracked rotor. This coupling is affected by the crack depth and the position of the unbalance. The effects of the unbalance angle on the natural frequency are summarized in Fig. 6. The results are obtained from a Matlab program run.

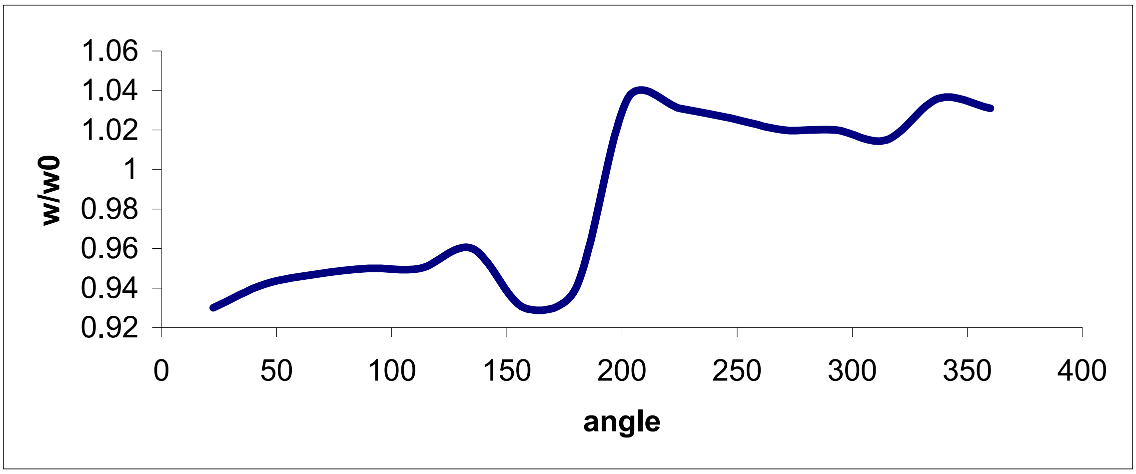

Fig.6. Variation of natural frequency ratio due to changing unbalance angle.

EXPERIMENTAL RESULTS BY TEST RIG FOR CRACKED SHAFT

Stiffness Change Due To Crack

The main controller in crack behavior is stiffness change, so a stiffness measurement was conduct to measure this change. An experimental apparatus is used consist of a disk is mounted at the mid point of a shaft which is supported by two deep groove ball bearings.

The experimental operated in two crack model, the first one by using rigid support and the second one by using flexible support shaft. The shaft is 700 mm length. The crack was made by iron saw; a small reflector tape was attached to the shaft to indicate the phase angle. The diameter of shaft in the case of rigid support is 20mm diameter and in the case of flexible support is 12mm diameter.

Stiffness Change due to Crack for Rigid Support

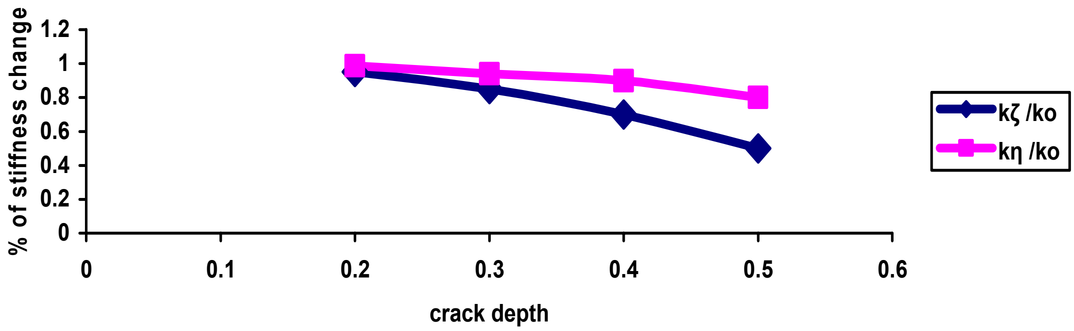

Fig. 7. Total change for stiffness in the two ζ & η due to changing crack depth.

Stiffness Change due to Crack for Flexible Support

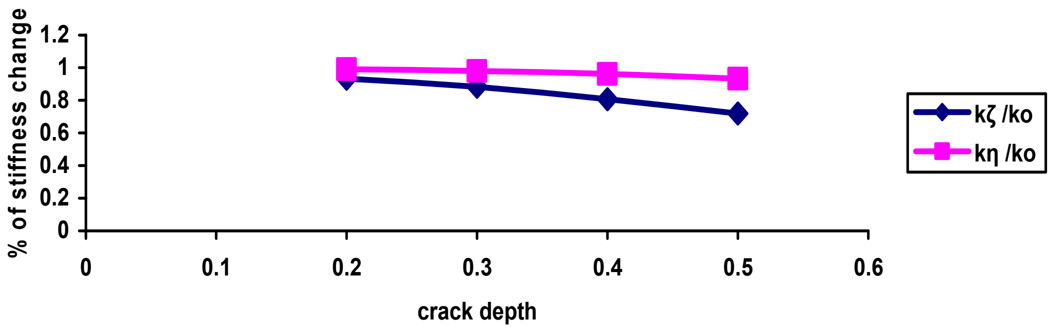

Fig. 8. Percentage of stiffness change due to crack in ζ and η direction against crack depth.

SPECTRUM ANALYSIS FOR EXPERIMENT OF RIGID SUPPORTED MODEL

Cracked Flexible Shaft

Figures 9 and 10 show that, The 1X and 2X appear and the harmonics of the rotating speed and the sub-harmonics. Figure 10 shows the harmonics of the rotating speed and the half of the rotating speed with high intensity.

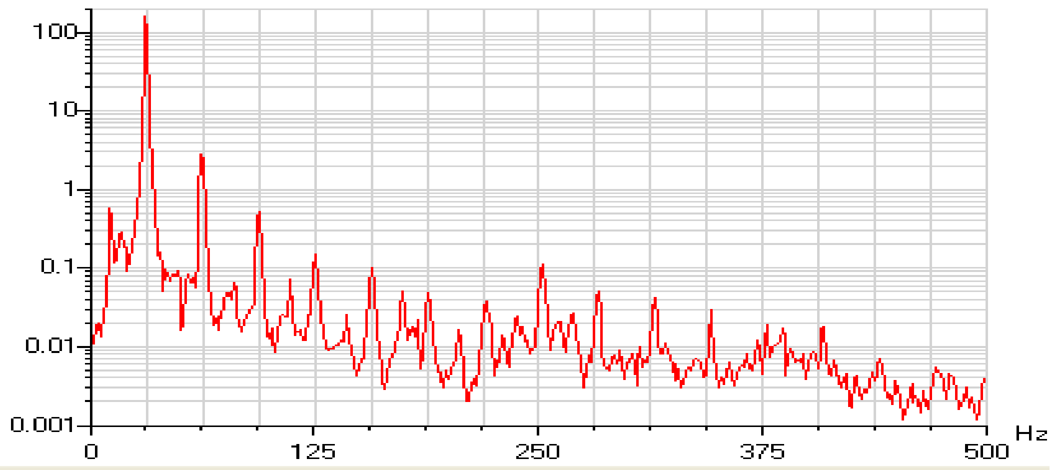

Fig. 9. Flexible support cracked shaft (0.4) at horizontal direction.

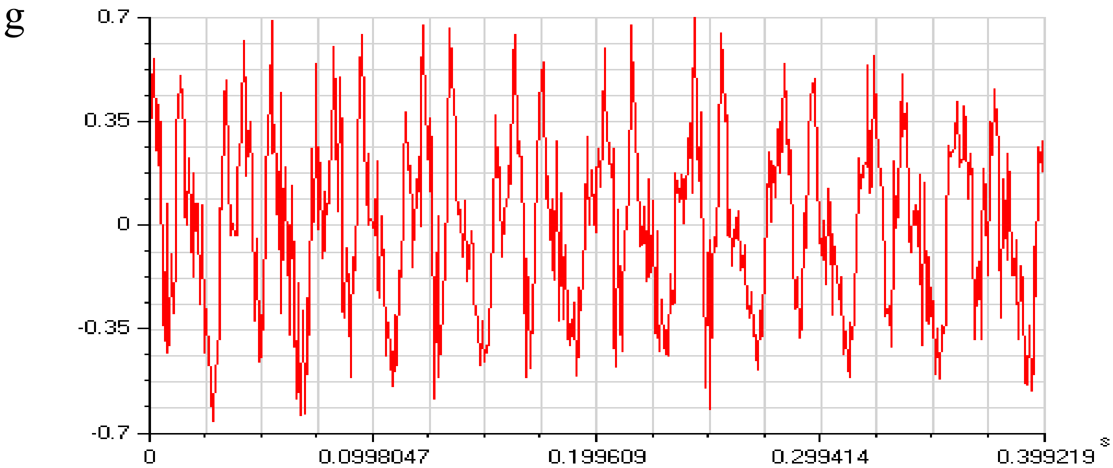

Fig. 10. Rigid support cracked shaft (0.45) time wave form at horizontal direction.

DISCUSSION FOR THEORETICAL AND EXPERIMENTAL RESULTS

It's an important result that the proper diagnostic methods are to be used for diagnosing a shaft crack.

1) Change in the amplitude and phase of the shaft rotating speed (1X) (Fig. 11).
2) The occurrence of twice rotating speed (2X) vibration, which may occur at operating speed (Fig. 12).
3) Appearance of sub-harmonic and harmonics of the rotating speed (Fig. 13).
4) Change in phase signal reading (Fig. 14).
5) Changes in the nature frequency illustrated in run-up and coast-down (Fig. 15).

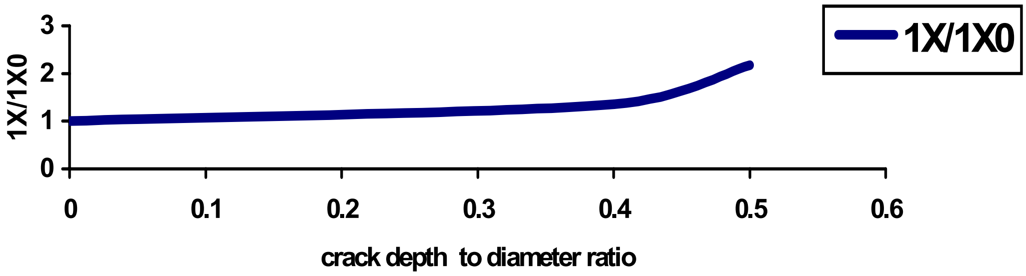

Fig. 11. Results explain how 1X dominates the total value of vibration.

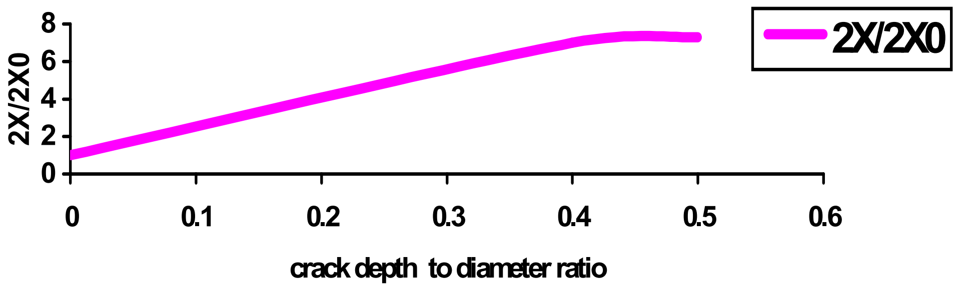

Fig. 12. Results explain the relation between 2X/2X0 and crack depth.

Sub-Harmonics Of Rotating Speed

Sub-harmonics of rotating speed are one feature of a cracked rotor, unlike in the case of a rotor with no crack, the vibration of a cracked rotor due to Sub-harmonics related to the non-linearity behavior for the cracked. The sub-harmonics can be separated into two stages for flexible support. In the first stage, sub-harmonics increases for increasing values of the crack depth. In the second stage, Sub-harmonics decreases in the case of increasing the shaft crack i.e. 1X, 2X, 3X increasing. Also for rigid supported shaft the crack effect changes the values of the sub-harmonics.

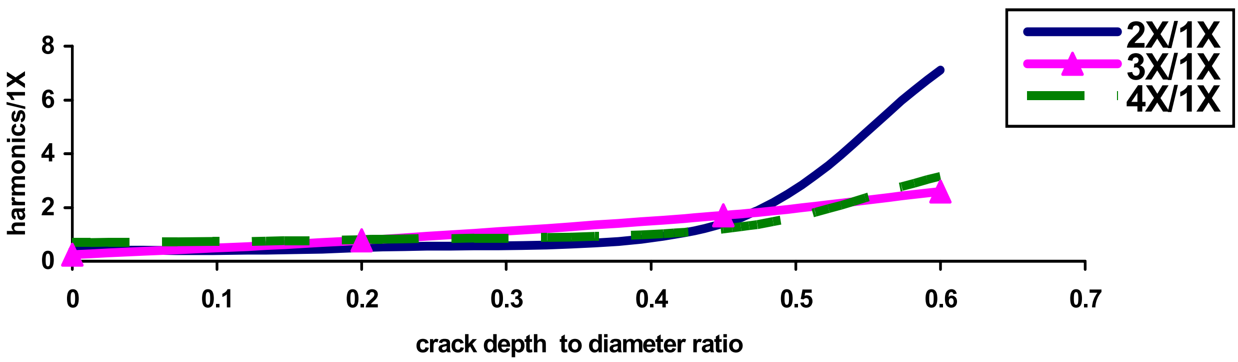

Fig.13. Harmonics/1X against crack depth.

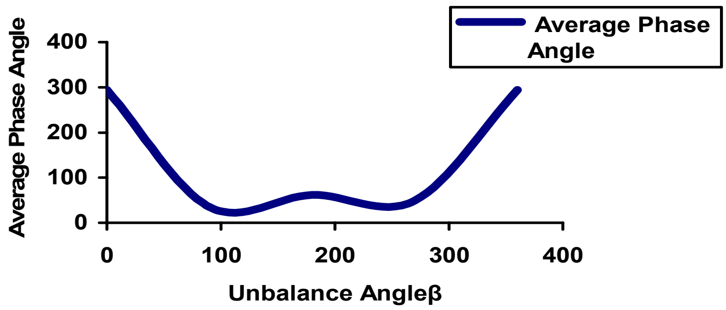

Fig. 14. The influence of the unbalance angle on the phase angle.

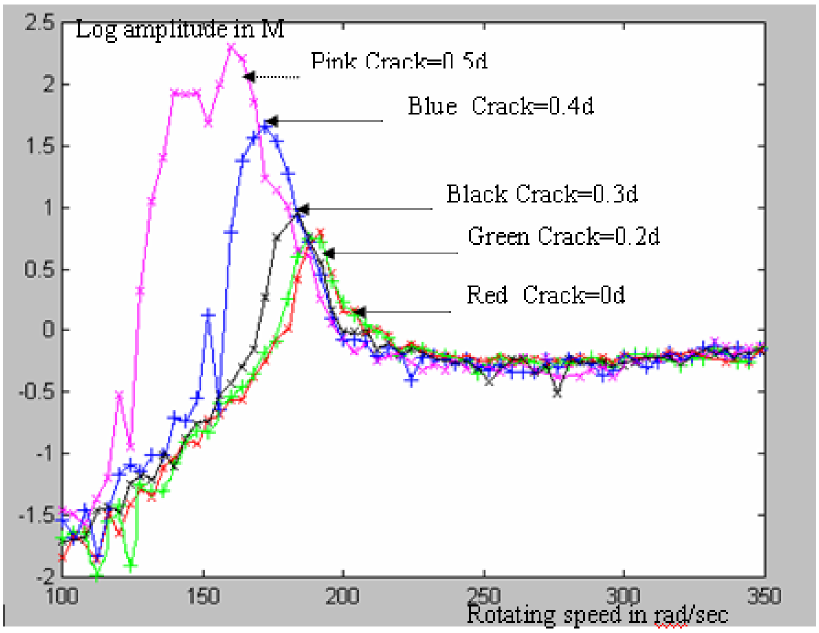

Fig. 15. Run-up and coast down for various crack depth.

CONCLUSIONS

Detecting cracks by vibration analysis is the only method for crack detection monitoring inline machines.

- The theoretical and experimental results show the effect of changing the phase angle and how it can be used for decreasing the crack effect .
- Sub-harmonics is one of the crack symptoms, the experimental and theoretical results show the appearing of the sub-harmonics at about 0.25 crack depth for diameter.
- The 2X is the best way for predicting the crack in vertical and horizontal direction and the rapid increasing is the best way for predicting the criticality with the other symptoms.
- Existing of 4X is one of the cracked shaft characteristics and its monitoring with the other harmonics is essential for knowing the crack depth.

REFERENCES

[1] Rao, J. S., Rotor Dynamics, Wiley, New York. (1991).

[2] Imam, I., and Azzaro, S. H., ‘‘Development of an On-Line Rotor Crack Detection and Monitoring System,’’ ASME J. Vibration and Acoustics, 111, pp. 241–250 (1989).

[3] Herbert, R. G., ‘‘Turbine-Alternator Run-Down Vibration Analysis: Automated ’’Crack Detection,’’ in 11th Biennial Conference on Mechanical Vibration and Noise, Vol. 2, pp. 631–636. (1987).

[4] Grabowski, B., ‘‘The Vibrational Behavior of a Turbine Rotor Containing a Transverse Crack,’’ ASME J. Mech. 121, pp. 140–146. (1980).

[5] Nelson, H. D., and Nataraj, C., ‘‘The Dynamics of a Rotor System With a Cracked Shaft,’’ ASME J. Vibration and Acoustics, 108, pp. 189–196. (1986).

[6] Gasch, R., ‘‘A Survey of the Dynamic Behavior of a Simple Rotating Shaft With a Transverse Crack,’’ J. Sound Vib., 160, pp. 313–332. (1993).

[7] Jun, O.S., Eun, H.J., Earmme, Y.Y., and Lee C.W.,”Modling And Vibration Analysis Of A Simple Rotor With A Breathing Crack,” Journal Of Sound And Vibration , Vol.155(2), pp.273-290. (1992).

[8] Green, I., and Casey, C.,” Crack Detection in a Rotor Dynamic System by Vibration Monitoring”, ASME J. Of Engineering for Gas turbines And Power , Vol.127. (2005)