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CAUCHY PROBLEM FOR SHAPED CHARGE DESIGN OPTIMIZATION

H. I. Hristov*

ABSTRACT

The main disadvantage of the cumulative jet is the velocity gradient on the jet length. Due to the velocity gradient, the jet has effectiveness only in a strong focus distance from the target. One possibility to resolve this problem is by the Cauchy problem.

The Cauchy problem for a shaped charge design optimization is presented in the report. The optimization includes the profile determination of main surfaces of the shaped charge elements, which influences into the forming of the cumulative jet under condition of the no-gradient velocity of the jet length.

The solution of the task is fulfilled within the hypothesis of the radial-flat scheme of the hydrodynamic model of Orlenko-Staniukovitch. On this basis, the functions of the shaped charge main elements profiles (shell, liner and explosive charge between them) enter into the first order ordinary differential equation.

In the end, the Cauchy problem for the first order ordinary differential equation is formulated considering the unknown function of the cumulative charge geometry on condition if other function is known. The solutions allow the cumulative liner profile design in case of a known shell profile, or the shaped charge shell profile design in case of a known cumulative liner profile. In both cases the designed profiles ensure no-gradient velocity jet forming and its saving in a distance different from the focus. The experiments with the designed charges were verified into an armor in a distance between shaped charges and armor up to 7 calibers.

KEY WORDS

Shaped Charge, Design, and Cauchy problem.

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* Prof., Head of Armament and Ammunition Dept., Defense Institute, Sofia, Bulgaria.

INRODUCTION

The development of scientific programs dealing with studies on effectiveness rising are connected with financial investments in experimental researches on one hand, and, on other hand, with math's modeling of the process of cumulative jet formation. The shaped charge consists of main elements like shell, explosive charge and liner which their longitudinal profiles could be written by some continuous functions. If these functions are elements from an equation, then the optimization tasks could be formulated, for example the task with a variation calculation [1, 2] or a Cauchy problem.

The main disadvantage of the cumulative jet is the velocity gradient on the jet length. Due to the velocity gradient, the jet has effectiveness only in a strong focus distance from the target surface. If the velocity gradient is zero then the jet is able to work in large ranges from the target.

Possibility to resolve this problem is by the Cauchy problem. The optimization may include the profiles determination of the main surfaces of the shaped charge elements, which have influence into jet forming. We consider this task within the hypothesis of the radial-flat scheme from the hydrodynamic model of Orlenko-Staniukovitch [3, 4]. On this basis, the functions of the shaped charge main elements profiles (shell, liner and explosive charge between them) enter into the differential equation. This equation is unitary to formulate a Cauchy problem and to solve it about one function when other are defined, in several cases under additional conditions [5].

CAUCHY PROBLEM

It is known [5], that the characteristics of the shaped jet, its length in particular, depend on the geometry of the shaped charge. The influence of this factor is analyzed later. The solution of the task is fulfilled within the hypothesis of the falt-radial scheme of the hydrodynamic model of Orlenko-Staniukovitch - Fig.1. In the Cartesian co-ordinate system *z0y* there are determined the following equations of the curve of the surfaces of the basic details of the shaped charge with height H: $y_1=F(z)$, $y_2=\Phi(z)$, $y_3=\varphi(z)$ and $y_4=f(z)$ which describe the external and internal surface of the body, and the external and internal surface of the lining of the shaped charge, respectively. The following limitations are imposed on the functions listed here above - they are continuous and have continuous first derivatives. Besides, the following conditions are always fulfilled:

$$F(z) \geq \Phi(z) \geq \varphi(z) \geq f(z);\ h_2 > 0;\ z \in [0;\ h_2];\ y \geq 0.$$

It is assumed that the explosive charge is isotropic and homogeneous. The front of the detonation wave moves from left to right and it is flat and perpendicular to axis *Oz*. In time $t = 0$, the detonation wave reaches top of a cumulative liner and at $t > 0$, the wave starts its movement along a liner with detonation velocity *D*. The schema of a cumulative liner collapse is shown on Fig.2.

The elementary part of a liner with coordinate *z* and length *dz* is selected. In time *t = z/D,* the point *A* of this part starts moving to the *Oz* axis with velocity $W_0(z)$. During time *dt = dz/D* the detonation wave reaches a point *B* and the point *A* is displaced in a direction to axis *Oz* on distance *dy*

$$dy = W_0(z)dt = W_0(z)\frac{dz}{D}. \tag{1}$$

Then the point *A* will be from the axis *Oz* at a distance

$$R = f(z) - dy = f(z) - W_0(z)\frac{dz}{D}. \tag{2}$$

Let the collapse velocity of the liner *Wo(z)* do not depend on time and it is a function of the *z* coordinate only. Then, the point *A* covers the distance *R* after time *T* which is represented by:

$$T = \frac{R}{W_0(z)} = \frac{1}{W_0(z)}\left[f(z) - W_0(z)\frac{dz}{D}\right]. \tag{3}$$

For the same time *T*, the point *B* displaces to the axis *Oz* with velocity $W_0(z) + dW_0(z)$ with distance L_1 where:

$$L_1 = T[W_0(z) + dW_0(z)] = \left[f(z) - W_0(z)\frac{dz}{D}\right]\frac{W_0(z) + dW_0(z)}{W_0(z)}. \tag{4}$$

Then the point *B* stays from the axis *Oz* with space L_2

$$L_2 = [f(z) + df(z)] - L_1. \tag{5}$$

The cumulative liner part collapse angle from Fig.2 is determined by the formula

$$tg\alpha(z) = \frac{L_2}{dz} = \frac{f(z) + df(z) - \frac{1}{W_0(z)}\left[f(z) - W_0(z)\frac{dz}{D}\right][W_0(z) - dW_0(z)]}{dz}. \tag{6}$$

or

$$tg\alpha(z) = \frac{df(z)}{dz} - \frac{f(z)}{W_0(z)}\frac{dW_0(z)}{dz} + \frac{W_0(z)}{D}.$$

The collapse velocity $W_0(z)$ of the elementary part of the liner has related to the cumulative jet velocity $W_1(z)$ by the kinematics proportion [2, 5]**:**

$$W_0(z) = W_1(z)tg\frac{\alpha(z)}{2} = \frac{k_i D}{2}\sqrt{\beta(z)[2 + \beta(z)]^{-1}}. \tag{7}$$

Then

$$tg\alpha(z)=\frac{df(z)}{dz}-\frac{f(z)}{W_1(z)tg\frac{\alpha(z)}{2}}\left[\frac{dW_1(z)}{dz}tg\frac{\alpha(z)}{2}+W_1(z)\frac{d(tg\frac{\alpha(z)}{2})}{dz}\right]+\frac{W_1(z)tg\frac{\alpha(z)}{2}}{D}. \quad (8)$$

In accordance with ratiocination above we place a condition $W_1(z) = W_1 = const.$ and the equation (8) transforms in the following equation:

$$tg\alpha(z)=\frac{df(z)}{dz}-f(z)\frac{1}{\sin\alpha(z)}\frac{d\alpha(z)}{dz}+\frac{W_1}{D}tg\frac{\alpha(z)}{2}. \quad (9)$$

where

$$tg\alpha(z)=\frac{2.\in(z)}{1-\in^2(z)}; \quad (10)$$

$$\sin\alpha(z)=\frac{2\in(z)}{1+\in^2(z)}; \quad (11)$$

$$tg\frac{\alpha(z)}{2}=\in(z); \quad (12)$$

$$\frac{d\alpha(z)}{dz}=2\frac{1}{1+\in^2(z)}\frac{d\in(z)}{dz}, \quad (13)$$

$$\in(z)=\frac{D}{2W_1}\sqrt{\beta(z)[2+\beta(z)]^{-1}}. \quad (14)$$

From equations (10), (11), (12), (13) and (14), equation (9) can be rewritten as:

$$\frac{2\in(z)}{1-\in^2(z)}=f'(z)-\frac{f(z)D}{2\in(z)W_1}\left\{\sqrt{\beta(z)[2+\gamma(z)]^3}\right\}^{-1}\beta'(z)+\frac{W_1\in(z)}{D}. \quad (15)$$

The densities of the materials of the main charge elements are constant values. Equation (15) can be written as following equation:

$$\frac{df(z)}{dz}-E(z)A(z)a(z)\frac{dF(z)}{dz}-E(z)[A(z)d(z)+C(z)c(z)]\frac{d\Phi(z)}{dz}+$$

$$+E(z)[B(z)b(z)+C(z)g(z)]\frac{d\varphi(z)}{dz}+E(z)B(z)e(z)\frac{df(z)}{dz}+\frac{W_1\in(z)}{D}-\frac{2\in(z)}{1-\in^2(z)}=0. \quad (16)$$

where $$E(z)=\frac{f(z)D}{2\in(z)W_i}\left\{\sqrt{\beta(z)[2+\beta(z)]^3}\right\}^{-1}.$$

And $A(z)$, $B(z)$, $C(z)$, $a(z)$, $b(z)$, $c(z)$, $d(z)$, $e(z)$, $g(z)$ are functions which include constant values of the mechanical properties of the main charge elements and function's values $F(z) \geq \Phi(z) \geq \varphi(z) \geq f(z)$ in a cross-section z. The equation (16) is the

first order differential equation. If one of the functions *F(z), Φ(z), φ(z)* or *f(z)* is unknown and the other three functions are given, then this equation is a basis for the Cauchy problem formulation for determination the geometrical profiles of the shaped charge, which ensures no-gradient forming of a cumulative jet. In the following, two examples for no-gradient cumulative jet are presented; first is concerned with liner profile design whereas the second is concerned with technological shaped charge design.

EXAMPLES

Liner Profile Design with No-Gradient Cumulative Jet

Let study one of the possible versions of the problem (16). It is introduced a new designation for an equation of thickness of a cumulative liner $\delta(z) = \varphi(z) - f(z) = const$. The problem is splved by accepting the assumptions and limitations above.

$$-\frac{2\in(z)}{1-\in^2(z)}+\frac{df(z)}{dz}-\frac{f(z).D}{2\in(z)W_i}\left\{\sqrt{\beta(z)[2+\beta(z)]^3}\right\}^{-1}$$

$$\mathrm{X}\quad\left\{A(z)a(z)\frac{dF(z)}{dz}+[A(z)d(z)+C(z)c(z)]\frac{d\Phi(z)}{dz}+\right.$$

$$\left.+[B(z).e(z)+C(z).g(z)].\frac{d\delta(z)}{dz}+[B(z).b(z)+C(z).g(z)].\frac{df(z)}{dz}\right\}+\frac{W_i\in(z)}{D}=0\,. \qquad (17)$$

or

$$\frac{df(z)}{dz}=\left\{\frac{2\in(z)}{1-\in^2(z)}-\frac{W_i\in(z)}{D}+E(z)\left\{A(z)a(z)\frac{dF(z)}{dz}+[A(z)d(z)+C(z)c(z)]\frac{d\Phi(z)}{dz}+\right.\right.$$

$$\left.\left.+[B(z)e(z)+C(z)g(z)]\frac{d\delta(z)}{dz}\right\}\right\}Q(z)\,.$$

where $$Q(z)=\{1-E(z)[B(z)b(z)+C(z)g(z)]\}^{-1}\,.$$

In result here the Cauchy problem is formulated for a first order differential equation concerning the unknown function *f(z)*. The numerical solution of a problem allows designing a profile of a liner which influences to no-gradient forming of a jet.

Figure 3 shows a solution of the current example for no-gradient jet with several liner thicknesses $\delta(z) = \delta = const$: δ_1 = 1 mm; δ_2 = 2 mm; δ_3 = 3 mm; δ_4 = 4 mm, in case the shaped charge is with constant thickness of the cylindrical shell.

Figure 4 shows a solution of the current example for no-gradient jet with velocities of the jet: $W_{1,1}$ = 5000 m/s; $W_{1,2}$ = 7000 m/s; $W_{1,3}$ = 9000 m/s; $W_{1,4}$ = 11000 m/s; $W_{1,5}$ = 13000 m/s for a shaped charge with constant thickness of the cylindrical shell.

But, the liner which was designed is not technological. It is expensive to manufacture the liner with difficult shape. In this case, the task for liner surface determination has to be prolonged. Therefore we have to put in the equation (16) under a condition for a conical liner *df = const* and *dφ = const*.

Technological Shaped Charge Design with No-Gradient Cumulative Jet

Let study another example. In this case we require technological and conical type of the liner and the cylindrical type of the shell. In this case *df = const*, *dφ = const* and *dF(z) = dΦ(z) = 0*. Then the equation (16) could be rewritten:

$$E(z)[B(z)b(z)+C(z)g(z)]\frac{d\varphi(z)}{dz}+[1+E(z)B(z)e(z)]\frac{df(z)}{dz}+\frac{W_1\in(z)}{D}-\frac{2\in(z)}{1-\in^2(z)}=0\,. \quad (18)$$

or

$$\frac{d\delta(z)}{dx}=\left\{\frac{2.\varepsilon(z)}{1-\varepsilon^2(z)}-\frac{W_1(z).\varepsilon(z)}{D}+\frac{df}{dz}+[B(z)b(z)+C(z)g(z)]\frac{df}{dz}\right\}.Q(z)\,.$$

In the result, the Cauchy problem is formulated for a first order differential equation concerning an unknown function *δ(z)*. The numerical solution of the problem predicts the design a conical profile of a liner for no-gradient velocity forming of the jet and the shell is cylindrical. Figure 5 shows the solution for the current example.

The experiments with the designed charges were verified into a thick armor in a distance between shaped charges and armor up to 7 calibers.

CONCLUSION

The report shows the Cauchy problem like one possibility to raise effectiveness of the shaped charges by profiles design of its main elements. The equation (16) is common and allows designing the liners profile as well as the shells profile under the condition for no-gradient velocity jet.

This is one way to eliminate the main disadvantage of the cumulative jet - its velocity gradient on the jet length which has influence on the jet destroying. The elimination of this disadvantage will allow designing and manufacturing the shaped charges with no-gradient velocity in time of jet forming or we come to theme for designing of long-focused shaped charges.

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INFORMATION ABOUT AUTHOR:

Prof., DSc, Hristo Ivanov Hristov, Dipl. Eng.,
Head of “Armament and Ammunition” Dpt., Defense Institute, Sofia, Bulgaria,
Dipl. Eng. in Ammunition from Tula State University, Tula, Russia, 1984,
PhD in Ammunition from Tula State University, Tula, Russia, 1993,
DSc in Ammunition from “G.S.Rakovski” Defense and Staff College, Sofia, Bulgaria, 2000.

Figures

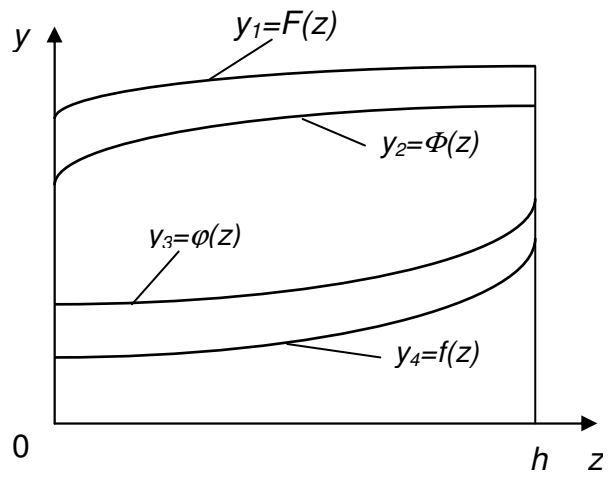

Fig.1. Shaped charge schema.

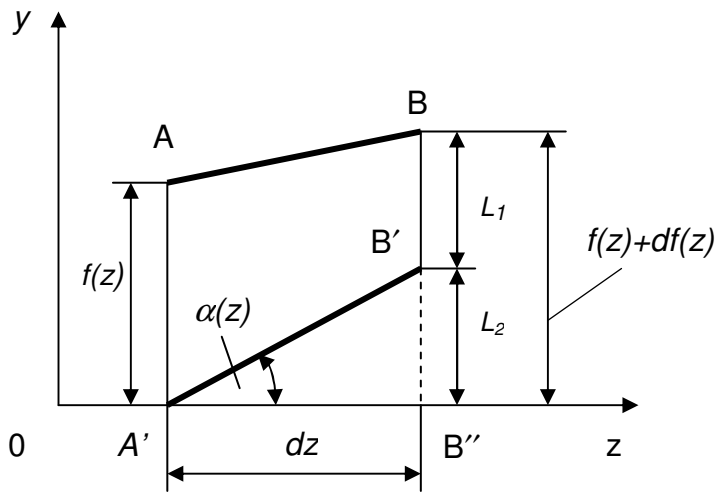

Fig.2. Collapse schema for cumulative jet forming.

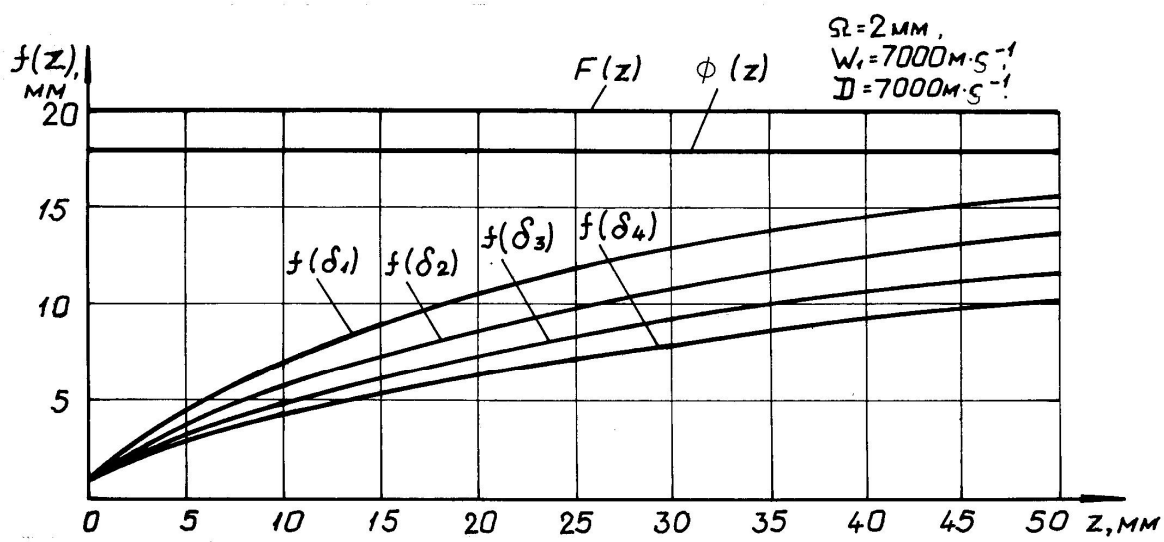

Fig.3. Several solutions for a liner which forming no-gradient jet in case of a cylindrical shell, where $\Omega = F(z) - \Phi(z)$ and different thickness of the liner are begun to give $\delta(z)=\delta=const$: δ_1=1 mm; δ_2 =2 mm; δ_3 = 3 mm; δ_4 = 4 mm.

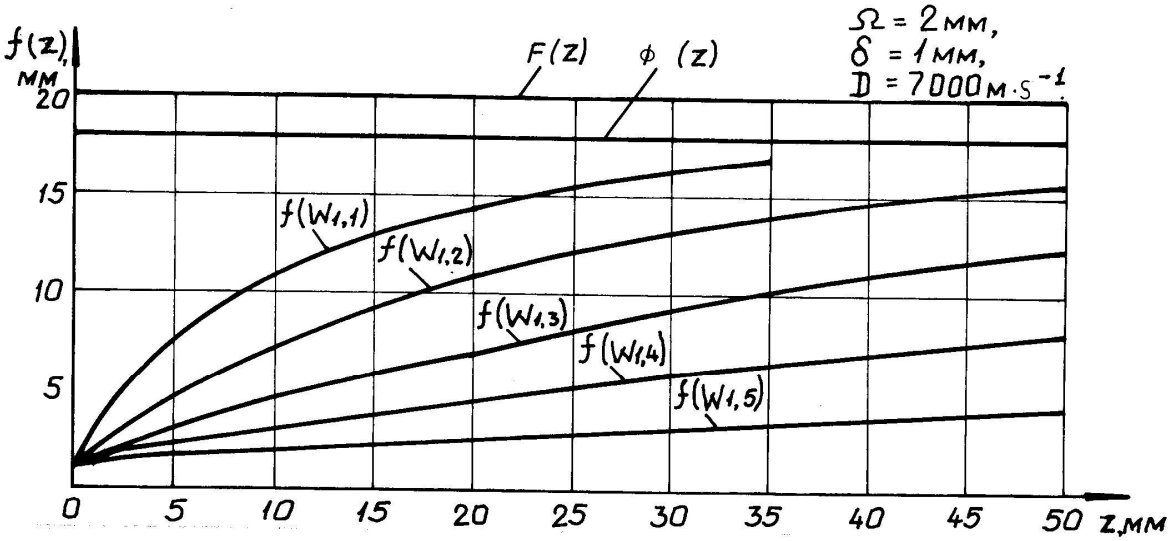

Fig.4. Several solutions for a liner which forming no-gradient jet in case of a cylindrical shell, where $\Omega = F(z) - \Phi(z)$ and different velocities W_1 for a jet are begun to give.

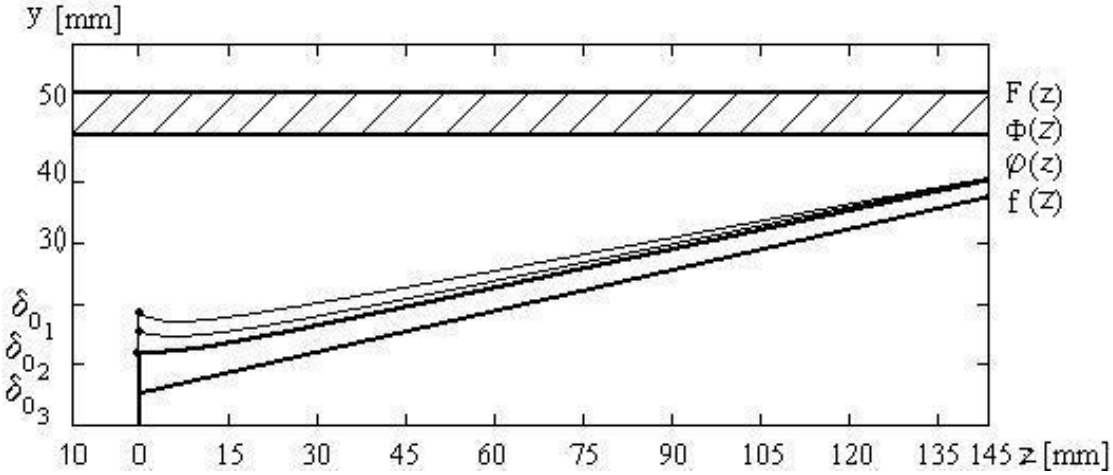

Fig.5. Technological shaped charge with a conical liner and a cylindrical shell which both influence to forming a no-gradient jet.